Analysis of Multi-Source Multi-Hop Cooperative Networks Employing Network Coding

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***Abstract*—In this paper, the performance of a multi-hop network is investigated in which M sources have independent information to be transmitted to a far off common destination. Linear network coding technique is used by the intermediate relays to transmit the combined information of M sources. Channel model includes Rayleigh fading and path loss. The multi-hop transmission process is modeled by a quasi-stationary Markov process, whereas the relay nodes use decode and forward (DF) mechanism at each hop. By finding the outage probability of each node and studying the properties of Markov process, the network coverage is analyzed for a given signal-to-noise ratio (SNR) margin.**

I. Introduction

Cooperative transmission (CT) has become an area of prime interest in wireless systems which are affected by multi path fading. By having spatially separated radios transmitting the same message signal through uncorrelated fading channels, CT reaps the advantages of distributed multi-input multiple-output (MIMO) systems such as range extension [2] and energy-efficiency [3]. Opportunistic large array (OLA) network is a form of physical layer CT technique where groups of nodes forward the data to other groups of nodes without any coordination with one another [1]. This is a decentralized scheme where the source and destination are far apart and multi-hop mechanism is employed to reach far off distances.

The modeling of general OLA networks has always been critical owing to the random node locations and random boundaries of each hop. Initial studies used the continuum assumption implying an infinite node density per unit area with fixed transmit power [1]. A finite node density analysis of linear networks was studied in [4] and many variants on 2D random networks and effects of various channel impairments were carried out [5-6]. All these works study a single source with one message traversing the multi-hop network. Multi-packet insertion by the same single source was studied in [7-8] for continuum networks. A similar concept of nodes broadcasting/unicasting a message was studied in [9] and optimization on the distance has been carried out to place nodes for better range extension.

In above-mentioned works, a single source excites the whole network. However, due to distributed nature of the nodes (and assuming no MAC protocols) there is always a possibility that simultaneous transmission of more than one sources appears. In this case, a relay node at some arbitrary hop may receive two (or more) distinct signals, which may cause interference. To cope with this, network coding (NC) has been suggested in literature to combine the information. In this paper, we use linear NC at a relay node to combine informations of different sources and investigate the multi-hop propagation of these codewords in opportunistic networks.

In earlier literature, network coding [10] was proposed for networks where wireless links are assumed to be noiseless. XOR is the simple example of binary network coding followed by non-binary and linear NC techniques. These techniques provide better diversity than conventional binary NC. As in these techniques, we can recover the information with the help of codewords only, no direct transmission or information is required. To achieve diversity in NC, many code design techniques are proposed e.g., LDPC-based coding [11], and Reed-Solomon codes [12]. Random linear network coding within a defined field is proposed in [13]. In [12], the destination is not far away and directly receive sources information and also only two-hop network is studied, which can be useful for cellular systems. The outage probability and diversity is calculated at the destination node in the presence of NC in [12].

In this paper, we consider a variant of OLA network where M sources transmit their individual information to a common destination via cooperative multi-hop mechanism. We study two variants of opportunistic networks; a distributed topology where the nodes are distributed as a strip network and a co-located topology where a cluster of co-located nodes transmit the message to another cluster of co-located nodes. In both cases, NC is used at each node of each hop and the process of multi-hop transmission is characterized by a Markov Process. The outage probability for each node of the network is derived and then used in the formulation of Markov process, which together with Perron-Frobenius theorem is used to calculate the coverage of the network for a given signal-to-noise ratio (SNR) margin.

The rest of the paper is organized as follows. In Section II, the system parameters are described along with network coding scheme. The decoding of codewords, Markov modeling and formation of transition matrix are explained in Section III. In Section IV, the analytical and simulation results are

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presented and Section V concludes the paper.

II. System Description

Consider a network topology in which multiple sources M, where $M \geq 2$, are transmitting their information to a far off destination D. The transmission traverses a multi-hop network where each hop (or level) contains N relay nodes, which are at a distance d from the adjacent cluster of relay nodes as shown in Fig. 1. To keep a deterministic path loss, the distance between the adjacent nodes of each cluster is kept at w in this paper. This assumption, however, can be relaxed by having random node locations and averaging the results over Monte-Carlo trials or by assuming a Poisson point process (PPP) model, which could be a future direction to this work.

