

# On the Impacts of Lognormal-Rice Fading on Multi-Hop Extended Networks

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**Abstract**—This paper presents an analytical model for a multi-hop two-dimensional (2-D) network with finite density of nodes communicating with one another by forming an opportunistic large array (OLA). Transmission among these nodes is modeled via Markov Chain, where the wireless channel is considered as a composite lognormal-Rice random process. We approximate the sum distribution of the received power at a node with lognormal random variable (RV) using a moment generating function (MGF)-based approach, which is acquired by using Gauss-Hermite integration. The transition probability matrix is derived to determine the one-hop success probability. Coverage aspects for different arrangement of nodes is quantified along with the effects of wireless channel on system performance.

**Index Terms**—Cooperative Communication, Wireless Sensor Networks, Lognormal-Rice fading

## I. INTRODUCTION

Multi-hop network is a common layout nowadays for wireless sensor networks (WSN). Data gathering, possibly in a multi-hop fashion, from a wide spread of unknown amount of sensors to a sink node is an important application of WSN. Each sensor node in the path of a transmission is responsible for relaying the message to another node. However, issues of long delays, low reliability, and increased latency raised serious questions on the performance of single-input single-output (SISO) multi-hop networks. In addition to that, the channel impairments like multipath fading and shadowing limit the performance of the system.

Cooperative transmission (CT), as a physical layer transmission technique, was introduced to boost the system efficiency by employing spatial diversity. The system capacity increases by providing distributed transmit diversity as multiple nodes transmit the same message and in lieu of that a node receives multiple copies of the same message, which increases the likelihood of decoding the message correctly. One of the most effective CT networks is the Opportunistic Large Arrays (OLAs) [1] that perform in a multi-hop manner where all nodes in a network form groups, and these groups of nodes transmit the same message at the same time to the other group. The promising feature of range extension [2]-[3] due to OLA broadcasting and energy efficiency [4] makes it a desirable option for densely populated networks.

A considerable amount of research on OLA is done, exploiting the various aspects of OLA transmission. In [5], Mergen et al. used the approach of *continuum* approximation to model OLA, i.e., infinite density of nodes along with a fixed transmit power per unit area. Their model guarantees that the message will be delivered to the end node, no matter how far the destination is. A linear network of finite number of nodes is considered in [6] and [7], where each node is placed at a fixed distance or in groups, respectively. The successful number of hops for the message was quantified before the transmission dies out. Rayleigh fading along with the path loss was used to model the channel.

In a wireless channel, the radio propagation under-goes sufficient amount of distortion and attenuation from its surroundings. In literature, various studies were made to analyse the effects of multipath fading in cooperative networks, most of them had considered Rayleigh fading as the only channel impairment along with path loss [8]-[11]. However, in a wireless system the effect on signal propagation in the presence of lognormal shadowing is significant, which is another important step while moving towards the practical scenarios. A line network with composite Rayleigh fading and independent shadowing was studied in [12], whereas the effect of correlated shadowing are quantified in [13] for the same 1-dimensional network. In addition to shadowing, if we move to the more realistic scenario of deploying a chain of nodes communicating cooperatively, we can see that the possibility of having a line-of-sight (LOS) component increases specially in outdoor environment. Even in indoor, we can have a LOS component as shown in [14]. This LOS causes a great shift in mean estimations for decoding signal correctly. To our knowledge, no research is been conducted to consider the composite Rician fading and lognormal shadowing for analysing the performance of OLA networks.

