Analysis of Composite Fading in a Single Cell Downlink Cooperative Heterogeneous Networks

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***Abstract*—Shadowing and multipath fading are two fundamental channel characteristics that impact the performance of a wireless communication system. In this paper, we analyze the performance of device to device (D2D) cooperative heterogeneous networks where idle femtocell users can be engaged to provide better coverage to a macrocell user using amplify-and-forward cooperative strategy. We consider multipath fading and lognormal shadowing along with path loss and model their effects on the performance of the network. Closed-form expressions for the signal-to-noise ratio (SNR) and outage probabilities are derived and it has been shown that the downlink performance of the macrocell user can be enhanced by employing femto user cooperation. Analytical results are validated through simulations.**

I. Introduction

Increased coverage, high data rates, and improved spectral efficiency are becoming critical parameters in the next generation cellular networks. Device to device (D2D)-based heterogeneous networks (HetNets) have gained enormous attention because of their capability of achieving these goals. In a general HetNet, small base stations (BS), also known as femtocells, are deployed within a macrocell that provide coverage to their intended users through different access modes. D2D communication is also considered as a key component of Long Term Evolution-Advanced (LTE-A) in which devices directly communicate with each other resulting in improved local services [1]. D2D communication occurs when devices are in close proximity with each other, reducing not only the communication cost by eliminating redundant cellular direct transmission but also improving the spectral efficiency.

Cooperative communication in D2D-based HetNets also helps in achieving the desired goals, e.g., femtocell user equipment (FUE) in an idle state can utilize capabilities to enhance the coverage of a macrocell user equipment (MUE), which is distant to its serving BS. The BS broadcasts the signal to its intended MUE, however the overheard signal by the idle FUEs can be relayed to the MUE and can be used to enhance its decoding capability. Carrying out BS controlled D2D communication in a HetNet offers many challenges, which include interference, path loss, multipath fading, and shadowing.While modeling large-scale networks, shadowing should be taken into consideration as it significantly degrades the quality of the signal. Until now, most of the work in D2D communication has been done considering deterministic models, e.g., path loss, or only fading is considered as the only channel impairment as in [2].

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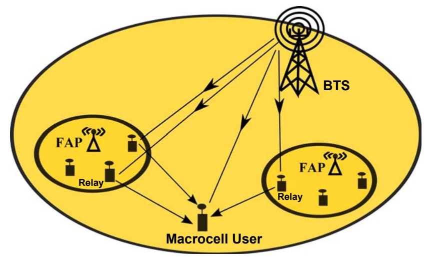

Fig. 1: A single cell HetNet with cooperation

Multipath fading is generally modeled through Rayleigh, Nakagami or Rice distributions [3]. However, in many cases both multipath fading and shadowing occur at the same time and lead to a composite distribution. The composite Rayleigh-lognormal distribution [3] is widely used to model the composite fading. The objective of this paper is to analyze how D2D communication helps a macro user, which is weakly connected to its BS in the presence of composite fading. For that we derive the analytical expression for the signal-to-noise ratio (SNR) at the destination node in presence of FUEs cooperation [4]. We then further derive the outage probability of the destination node assuming a lognormal distribution of the received SNR. At the end, we show that an increase in the severity of shadowing badly impacts the performance of system under consideration. In the next section, we briefly discuss the system model. In Section III, system outage will be derived. The derived models will be verified through simulations in Section IV, which is then followed by the conclusion in section V.

II. System Model

Consider an example of a heterogeneous two-tier network having both network and device centric entities participating, as depicted in Fig. 1. The base transceiver station (BTS) transmits the message signal with a certain transmit power to its MUE. In femtocell coverage regions, the femtocell access points (FAPs) are in transmission mode as well. We assume that all the FUEs located in the femtocell region are not in busy state, i.e., not having any data from FAP to send or

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receive. These users can establish D2D communication by acting as relays and enhance the SNR of the macrocell user via cooperation. The destination will decide on the basis of its signal-to-interference plus noise ratio (SINR) whether to employ cooperation or not. The macrocell user, after realizing that the SINR is below a certain threshold will broadcast a message. These idle femtocell users, on reception of that message will broadcast their messages mentioning the SINR values that they are receiving from both FAP and the BTS. The end user can select the relays having the highest SINR values from the BTS and those idle femtocell users start relaying the signal towards destination through amplify-and-forward (AF) mechanism [5].