We assume that all the nodes including the sources transmit with a power P_t on orthogonal channels [15]. Hence there is no interference while receiving the information of multiple sources at a relay node. A relay node in the first cluster decodes the information of all the sources independently. The decode-and-forward (DF) technique is used by the relay nodes to broadcast the decoded information to the next cluster of nodes. Only those nodes in a level participate in transmission that decoded the information of all sources. The DF nodes of the first hop perform suitable processing at the physical layer to combine the information of all sources to make codewords and transmit the network coded data to the next cluster of nodes. The nodes of the second hop and onwards decode the codewords to extract the information of the sources. For example, in Fig. 1, the nodes represented by filled circle are DF nodes, whereas the nodes that decode either L sources information, where $L < M$, or do not decode any information are represented as hollow circles.

A DF node at a hop will relay the network codeword, $C_{l,m}^{(n)}$, with power P_t to the next cluster of nodes. The $C_{l,m}^{(n)}$ is the codeword from node l at level $(n-1)$ to a node m at level n. The received signal at node m at level n is given as

$$Y_{l,m}^{(n)} = h_{l,m} C_{l,m}^{(n-1)} + w_{l,m}, \tag{1}$$

where l,m denote the transmitting and receiving nodes, respectively $l, m \in \{1, 2, 3, ...N\}$, while the superscript denotes the receiving level. Here $w_{l,m}$ is the additive white Gaussian noise with double-sided power spectral density $N_o/2$ and $h_{l,m}$ takes into account the flat fading channel and path loss, i.e., $h_{l,m} = \frac{g_{l,m}}{d_{l,m}{}^{\beta}}$, where $\mid g_{l,m} \mid \sim$ Rayleigh distributed and $d_{l,m}$ is the Euclidean distance between nodes l and m. The β is the path loss exponent. The receiving nodes at the next hop can be DF if and only if the number of decoded codewords are greater than or equal to the number of transmitting sources. This process continues until the number of nodes to relay information is lesser than the number of sources. Therefore, to propagate the information of all sources we need at least M relays at each hop i.e., $N \geq M$.

In this study, we assume that a node is DF if it decodes the information of all sources. A variant of this process where partial information is decoded and forwarded is not taken into account.

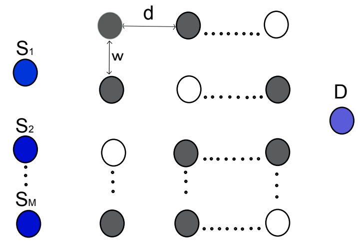

Fig. 1: Realization of deterministic network topology for M Sources

A. Network Coding Scheme

We use the network coding coefficient matrix [10] to design the network codes (linear finite field codes) for our system. The network coding coefficient matrix T is a $N \times M$ matrix given as

$$T = \begin{pmatrix} a_{1,1} & a_{1,2} & \cdots & a_{1,M} \\ a_{2,1} & a_{2,2} & \cdots & a_{2,M} \\ \vdots & \vdots & \ddots & \vdots \\ a_{N,1} & a_{N,2} & \cdots & a_{N,M} \end{pmatrix},$$

where each row makes a codeword for one relay, which is a linear combination of source informations $I_i \; \forall i \in \{1, ..., M\}$. The coefficient $a_{l,m}$ is selected within a defined finite field (Galois field). To acheive diversity in this system, the selected codewords should be linearly independent from one another. Hence to make linearly independent codewords, the network coding coefficient matrix is maximum distance separable (MDS). The Reed-Solomon (RS) codes are commonly used as a variant of MDS codes [12], which are used in this study.