In this paper, we consider the model for a two-dimensional (2-D) extended networks under composite lognormal-Rice fading, whereby, a group of nodes cooperatively transmit the same message to another group of nodes and model the transmission from one group to another as a discrete-time Markov chain. The received signal, at a particular node, is a sum of multiple transmitted signals over orthogonal channels [15]; each of this signal experiences small-scale fading as well as shadowing. To find the success probability of a node, the required probability density function (PDF) for the

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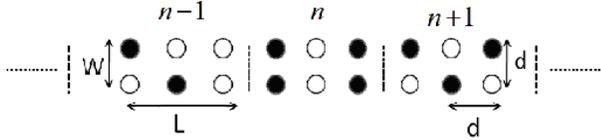


Fig. 1: A 2D grid network with  $L = 3$ ,  $W = 2$ ;  $M = 6$

sum of composite lognormal-Rice random variables (RVs) is computed using Gauss-Hermite numerical computation, which is composed by using a moment generating function (MGF)-based approach [16]. By using Gauss-Hermite integration, the MGF of the sum distribution is approximated to a single lognormal RV, where we use its mean and variance to determine the effect on network's coverage area. The effect on network's coverage area is quantified as a function of path loss exponent, Rice K-factor and standard deviation of shadowing.

The rest of the paper is organized as follows. Section II describes the system model. In Section III, we model our network as a discrete-time Markov chain. We derive the transition probability matrix of the Markov chain in Section IV. Section V validates our analytical results and Section VI concludes the paper along with future work.

## II. SYSTEM MODEL

Consider an extended network in the form of a horizontal strip, with finite number of nodes, placed in a two-dimensional formation as shown in Fig. 1. Each node is placed at a distance  $d$  from its neighbouring nodes in both dimensions. Let  $L$  denotes the number of nodes per hop along the horizontal stretch and  $W$  be the width of the network, representing the number of nodes in the vertical dimension, as shown in Fig. 1. Correspondingly, we define  $M \triangleq L \times W$  to be the number of nodes in one level forming an OLA that cooperates synchronously to transmit the same message to next level (or OLA) containing other  $M$  number of nodes. The cooperative mechanism of decode-and-forward (DF) is used.

The message will propagate from hop to hop, when a node, in any level, receives a message and at least one node from the previous level transmits. However, error-free decoding of the message depends upon the received signal-to-noise ratio (SNR) being greater than or equal to a certain threshold,  $\tau$ . In Fig. 1, a node that satisfies the threshold criterion and becomes a DF node is filled with black, while the hollow circles show the nodes that failed to decode the message. The different levels are represented as  $n-1$ ,  $n$ ,  $n+1$  etc. Our assumptions include that all the nodes have same transmit power,  $P_t$ ; each transmission is done over orthogonal fading channel, and all receivers have perfect timing and frequency synchronization.

To represent the nodes in a system, let us define some notations, which would be used throughout this paper. The nodes in the network are labeled from top to bottom and left to right, in a level. At a level  $n$ , the indices of the nodes that decoded the message without any error are represented by a set  $\mathbb{N}_n$ . For example,  $\mathbb{N}_n = \{1, 2, 5, 6\}$  and  $\mathbb{N}_{n+1} = \{1, 4, 5\}$ ,

for the case of Fig. 1. If a node  $m$  at level  $n$  is transmitting to a node  $j$  at level  $n+1$ , then the received power,  $P_r$ , at the  $j^{th}$  node is given as

$$P_{r_j}(n+1) = \frac{P_t}{d^\beta} \sum_{m \in \mathbb{N}_n} \frac{\nu_{mj}}{(\sqrt{\delta_{mj}})^\beta}. \quad (1)$$

In the above equation  $\nu_{mj}$  is a composite fading coefficient that represents the small-scale as well as large-scale channel fading between node  $j$  in the current level and node  $m$  in the previous level. The large-scale shadowing is modeled as lognormal random process, whereas Rice fading is used to model the effects of small-scale fading. The Path loss exponent, represented as  $\beta$ , ranging between 2-4. The distance between nodes  $m$  and  $j$  is given by a Euclidean metric  $\sqrt{\delta_{mj}}$ .