Empirical analysis has proved that shadowing can be modeled by a lognormal random variable (RV) [6] and the mean envelope power Ω_p corresponds following probability density function (PDF)

$$P_{\Omega_p}(x) = \frac{1}{x\sigma_\Omega \zeta\sqrt{2\pi}} \exp\left\{ \frac{-(10log_{10}\{x\} - \mu_{\Omega_p(dbm)})^2}{2\sigma_\Omega^2} \right\}, \quad (1)$$

where $\zeta = \frac{ln(10)}{10}$, μ_{Ω_p} is the mean and σ_Ω is the shadow standard deviation. We consider a binary phase-shift keying (BPSK) modulation technique and assume that the channel is slow, flat faded and Rayleigh distributed where shadowing is also present. The power received at the destination because of BTS transmission is given as

$$P_r = \frac{P_t}{d_{ij}^{\beta}} h_{ij} S_{ij}, \quad (2)$$

where P_t is the power transmitted from the BTS and i and j are the source and destination nodes, respectively. S_{ij} is the large-scale fading channel gain, $S_{ij} \in S$, where S represents a lognormal RV, with mean μ and standard deviation σ. The flat fading channel gain is represented as $h_{ij} \in h$, where h represents a unit mean exponential RV, which is the squared envelope of the signal experiencing Rayleigh fading, the d_{ij} is the distance from the BTS to the destination and β is the path loss exponent.

A simplistic diagram of the system is depicted in Fig. 2. We begin our analysis by considering that the end user is receiving a BPSK modulated signal from the BTS. This signal is also being relayed to the destination user from different relays that are selected by the procedure mentioned earlier. The received signals at destination d from source s can be expressed as

$$Y_{sd} = h_{sd}\sqrt{S_{sd}}\sqrt{E_b}X_s + n_1, \quad (3)$$

where E_b is the bit energy sent by the source, X_s is the BPSK modulated signal and $n_1 \sim \mathcal{N}(0, \sigma_N^2)$ represents the Gaussian noise at the destination receiver. Similarly, the signal from source s to relay i can be written as

$$Y_{si} = h_{si}\sqrt{S_{si}}\sqrt{E_b}X_s + n_2. \quad (4)$$

We assume that each relay i on receiving the BTS signal forwards it to the destined end user d through AF principle,

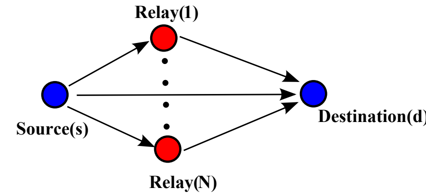

Fig. 2: Simplistic model of cooperative communication through relays

which can be expressed as

$$Y_{id} = \alpha_i(h_{si}\sqrt{S_{si}}\sqrt{E_b}X_s + n_2)h_{id} + n_3, \quad (5)$$

where amplification factor α_i of the relay i is expressed as

$$\alpha_i = \sqrt{\frac{E_i}{E_b | h_{si} |^2 S_{si} + \sigma_N^2}}. \quad (6)$$

All the received signals are combined at destination using maximal-ratio-combining (MRC) and the resultant signal Y_s becomes

$$Y_s = \frac{| h_{sd} |^2 S_{sd} E_b X_s}{\sigma_N^2} + \sum_{i=1}^{N} \frac{\alpha_i^2 | h_{si} |^2 | h_{id} |^2 S_{si} S_{id} E_b X_s}{\sigma_N^2(\alpha_i^2 | h_{id} |^2 S_{id} + 1)}, \quad (7)$$