III. Decoding of codewords

Let M sources transmit their respective information I_1,I_2,...,I_M, to the first hop nodes. The outage probability of a node m at the first hop for any of the M sources is given as

$$P_{o_{l,m}}^{(1)} = \mathbb{P}\left\{P_{r_{l,m}}^{(1)} < \tau\right\}, l \in \{S_1, S_2, ..., S_M\}, \tag{2}$$

where $P_{r_{l,m}}^{(1)}$ is the received power from node l to node m at the first hop and τ is the modulation dependent threshold. Assuming $\mid h_{l,m} \mid^2 \sim$ Exp $(d_{l,m}^{\beta})$, (2) can be written as

$$P_{o_{l,m}}^{(1)} = 1 - \exp(-d_{l,m}^{\beta} \tau). \tag{3}$$

The nodes at level 1 can be considered as DF nodes if they decode all M sources information I_1,I_2,...,I_M, i.e., $P_{r_{l,m}}^{(1)} \geq \tau$, where $l \in \{S_1, S_2, ..., S_M\}$ and m denotes the receiving node index at level 1. In other words,

$$\mathbb{P}\{node\ m\ is\ DF\ at\ level\ 1\} = \prod_{l=1}^{M} \mathbb{P}\{P_{r_{l,m}}^{(1)} \geq \tau\},$$

These DF nodes form network codes using the rows of matrix T. These codewords are linearly independent from one another so that only M relayed signals are sufficient to recover M sources information. For example, in case of two sources, I_1+I_2 and $2I_1+3I_2$ are linearly independent codewords, where coefficients taken from matrix $T=[\begin{smallmatrix}1&1\\2&3\end{smallmatrix}]$; one can recover both sources information using a simple Gaussian elimination method. This condition can also be checked by evaluting the rank of matrix T, which should be equal or greater than the number of sources to perfectly decode the signals.

Denoting $\xi^{(n-1)}$ to be a set of indices of DF nodes at previous hop or level $n-1$, the probability of outage at current hop or level n ($n \geq 2$) for a node m can be calculated as

$$\begin{aligned} P^{(n)}_{o_{(m|\xi^{(n-1)})}} &= \prod_{l\in\xi^{(n-1)}} P^{(n)}_{o_{l,m}} \\ &+ \sum_{z=1}^{M-1}\left(\sum_{v=1}^{\chi}\left(\prod_{a=1}^{z} P^{(n)}_{o_{\varphi,m}} \prod_{b=z}^{M-z}(1-P^{(n)}_{o_{\vartheta,m}})\right)\right), \end{aligned} \tag{4}$$

where $\chi \doteq |\ \xi^{(n-1)}\ |$ is the cardinality of set $\xi^{(n-1)}$. For successful transmission $|\ \xi^{(n-1)}\ | \geq M$, $\varphi \doteq \left(\xi^{(n-1)}(v+a-1)\right)_\chi$ and $\vartheta \doteq \left(\xi^{(n-1)}(b+v)\right)_\chi$ denotes the transmitting node from level $(n-1)$ and $(.)_\chi$ shows modulo-χ operator. The $P^{(n)}_{o_{l,m}}$ is the outage probability of the codeword from node l at level $(n-1)$ to node m at level n given in (3). These probabilities are different because of different path loss and will be same if we consider all nodes at each hop to be co-located, which is described in next section.

A. Markov Chain Modeling

At any time instant, a relay node at level n will transmit the codeword to the next hop with a condition that the node had already decoded the M sources information. On the other hand, it will not participate in further transmission, if it had not decoded at least M codewords. The state of the system at the nth level can be represented as $\mathbb{X}(n) = [\mathbb{I}_1(n), \mathbb{I}_2(n), \mathbb{I}_3(n), ..., \mathbb{I}_N(n)]$, where $\mathbb{I}_m(n)$ is a binary indicator function for the mth node at level n and can be represented as

$$\mathbb{I}_m(n) = \begin{cases} 0 & \textit{node m does not decode data} \\ 1 & \textit{node m decodes data} \end{cases} \tag{5}$$

Therefore, each node of a particular hop can be represented by either 0 or 1 depending whether it has successfully decoded the M codewords or not. For N nodes in a cluster, there are 2^N states within the whole system. We use discrete-time finite-state Markov chain to model the system. In this case, there is always a non-zero probability that the Markov chain can go into an absorbing state and the process of transmissions is stopped. This happens when all the nodes of a hop cannot decode the information. Hence we define two matrices to represent the Markov chain. One is a full transition probability matrix $\mathbf{P}'$ for all absorbing and irreducible states of the system and the other matrix $\mathbf{P}$ models the transient states only. The rows of the matrix $\mathbf{P}'$ sum to 1. The matrix $\mathbf{P}$ is a submatrix of $\mathbf{P}'$ and due to the killing probabilities, the row sum is always less than unity. These killing probabilities can be expressed as