## III. MODELING BY MARKOV CHAIN

In this section, we develop a one-hop transmission model, keeping in view that a node at level  $n$  can only participate if at least one node in level  $n-1$  has transmitted and that node also decodes the message correctly. At a particular time instant, all nodes in a level can decode the data or not, therefore, we can represent the nodes as in 'on' or 'off' state, either '1' or '0', respectively. As mentioned earlier the nodes are labeled from top to bottom and left to right, the state of a node in a certain level  $n$  can be given as

$$\tilde{\mathcal{X}}(n) = \begin{bmatrix} \mathbb{I}_{1,1}(n) & \mathbb{I}_{2,1}(n) & \cdots & \mathbb{I}_{L,1}(n) \\ \mathbb{I}_{1,2}(n) & \vdots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ \mathbb{I}_{1,W}(n) & \mathbb{I}_{2,W}(n) & \vdots & \mathbb{I}_{L,W}(n) \end{bmatrix} \quad (2)$$

where  $\mathbb{I}_{i,j}(n)$  is the binary indicator random variable representing the state of a node in  $i^{th}$  position horizontally and  $j^{th}$  position vertically, where  $i = \{1, 2, \dots, L\}$  and  $j = \{1, 2, \dots, W\}$ . The subscript act as a coordinates for row and column in which the node is present. From Fig.1  $\mathbb{I}_{1,1}(n) = 1, \mathbb{I}_{1,2}(n) = 1, \mathbb{I}_{2,1}(n) = 0$ ;  $\tilde{\mathcal{X}}(n) = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$ . We can now convert our 2-D model into 1-D by defining  $\mathcal{X}(n) = \{vec[\tilde{\mathcal{X}}(n)]\}^T$ , here a vector operation,  $vec$ , is performed to convert the 2-D matrix in to single dimension column vector and  $T$  denotes the transpose operator. For instance, from Fig.1,  $\mathcal{X}(n) = [110011]$ .

This tends to confer an  $M$ -bit binary outcome for each hop, forming a state, and there are  $2^M$  possible states. Since at a certain time instant  $n$ , the current state node only depends upon the previous state, hence  $\mathcal{X}(n)$  can be regarded as a discrete-time Markov Process. The discrete-time assumption comes from the fact that the transmissions occur at every distinct slots synchronously [17]. Markov chain with state space,  $\mathbf{S}$ , given  $M$  number of nodes per state, can be represented as  $\mathbf{S} = \{0, 1, 2, \dots, 2^M - 1\}$ . Here we assume that the channel between levels exhibits the same statistics (in terms of channel fading) and each node have the same transmit power; hence the Markov chain is regarded as homogeneous.

Furthermore, there is a probability that at some point, none of the nodes in a level is able to decode the message correctly, implying that the Markov chain is in state  $\{0\}$  resulting in no further transmission,  $\lim_{n \rightarrow \infty} \mathbb{P}\{\mathcal{X}(n) = 0\} \nearrow 1$ . We are now in a position to define the state transition matrix for our Markov chain. Our state space is defined as  $\mathbf{S} = \{0\} \cup \mathbf{A}$ , where  $\mathbf{A} = \{1, 2, \dots, 2^M - 1\}$  is irreducible transient space set and 0 is absorbing state. If we define  $\hat{\mathbf{P}}$  as transition probability matrix with state space  $\mathbf{S}$  then by removing the first column and row that involves transitions to and from absorbing state, we are left with matrix  $\mathbf{P}$ , which is a sub-matrix of  $\hat{\mathbf{P}}$  comprising of states  $\mathbf{A} \subseteq \mathbf{S}$  with dimension  $(2^M - 1) \times (2^M - 1)$ .