and the resultant noise signal Y_n can be expressed as

$$Y_n = \frac{h_{sd}^*\sqrt{S_{sd}}\sqrt{E_b}n_1}{\sigma_N^2} + \sum_{i=1}^{N} \left\{ \frac{\alpha_i^2 | h_{id} |^2 h_{si}^* S_{id}\sqrt{S_{si}}\sqrt{E_b}n_2}{\sigma_N^2(\alpha_i^2 | h_{id} |^2 S_{id} + 1)} + \frac{\alpha_i^* h_{id}^* h_{si}^* \sqrt{S_{id}}\sqrt{S_{si}}\sqrt{E_b}n_3}{\sigma_N^2(\alpha_i^2 | h_{id} |^2 S_{id} + 1)} \right\}, \quad (8)$$

where N is the number of relays participating in communication. The total SNR, χ_{total}, at the destination is given as

$$\chi_{total} = \chi_{sd} + \sum_{i=1}^{N} \frac{\chi_{si}\chi_{id}}{\chi_{si} + \chi_{id}}, \quad (9)$$

where $\chi_{ij} = \frac{|h_{ij}|^2 S_{ij}}{\sigma_N^2}$. The received SNR is the addition of two different SNRs; one from the direct link between the source and destination while the other is the sum of individual SNRs of the relayed links.

III. System Outage Analysis

In this section, we show that the received SNR at the destination node, shown in (9), can be expressed as a lognormal RV with mean μ_{x_t}, and variance $\sigma_{x_t}^2$. Having said that, the outage probability is given as

$$P\{\chi_{total} \le \tau\} = Q\left(\frac{\mu_{x_t} - \tau}{\sigma_{x_t}}\right), \quad (10)$$

where Q represents the Q-function; where $Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} \exp\frac{-y^2}{2} dy$ and τ is the modulation dependent threshold. We now proceed step by step to show that χ_{total} is a lognormal RV. Consider the following lemma:

Lemma 1. *Let $Z = XW$, where X is a lognormal RV with mean μ_X and variance $\sigma^2_{X(dB)}$ and W is a unit mean exponential RV, then Z is also lognormal with mean $\mu_Z = \frac{C}{\zeta} + \mu_{X(dB)}$ and variance $\sigma^2_Z = \frac{1}{\zeta^2}\left(\frac{\pi^2}{6} + \zeta^2\sigma^2_X\right)$, where $C = 0.5772$ and $\zeta = \frac{ln(10)}{10}$.*

Proof. The products involving a lognormal RV can be well approximated by another lognormal RV [7]. The composite distribution of squared envelope of an exponential-lognormal RV can be expressed as

$$P_Z(z) = \int_0^\infty \frac{1}{x} e^{\frac{-z}{x}} \times \frac{1}{x\sigma_X\zeta\sqrt{2\pi}} \exp\left\{\frac{-(10log_{10}\{x\} - \mu_{X(db)})^2}{2\sigma^2_X}\right\} dx. \tag{11}$$

We start our procedure by finding the first moment, i.e.,

$$\mathbb{E}\left[10log_{10}(Z)\right] = \int_0^\infty 10log_{10}(z) \int_0^\infty \frac{1}{x} e^{\frac{-z}{x}} \times \frac{1}{x\sigma_X\zeta\sqrt{2\pi}} \exp\left\{\frac{-(10log_{10}\{x\} - \mu_{X(dB)})^2}{2\sigma^2_X}\right\} dzdx. \tag{12}$$

By change of variables and solving the inner integral, the above expression can be written as

$$\mathbb{E}\left[10log_{10}(Z)\right] = \frac{1}{\sigma_X\zeta\sqrt{2\pi}} \int_{-\infty}^\infty \exp\left\{\frac{-(y - \mu_{X(dB)})^2}{2\sigma^2_X}\right\} dy. \tag{13}$$

Solving this equation gives us the first moment of the related normal RV, i.e.,

$$\mu_Z = \mathbb{E}\left[10log_{10}(Z)\right] = -\frac{C}{\zeta} + \mu_{X(dB)}, \tag{14}$$

where $C \simeq 0.5772$ is the Euler's constant. Similarly the second order moment can be found by same procedure and expressed as

$$\mathbb{E}\left[(10log_{10}(Z))^2\right] = \frac{1}{\zeta^2}\left\{C^2 + \frac{\pi^2}{6} - 2\zeta\mu_{X(dB)}C + \zeta^2({\mu_{X(dB)}}^2 + \sigma^2_{X(dB)})\right\}. \tag{15}$$