$$\kappa_i = 1 - \sum_{j\in\mathcal{B}} \mathbf{P}_{ij}, \ \ i \in \mathcal{B} \tag{6}$$

where $\mathbf{P}$ is a square irreducable and non negative matrix and $\mathcal{B}$ denotes the irreducible state space of the system. According to Perron-Frobenius theorem [14], there exists an exclusive and maximum eigenvalue, ρ, such that the related eigenvector is unique and contains only positive entries [14]. The theory of Markov chain says that if $\mathbf{u}$ is the left eigenvector of matrix $\mathbf{P}$, then the distribution $\mathbf{u}$ $=u_i, i \in \mathcal{B}$ is called ρ-invariant distribution, where ρ is corresponding eigenvalue of matrix $\mathbf{P}$, i.e., $\mathbf{uP} = \rho\mathbf{u}$.

As the probability of going into the absorbing state is greater than zero, therefore eventually killing of process is occurred. However, the probability for the system to be in state j at level n is given as

$$\mathbb{P}\{\mathbb{X}(n) = j\} = \rho^n u_j, \ \ j \in \mathcal{B}, \ \ n \geq 0. \tag{7}$$

Hence ρ can be used to characterize the coverage of the network using the properties of transition matrix.

B. Transition probability matrix

To compute the entries of transition probability matrix $\mathbf{P}$, we have to find probability of success/outage at certain level n. Let i and j be the states of system such that $i, j \in \mathcal{B}$. For instance, if N=4, then $i = [1101]$ at level 1 from Fig. 1. A sucessful hop occurs when a minimum of M nodes are in state 1, i.e., $\sum_{m=1}^{N} \mathbb{I}_m(n) \geq M$, where the probability of a node m to go in state 1 is given by $1 - P^{(n)}_{o_{(m|\xi^{(n-1)})}}$ and $P^{(n)}_{o_{(m|\xi^{(n-1)})}}$ is calculated using (4). The probability of going from any transient state i to any transient state j depends upon the position of the nodes, which decode the informations. The one-step transition probability from state i to j can be found by using

$$P_{ij} = \prod_{m\in\xi^{(n)}} (1 - P^{(n)}_{o_{(m|\xi^{(n-1)})}}) \times \prod_{m\in\bar{\xi}^{(n)}} (P^{(n)}_{o_{(m|\xi^{(n-1)})}}) \tag{8}$$

where $\xi^{(n)}$ and $\bar{\xi}^{(n)}$ are the sets of indices of those nodes which are 1 and 0, respectively, in state j at level n.

C. Co-located Topology

We consider another network topology in which the distance w between the adjacent nodes of a cluster is negligible hence the N relay nodes form a co-located group. The only distance left is the inter-group distance D. Therefore, one co-located group is D distance apart from another co-located group. As previously, those nodes that decode M messages, make network codewords and transmit these codewords to the next level using orthogonal channels. The probability of outage of node m at current level n can be calculated as

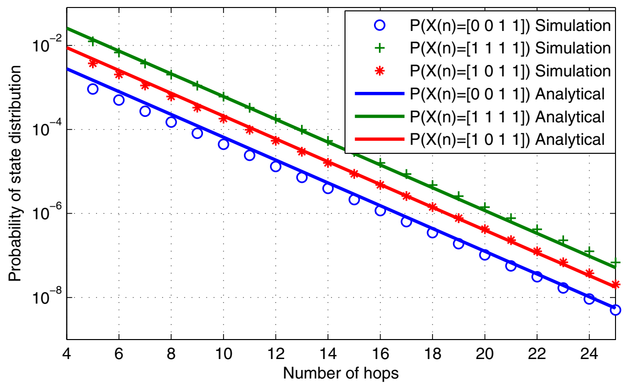

Fig. 2: Probability of state distribution; $P_t = 1$, $w = 2$, $d = 1$, $\beta = 2$, $\tau = 0.1$, $M = 2$, $N = 4$.