Since  $\mathbf{P}$  is a square irreducible and non-negative matrix, then by Perron-Frobenius theorem [18], a unique maximum eigenvalue,  $\rho$ , exists with a unique eigenvector associated with it. As  $\mathbf{P}$  is not truly right stochastic due to eliminating of absorbing state, the Perron eigenvalue is always less than one, i.e.,  $\rho < 1$ . If  $\mathbf{u}$  is the left eigenvector of transition matrix  $\mathbf{P}$ , then  $\mathbf{u} = (u_i, i \in \mathbf{A})$  is called  $\rho$ -invariant distribution and it describes the state of the system before going into absorbing or killing state and is called quasi-stationary distribution. At time  $n$ , probability of being in a state  $j$  is

$$\mathbb{P}_r\{\mathcal{X}(n) = j\} = \rho^n u_j, j \in \mathbf{A}, n \geq 0 \quad (3)$$

If we define  $\mathcal{K} = \inf\{n \geq 0 : \mathcal{X}(n) = 0\}$  as end of transition time then probability, until killing occurs is,  $\mathbb{P}_r\{\mathcal{K} > n + h | \mathcal{K} > n\} = \rho^h$ . Hence quasi-stationary distribution for the Markov chain becomes

$$\mathbb{P}\{\mathcal{X}(n) = j | \mathcal{K} > n\} = u_j, j \in \mathbf{A}, \quad \forall n \quad (4)$$

#### IV. FORMULATION OF THE TRANSITION MATRIX

In this section, we find the transition probability matrix,  $\mathbf{P}$ , for our model, the eigenvector of which will give us the quasi-stationary distribution. Let us define the source and destination states as  $i$  and  $j$ , respectively. Note that  $i$  and  $j$  are the decimal equivalent representations of the pair of states of the system, such that  $i, j \in \mathbf{A}$ . For instance, for  $M = 6$ , as show in Fig. 1, if transition is going from  $\{100100\}$  to  $\{110011\}$  then  $i = 36$  and  $j = 51$ .

At a particular node, we receive signals from various paths, each of which undergoes shadowing and multipath fading; the number of paths depends upon the number of transmitting nodes (number of 1's) in the previous level (state). The received SNR at the  $k$ th node of level  $n$  can be given by  $\gamma_k(n) = P_{rk}(n)/\sigma_N^2$ , where,  $\sigma_N^2$  is variance of white noise, which is assumed equal for all nodes in a level, and  $P_r$  is the received power as given in (1). The conditional probability for the receiving node  $k$  to successfully decode the message at time  $n$  is given as

$$\begin{aligned} \mathbb{P}_r\{\text{Success of node } k|\varepsilon\} &= \mathbb{P}_r\{\mathbb{I}_k(n) = 1|\varepsilon\} = \\ \mathbb{P}_r\{\gamma_k(n) > \tau|\varepsilon\} &= \int_{\tau}^{\infty} p_{\gamma_k|\varepsilon}(z) dz \quad , \end{aligned} \quad (5)$$

Here the event,  $\varepsilon \in \mathcal{X}(n-1)$ , indicates that previous state is transition state and,  $p_{\gamma_k|\varepsilon}(z)$  is the conditional probability

density function (PDF) of the received SNR. Similarly, the probability for the receiving node being in outage is given by  $1 - \mathbb{P}_r\{\gamma_k(n) > \tau|\varepsilon\}$ . It can be noticed that the received SNR is a sum of lognormal-Rice RVs, for which the distribution for sum of lognormal-Rice RVs does not exists in closed-form [19]. Therefore, a moment generating function (MGF) technique is used to find the sum of lognormal-Rice RV. In this method, the sum of independent RVs can be expressed as the product of the MGFs of individual RVs.