The variance of Z can be calculated from (14) and (15), i.e.,

$$\sigma^2_Z = \mathbb{E}\left[10log_{10}(Z)^2\right] - \mathbb{E}\left[10log_{10}(Z)\right]^2 = \frac{1}{\zeta^2}\left(\frac{\pi^2}{6} + \zeta^2\sigma^2_X\right), \tag{16}$$

■

where $\zeta = \frac{ln(10)}{10}$. Lemma 1 provides a proof that the power received through direct path is lognormal distributed. We now focus on the relayed paths. Consider the following propositions for the numerator and denominator of the second term in (9).

Propostion 1.

If $X_i \sim lognormal(\mu_i, \sigma_i^2)$ are n independent and identically distributed (i.i.d) lognormal RVs, and $Y = \prod_{i=1}^n X_i$, then Y will also be a lognormally distributed such that

$$Y(\textstyle\sum_{i=1}^n \mu_i, \sum_{i=1}^n \sigma_i^2). \tag{17}$$

Proof. For proof, see [7] ■

Propostion 2.

Let $X_i \sim lognormal(\mu_i, \sigma_i^2)$ be i.i.d lognormal RVs then, $Y = \sum_{i=1}^n X_i$ is another lognormal RV with

$$\sigma^2_Y = ln\left[\frac{\sum_{i=1}^n (e^{2\mu_i + \sigma_i^2})(e^{\sigma_i^2} - 1)}{\sum_{i=1}^n (e^{\mu_i + \sigma_i^2/2})^2} + 1\right], \tag{18}$$

$$\mu_Y = ln\left[\sum_{i=1}^n (e^{\mu_i + \sigma_i^2/2})\right] - \frac{\sigma^2_Y}{2}. \tag{19}$$

Proof. For proof, see Fenton-Wilkinson [8] ■

Corollary 1.

If each lognormal RV X_i has identical variance parameter, i.e., $\sigma_i^2 = \sigma^2\ \forall i$, then the mean and variance of lognormal RV $Y = \sum_{i=1}^n X_i$ can be expressed as

$$\sigma^2_Y = ln\left[(e^{\sigma^2} - 1)\frac{\sum_{i=1}^n e^{2\mu_i}}{\sum_{i=1}^n (e^{\mu_i})^2} + 1\right], \tag{20}$$

$$\mu_Y = ln\left[\sum_{i=1}^n (e^{\mu_i})\right] - \frac{\sigma^2_Y}{2} + \frac{\sigma^2}{2}. \tag{21}$$

Proposition 1 and 2 will be used to approximate the product and sum of lognormal RVs in the second term of (9). For the ratio, consider the following lemma.

Lemma 2. *Let $Z = \frac{X}{Y}$, where X and Y are two lognormal RVs with mean μ_X, μ_Y and variances σ^2_X, σ^2_Y, respectively, then Z is another lognormal RV with mean $\mu_Z = \mu_X - \mu_Y$ and variance $\sigma^2_Z = \sigma^2_X + \sigma^2_Y - \sigma_{xy}$, where σ_{XY} is the covariance between X and Y.*

Proof. Consider the logarithm of the ratio of two lognormal RVs, i.e., $log(\frac{X}{Y}) = logX - logY$, where X and Y are lognormal RVs. This implies that $logX$ and $logY$ are normal RVs. Let's assume $logX$ and $logY$ have means μ_X and μ_Y, variances σ^2_X and σ^2_Y, respectively. Covariance can be represented as σ_{XY}, which will be 0 if X and Y are independent. The difference R is then normally distributed having mean $\mu_R = \mu_X - \mu_Y$ and variance $\sigma^2_R = \sigma^2_X + \sigma^2_Y - \sigma_{XY}$. The σ_{xy} can be expressed as

$$\sigma_{XY} = E[X^2]\mu_X + E[Y^2]\mu_Y - \mu_X\mu_Y, \tag{22}$$

To achieve $\frac{X}{Y}$, we know $\frac{X}{Y} = \exp\{R\}$, which shows that $\frac{X}{Y}$ itself is lognormally distributed with parameters μ_R and σ^2_R. The relationship between the means and variances of a lognormal and its corresponding RV can be expressed as