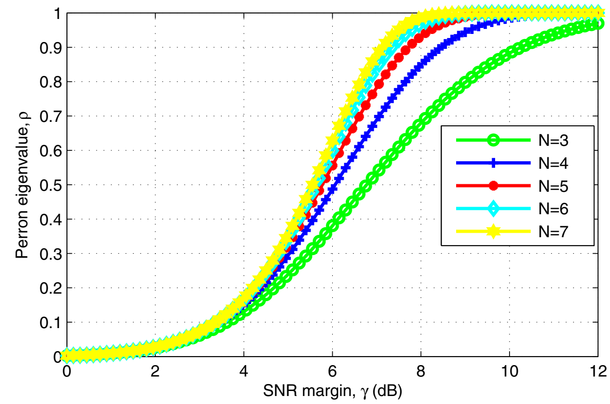

Fig. 3: Perron eigenvalue for different values of N; $M = 2$, $P_t = 1$, $\beta = 2$, $d = 1$, $w = 1$.

for χ=M,

$$P^{(n)}_{o_{m,\xi^{(n-1)}}} = \sum_{\alpha=1}^{\chi} {}^{\chi}\mathcal{C}_{\alpha}\, P_o^{\alpha}(1-P_o)^{\chi-\alpha}. \qquad (10a)$$

and for $\chi > M$,

$$P^{(n)}_{o_{m,\xi^{(n-1)}}} = \sum_{\alpha=\chi-(M-1)}^{\chi} {}^{\chi}\mathcal{C}_{\alpha}\, P_o^{\alpha}(1-P_o)^{\chi-\alpha}. \qquad (10b)$$

where ${}^{\chi}\mathcal{C}_{\alpha} = \frac{\chi!}{\alpha!(\chi-\alpha)!}$ denotes the 'combination' operator and $P_o = 1 - exp(-D^{\beta}\tau)$, is the outage probability of any node in a group. This probability of outage is same for all nodes because of the same path loss.

IV. Results and Analysis

In this section, we analyze the results generated through computer simulations and the analytical model. Fig. 2 shows the relation between the probability of a certain state of the system versus the number of hops. In this result, the distance w is kept at 2, while the inter-hop distance d is 1. Also the path loss exponent is taken to be 2 while $M = 2$ and the number of nodes at each hop is four, i.e., $N = 4$. The SNR margin, γ, which is the normalized SNR, i.e., $\gamma = P_t/\tau N_o$ is taken to be $10dB$. It can be seen that analytical results obtained by the use of Markov chain method almost match the simulation results. Analytically, the transition matrix is computed and the left eigenvector, corresponding to Perron eigenvalue, gives the state distribution probabilities. It can be seen that as the number of hops increases, the probability of achieving a certain transient state decreases. This is due to the killing probabilities as discussed earlier in the previous sections. To avoid repetition, only 3 states out of 11 transient states are shown in the graph.

Fig. 3 shows the relationship between the SNR margin and the Perron eigenvalue, ρ. The eigenvalue represents the one-hop success probability in our network. The one-hop success probability implies that at least M nodes are in success for onward transmission of data. As the system has more SNR margin to offer, the eigenvalue approaches to 1. It can also be seen from the figure that as N increases, the eigenvalue converges to 1 for less SNR margin. This is due to the fact that as N increases, the diversity gain is increased and thus the one-hop success probability is increased.

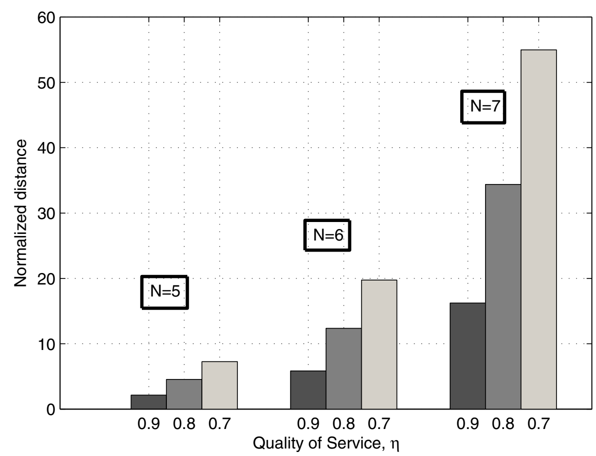

Fig. 4: Normalized distance for different value of N; $\gamma = 6dB$, $P_t = 1$, $M = 2$, $\beta = 2$, $d = 1$, $w = 1$.