We first define a composite RV,  $\nu = W\phi$ ; where  $W$  is a Rician RV and  $\phi$  is a lognormal RV. Then we approximate the sum of  $K$  lognormal-Rice RVs  $(\nu_1, \nu_2, \dots, \nu_K)$  by a single lognormal RV. One way to model this is to equate the PDF of the resultant lognormal RV,  $\phi$ , with the convolutions of the PDFs of individual  $\nu_i$ 's; however, a closed-form expression is prohibited. Another method is to equate the MGF of  $\phi$  with the product of MGFs of  $\nu_i$  and then find the mean and variance of  $\phi$  in terms of the parameter of individual  $\nu_i$ 's. We use the latter approach. Now for the sum of  $K$  lognormal-Rice RVs,  $\sum_{k=1}^K \nu_k = (\nu_1, \nu_2, \dots, \nu_K)$ , the MGF is given as

$$\Psi_{\phi}(\nu_1, \nu_2, \dots, \nu_K) = \prod_{k=1}^K \Psi_{\nu_k}(s; \mu_k, \sigma_k, \kappa_k) \quad (6)$$

Where  $\mu_k$  and  $\sigma_k$  are the mean and standard deviation of the  $k$ th lognormal RV and  $\kappa_k$ , is the Rice factor for the  $k$ th Rician RV. This approximation method requires a closed-form expression for the MGFs of both the lognormal-Rice and lognormal RVs. However, it is difficult to find them in a closed-form so for that we used a Gauss-Hermite integration, which provides an easy solution of their numerical computation. The MGF approximation of the lognormal RV  $\phi_k$ , by using the Gauss-Hermite integration is given as

$$\hat{\Psi}_{\phi}(s; \mu_X, \sigma_X) = \sum_{n=1}^N \frac{w_n}{\sqrt{\pi}} \exp \left[ -s \exp \left( \frac{\sqrt{2}\sigma_X a_n + \mu_X}{\xi} \right) \right] \quad (7)$$

Where  $\mu_X$  and  $\sigma_X$  are the mean and standard deviation of the Gaussian RV  $X = 10 \log_{10} \phi$ . In Gauss-Hermite integration, the integral is estimated by an approximate sum where the summation is defined by specific weights. The MGF of  $k$ th lognormal-Rice RV by Gauss-Hermite integration can be written as

$$\begin{aligned} \hat{\Psi}_{\nu_k}(s; \mu_k, \sigma_k, \kappa_k) &= \sum_{n=1}^N \frac{w_n(1 + \kappa_k)/\sqrt{\pi}}{1 + \kappa_k + s \exp(\frac{\sqrt{2}\sigma_k a_n + \mu_k}{\xi})} \\ &\times \exp \left( - \frac{s \kappa_k \exp(\frac{\sqrt{2}\sigma_k a_n + \mu_k}{\xi})}{1 + \kappa_k + s \exp(\frac{\sqrt{2}\sigma_k a_n + \mu_k}{\xi})} \right) \quad . \end{aligned} \quad (8)$$

The Hermite integration order,  $N$ , is used to achieve better estimate of mean and standard deviation the larger the value of  $N$  the better the estimate. The adjustable parameter  $s$  of the MGF with positive real values gives two independent equations from which  $\mu_k$ , and  $\sigma_k^2$  are calculated. Where  $\xi = 10/\ln 10$ , is a constant for scaling and  $w_n$  is the weight corresponding to the abscissas,  $a_n$ . In [20] the values for  $N$

along with the corresponding weights and abscissas can be found in a tabulated form. By finding individually for each RV, the  $\mu_X$  and  $\sigma_X$  as a function of  $\mu_k$  and  $\sigma_k$ , the value of these variables are acquired by equating the two equations i.e. (7) and (8)

$$\widehat{\Psi}_\phi(s; \mu_X, \sigma_X) = \prod_{k=1}^K \widehat{\Psi}_{\nu_k}(s_i; \mu_k, \sigma_k, \kappa_k), \text{ at } i = 1 \text{ and } 2. \quad (9)$$

The right hand side of the above equation consists of all the known elements, by solving for  $s_1$  and  $s_2$  we will only be left with unknown moments of X. The values for  $s_1$  and  $s_2$  are found by solving an optimization model [6],  $s_1 = 0.2$  is used to calculate the  $\mu_X$  and  $s_2 = 1.0$  for  $\sigma_X$ . Note that this method leaves us with two non-linear equations, which can be solved by using the standard `fsolve` function in MATLAB. Since the sum of  $K$  lognormal-Rice RVs are approximated by a single lognormal RV, the calculated  $\mu_X$  and  $\sigma_X$  will give us the resultant lognormal RV.