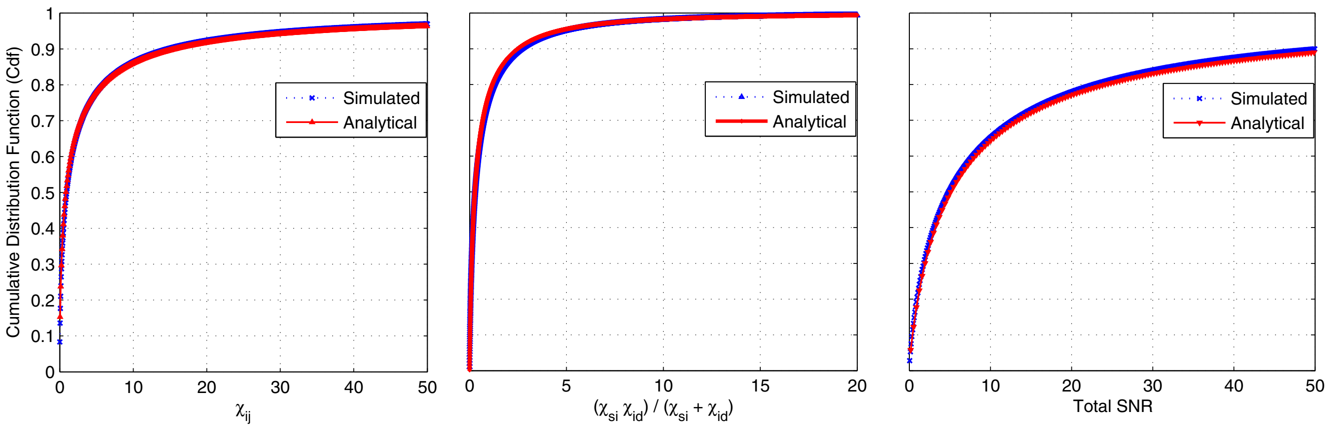

Fig. 3: The CDF of (a) χ_{ij}, product of exponential & lognormal RV., (b) SNR from relayed path., (c) Total SNR of the system.

$$\mathbb{E}\left[\frac{X}{Y}\right] = \mathbb{E}\left(e^R\right) = \exp\left\{\mu_R + \frac{1}{2}\sigma_R^2\right\} \quad (23)$$

$$\begin{aligned} Var\left[\frac{X}{Y}\right] &= Var\left(e^R\right) \\ &= \exp\left\{2\mu_R + 2\sigma_R^2\right\} + \exp\left\{2\mu_R + \sigma_R^2\right\} \end{aligned} \quad (24)$$

■

Theorem 1. *The total received SNR* $\chi_{total} = \chi_{sd} + \sum_{i=1}^{N} \frac{\chi_{si}\chi_{id}}{\chi_{si}+\chi_{id}}$ *can be approximated as a lognormal RV.*

Proof. From Lemma 1, each χ_{ij} is a lognormal RV where i, j denotes the transmitter and receiver nodes, respectively. From Lemma 2, Proposition 1 and Proposition 2, it can be proved that the ratio of lognormal RVs can be generalized with another lognormal RV. Hence each of the $N + 1$ terms in (9) have shown to be lognormal. In the final summation of lognormal RVs there may exist variables having different means and variances. From Proposition 2 again χ_{total} follows a lognormal distribution.

■

IV. Simulation Results and Performance Analysis

In this section, we verify the derived analytical expressions through computer simulations and provide the effects of various parameters on the performance of the system. We first verify the lognormal approximations that we performed in Lemma 1, Lemma 2, and Theorem 1, respectively. For the ease of notation, we refer to approximation of exponential-lognormal (Lemma 1) to χ_{ij} which is another lognormal. Fig. 3a compares the analytical and simulated results of cumulative distribution function (CDF) of RV χ_{ij}. The simulation curve is obtained by generating independent lognormal and exponential RVs and multiplying them. 10^7 iterations were performed to obtain the CDF of the resultant RV. The analytical result is obtained by simply generating a lognormal RV with mean and variance as obtained in Lemma 1. Both the curves match closely and the mean squared error between them was found to be of the order of 10^{-6}.