From the network point of view, a certain quality of service (QoS), η, is required to be maintained in the system. In other words, if we are interested in calculating the probability of delivering the message to n_{th} hop and require this probability to be greater than η, then $\rho^n \geq \eta$ where n is the hop count. Fig. 4 shows the relation between QoS and the normalized distance, where we define the normalized distance as $n \times d$, where d is distance between two adjacent hops. We can see that by increasing N, we can deliver our message far away with a particular QoS. Therefore, for a fixed η, a larger value of ρ implies a better coverage, which could be attained by

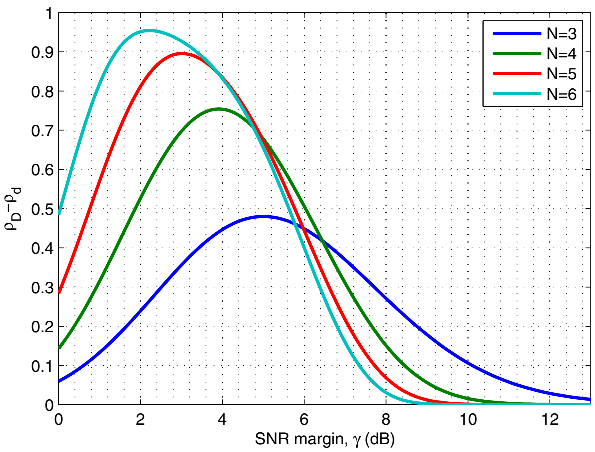

Fig. 5: Perron eigenvalue difference between co-located and distributed network topology; $M = 2$, $P_t = 1$, $\beta = 2$, $D = 1$, $d = 1$, $w = 1$.

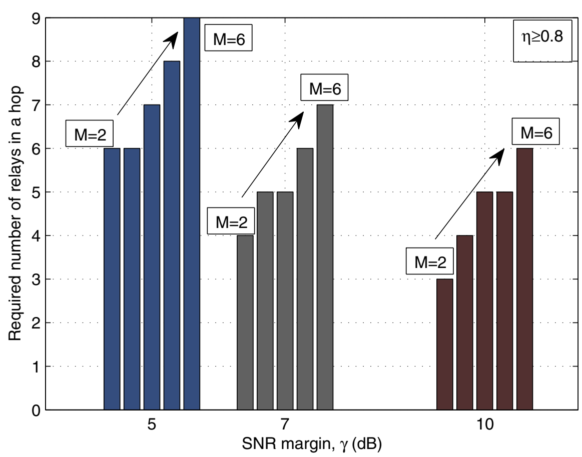

Fig. 6: Number of N required for different values of M at certain QoS ; $P_t = 1$, $\beta = 2$, $d = 1$, $w = 1$.

either increasing the SNR margin (Fig. 3) or by having larger number of relay nodes per hop (Fig. 4).

Fig. 5 shows the Perron eigenvalue difference of the co-located and distributed topologies, where ρ_D and ρ_d are the one-hop success probabilities of co-located and distributed topology, repectively. The difference is higher for the lesser values of SNR margin and as γ increases, the difference becomes small because the effects of path loss diminish with very high transmit power in both topologies. At lower SNR-margin, the co-located network topology performs better than the distributed one.

Fig. 6 represents the number of relays per hop, N, required to acheive a certain QoS, $\eta \geq 0.8$, versus the SNR margin. It can be observed that, as M increases, we require more relays per hop to maintain the QoS at a fixed SNR margin. The required N can be small if the SNR margin becomes high. For instance, if SNR margin is $5dB$, and M=5, then we need at least eight nodes (N=8) in each hop to acheive a success probability greater than 80%. However, for the same number of sources, if high SNR margin is used, e.g., $10dB$, then the required number of nodes per hop becomes 5.

V. Conclusion

In this paper, we investigated a multi-source multi-hop cooperative network and modeled the multi-hop transmission process with a Markov chain. We analyzed the network by calculating the outage probabilities of relay nodes in the presence of network coding. Using these probabilities, we formulated the state transition matrix and determined the coverage of the network under a given QoS constraint. We quantified the trade-off between SNR margin and required number of nodes per hop for delivering M sources information to a common destination. In future, we aim to investigate a network in which the relay nodes and the sources are randomly distributed.

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