Now to find that a node has decoded the message correctly, implies that, the received SNR is greater than or equal to a decoding threshold,  $\tau$ . Hence we shape our conditional probability in (8) accordingly. At node  $k$ , the SNR received is determined by the distribution of RV  $\phi$ ,  $\phi^{(k)} = 10^{0.1X^{(k)}}$ , (8) becomes

$$\begin{aligned} \mathbb{P}_r\{\gamma_k(n) > \tau | \varepsilon\} &= \mathbb{P}_r\{\phi^{(k)} > \tau | \varepsilon\} = \mathbb{P}_r\{10^{0.1X^{(k)}} \geq \tau\} = \\ &= \mathbb{P}_r\{X^{(k)} \geq 10 \log \tau\} = Q\left(\frac{10 \log \tau - \mu_{X^{(k)}}}{\sigma_{X^{(k)}}}\right). \end{aligned} \quad (10)$$

Therefore, the success probability depends upon the calculated  $\mu_X$  and  $\sigma_X$  from (12) and  $\tau$ . In a certain state, the one-hop probability is given by the product between probabilities of nodes that are in coverage multiplied with the probabilities of nodes that are in outage. If  $\mathbb{N}_n^{(j)}$  is the set of indices of nodes that has decoded at time  $n$  being in state  $j$  and  $\bar{\mathbb{N}}_n^{(j)}$  is a set for nodes that do not decode, then the one-step transition probability going from state  $i$  to  $j$  is given by

$$\begin{aligned} \mathbb{P}_{ij} &= \prod_{k \in \mathbb{N}_n^{(j)}} Q\left(\frac{10 \log \tau - \mu_{X^{(k)}}}{\sigma_{X^{(k)}}}\right) \times \\ &\prod_{k \in \bar{\mathbb{N}}_n^{(j)}} \left\{1 - Q\left(\frac{10 \log \tau - \mu_{X^{(k)}}}{\sigma_{X^{(k)}}}\right)\right\}. \end{aligned} \quad (11)$$

## V. RESULT ANALYSIS

In this section, we present some numerical results for various sets of parameters. We carried out numerical simulations to validate analytical models derived in the previous section. For the simulation environment, we consider  $M$  number of nodes in a level with some initial distribution of DF nodes for the first hop. The Euclidean distance between the current hop and the next hop nodes is computed. The received power is calculated at each node, which in turn sets the indicator

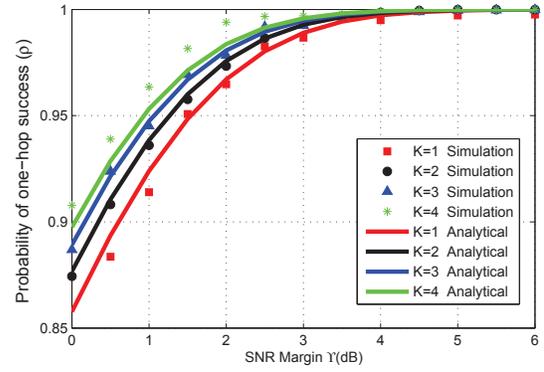


Fig. 2: Simulation versus analytical results,  $M = 6, \sigma = 10$

function to either 0 or 1. The new state distribution for the current hop is checked for an absorbing state (i.e. none of the node is able to decode) and the process continues.