Fig. 3b represents the analytical result proved in Lemma 2. It shows the CDF of a lognormal RV, which is an approximated result of the ratio of lognormal RVs. This ratio is itself a product of two lognormal RVs in its numerator and sum of lognormals in the denominator. The results follow almost identical CDF curves. Fig. 3c shows the CDF of total SNR proved in Theorem 1. The cumulative SNR expressed in (9) can be considered as a summation of individual lognormals as proved from Fig. 3a and Fig. 3b. These lognormals can also have different means and variances. The resulting parameters can be calculated from (18) and (19) and are plotted in the figure. In all of the above cases, the standard deviation of shadowing for each RV is identical and taken to be 8dB.

Fig. 4 shows the effects of increasing the number of relays on the coverage probability with respect to different path loss exponents, β. For each i, j link in Fig. 2, the d_{ij} is kept at 150 m, while the $\sigma_{ij} = \sigma = 8$ dB and threshold $\tau = -20$ dB. It can be observed from the figure that there is an increase in the coverage probability as the number of relays is increased. It can be further observed that as the number of relays increases from 0 to 2, there is a significant change in the coverage probability, however, this change becomes very small for larger number of relays. A diminishing trend in the diversity gain is seen as the number of relay increases.

The SNR threshold versus coverage probability trend is analyzed for different number of relays in Fig. 5. For $\beta = 2$ and $\sigma = 8$ dB, it can be observed that the coverage probability is decreased with an increase in the threshold. However, the coverage of the non-cooperative case is least amongst all cases and it increases as the number of relays is increased due to the increased diversity gain that additional relays offer. However, at higher values of threshold, the coverage becomes same for all cases because more and more nodes will start to be in outage until comes a point where every receiver node in the system will have received signal's strength below the SNR

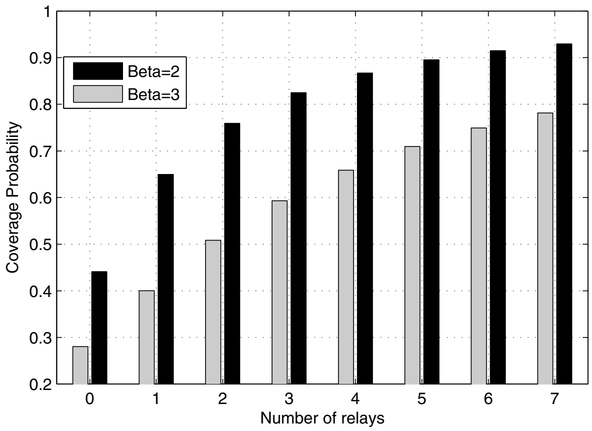

Fig. 4: Effects of β on coverage for various number of participating relays.

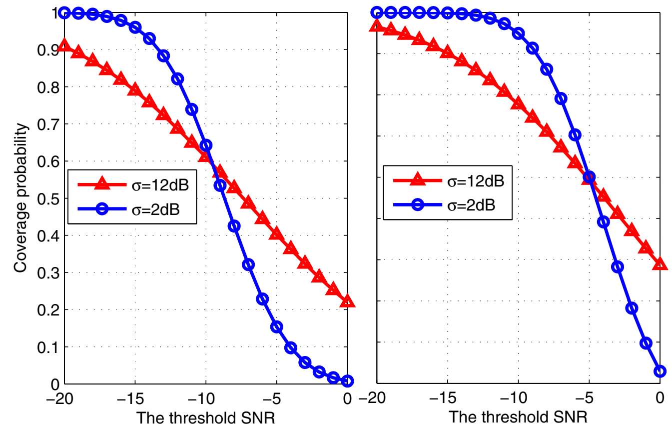

Fig. 6: Variation in system's coverage for values of σ in large (left) and small (right) cell environments