The case of composite Rician and lognormal fading channel is catered by generating these processes separately and then multiplying them together. The important parameters to handle are the K-factor of Rice distribution and the mean and standard deviation of shadowing. We obtained one-hop success probability,  $\rho$ , for both simulations and analytical model. In the case of simulations,  $\rho$  is the probability that at least one node in a level decodes the message successfully and we averaged out these simulations over 1 million trials. For illustration, the results are taken for a certain normalized SNR at each node, called the SNR margin,  $\Upsilon = \frac{P_t}{\tau d^\beta}$ . We assume  $d = 1$  and  $P_t = 1$  for all results and the path loss exponent,  $\beta = 2$ , is used unless mentioned otherwise. The length and width of the level are kept fixed at  $L = 3$  and  $W = 2$  until otherwise stated.

From Fig. 2, it is obvious that both the analytical and the simulation results are quite close to each other, which verify the accuracy of the proposed analytical model. As the SNR margin increases, the probability of being in a transient state also increases, which increases the probability for more number of nodes to decode in a certain level.

Fig. 3 shows the one-hop success probability for different parameters of shadowing and fading, at various SNR margins. It can be seen that as the SNR margin increases the probability of one-hop success also increases, which is intuitive because of an increased transmit power and/or reduced distance and threshold. In Fig. 3, we get two important notions depicting the behaviour of success as a function of  $\sigma$ . It can be observed that for low SNR margins, i.e.,  $\Upsilon \leq 9$  dB, an increase in  $\sigma$  results in an increase in  $\rho$ . This seems contradictory because an increase in  $\sigma$  depicts the severity of shadowing. In a wireless channel,  $\sigma$  depicts the variation in the received power across a certain mean and the mean in turn defines the path loss exponent; the larger the  $\sigma$ , the higher the variations. At low SNR margins, the only nodes present at the tail of a level will actively take part in transmission to the next hop due to minimum path loss between them and the next level nodes as compared to the

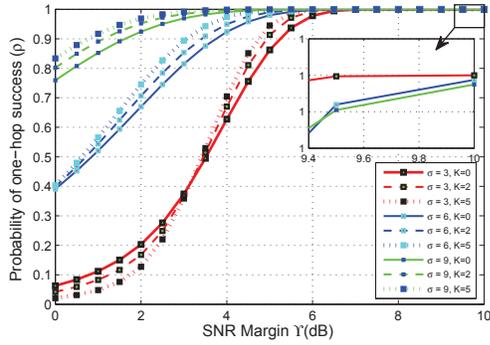


Fig. 3: Success probability vs. SNR margin for  $L = 3, W = 2; M = 6, \beta = 2$

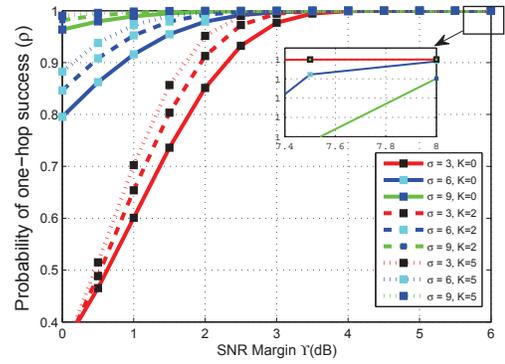


Fig. 4: Success probability vs. SNR margin for  $L = 2, W = 3; M = 6, \beta = 2$

nodes, which are at the starting edge with relatively higher path loss. Therefore, on low SNR margins, when path loss is high, we can get a favourable response when  $\sigma$  is increased, which eventually increases the one-hop success probability. As the SNR margin attains higher values, the path loss decreases and only the severity of shadowing  $\sigma$  causes the probability of success to drop for large values of  $\sigma$ , as shown in the zoomed portion of Fig. 3, where the behaviour of curves is reversed.

Also note that the effect of path loss, at a particular  $\sigma$ , suppresses the effect of K-factor. When we increase K for a certain  $\sigma$ , the probability of one-hop success decreases for low values of SNR margin. However, increase in K-factor reduces the occurrence of fade and act as a gain factor in the received signal but the signal becomes more vulnerable to the fact that only few number of nodes are actively cooperating in transmission because of higher path loss, which decrease the success probability. The increase in the SNR margin reduces the path loss and we get better one-hop probability for large values of K. Furthermore, to see more clearly the effect of path loss exponent in the presence of shadowing and fading, we acquired results for different deployment of nodes,  $L = 2$  and  $W = 3$ , as shown in Fig. 4. It can be seen that even for low SNR margins we have better one-hop success probability as compared to the case where we have two nodes at the tailing end.