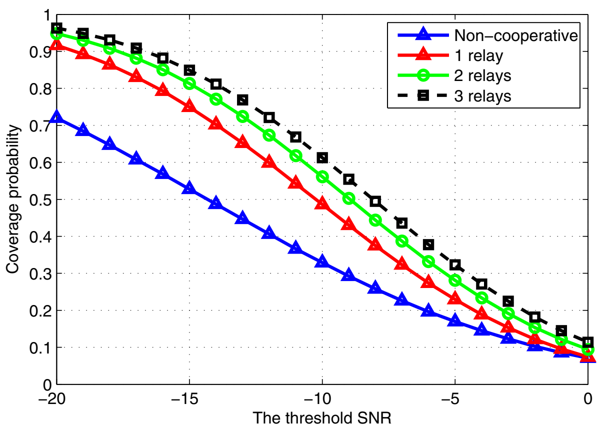

Fig. 5: Effects of threshold SNR on system's coverage.

threshold.

The difference in system's coverage probability in both large and small cells while considering different standard deviation of shadowing is shown in Fig. 6. Same values of shadowing has been assumed for all the links present in each particular scenario. For large cells, distance between the source and end user can be up to 1000 meters, while a relay can be selected if it is present within 100 meters from the destination. In case of a small cell, $d_{sd} = 400$ meters, while relays can be selected within 40 meters of radius. Three operational relays are considered with $\beta = 2$. It can be seen for both cases that at lower values of standard deviation, the coverage probability is higher. This is because the severity of shadowing is small and a coverage gain of almost 14% is seen at a threshold of -20dB for large cells. However, as the threshold is increased after -2 dB, the $\sigma = 12$ dB curve outperforms. This is because the shadowing models the fluctuations in received power. These fluctuations become large for larger σ. Therefore, even at higher thresholds, one can still get a favorable shadowing outcome, which eventually increases the coverage probability of the network. Hence in designing systems where shadowing becomes considerable channel impairment, a sufficient shadow margin is necessary to compensate for the losses. Overall the coverage of small cells outperforms the large cell environment because of the small distances between communication entities.

V. Conclusion

Enabling D2D cooperation in a heterogeneous network provides better coverage and high data rates. For efficient modeling of large-scale networks, shadowing must be considered along with channel fading and path loss. In this paper, we have modeled the effects of these three parameters on the coverage of a cooperative HetNet performing D2D, where idle FUEs cooperate with a MUE. It has been shown through analysis that the SNR of both individual link and the relayed links can be modeled by a lognormal RV. The bit error rate analysis and interference from neighboring macrocells in a multicell environment is left as a future work.

References

[1] Y. Li, C. Song, D. Jin, S. Chen, "A dynamic graph optimization framework for multihop device-to-device communication underlaying cellular networks," *IEEE Wireless commun.,* vol. 21, no. 5, pp. 52-61, 2014.

[2] O. Yilmaz, Z. Li, K. Valkealahti, M. A. Uusitalo, M. Moisio, P. Lunden, C. Wijting, "Smart mobility management for D2D communications in 5G networks," *IEEE Wireless commun. and networking conference (WCNC),* March 2014.

[3] M. Patzold, "Mobile Fading Channels," John Wiley and Sons, Ltd, U.K., 2002.

[4] A. M. Zaid, B. Hamdaoui, C. Xiuzhen, T. Znati, M. Guizani, "Improving macrocell downlink throughput in Rayleigh fading channel environment through femtocell user cooperation," *IEEE Trans. Wireless Commun.,* vol.12, pp. 6488-6499, Dec. 2013.

[5] T. M. Cover and A. A. El Gamal, "Capacity theorems for the relay channel," *IEEE Trans. Inform. Theory,* vol. 25, no. 5, pp. 572-584, Sept. 1979.

[6] G. L. Stuber, "Principles of Mobile Communication," 2nd Ed. Springer International, 2011.

[7] N. B. Mehta, J. Wu , A. F. Molisch, and J. Zhang, "Approximating a sum of random variables with a lognormal," *IEEE Trans. Wireless Commun.,* vol. 6, no. 7, pp. 2690-2699, July 2007.

[8] L. F. Fenton, "The sum of lognormal probability distributions in scatter transmission systems," *IEEE IRE Trans. Commun. Systems,* vol. 8, no. 1, pp. 57-67, 1960.