In Fig. 5 we show results for different K-factors at various SNR margins by keeping  $\sigma = 10dB$ . For a particular value of SNR margin, as we increase the value of K the probability of one-hop success also increases. This shows that for  $K=0$ , when there is no LOS component in our received signal, we need more SNR margin to reach same  $\rho$  as compared to higher values of K. Also the increase in path loss exponent,  $\beta$ , from 2 to 3 requires almost 5dB more SNR margin for the same value of K.

To enhance the performance of the network we need to control certain parameter like distance between nodes and their transmit power. The probability of end-to-end successful delivery of a message without going into absorbing state is defined by a certain quality of service (QoS) parameter,  $\eta$  i.e. if we need to reach a certain distance with 90% QoS then

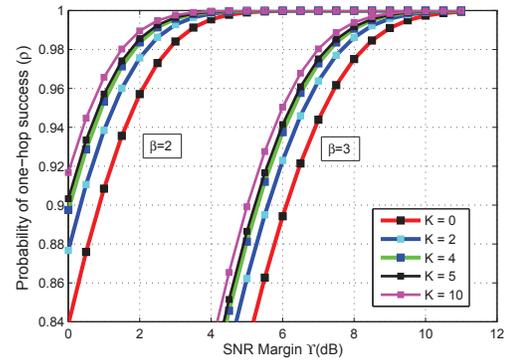


Fig. 5: Success probability for different value of K-factor

$\eta \cong 0.9$ . Since we know that, the propagation of a message for the maximum number of hops,  $h$ , until the killing occurs is given by,  $\rho^h$ . So maximum value of hop count is bounded, i.e.,  $\rho^h \geq \eta$ , which gives the maximum number of hops as  $h \geq \frac{\ln \eta}{\ln \rho}$ . If we take the product of number of hops with the number of nodes in horizontal dimension,  $L$ , it will give us the coverage range or maximum distance a message can go with the QoS. In Fig. 6, the normalized number of hops versus SNR margin for the topology of nodes in Fig. 1, is shown for varying  $\eta$ . We can see that an increase in the value of  $\eta$  reduces the number of hops, which is quite intuitive as we increase the QoS for same SNR margin, the probability of successful delivery also reduces.

In Table I, we quantify the coverage range for different arrangements of nodes while varying K-factor and keeping  $\sigma = 10dB$  fixed for specific SNR margins of  $\Upsilon = \{2, 5, 10\}dB$  to achieve a QoS of 90%. The cases considered are:

- CASE A  $L = 6, W = 1$ ; CASE B  $L = 3, W = 2$   
CASE C  $L = 2, W = 3$ ; CASE D  $L = 1, W = 6$

Notice that as we increase the number of nodes at the tailing edge of the transmission window, we get a considerable increase in the maximum achievable distance, i.e., the message can propagate to far off destinations. Furthermore, it can be seen from Table I that an increase in K-factor also improves the coverage range of the network. However, in a 2-D context

TABLE I: Coverage range for different topologies at  $\eta = 0.90$

K-factor	CASE	$\Upsilon = 2$	$\Upsilon = 5$	$\Upsilon = 10$
K=5	A	0	6	708
	B	4	7230	2.09e+09
	C	6726	1.07e+07	6.05e+13
	D	9.96e+06	8.02e+10	1.42e+15
K=10	A	0	6	1122
	B	60	1.38e+07	6.85e+09
	C	1.31e+04	2.89e+07	3.34e+14
	D	2.91e+07	3.54e+11	2.84e+15

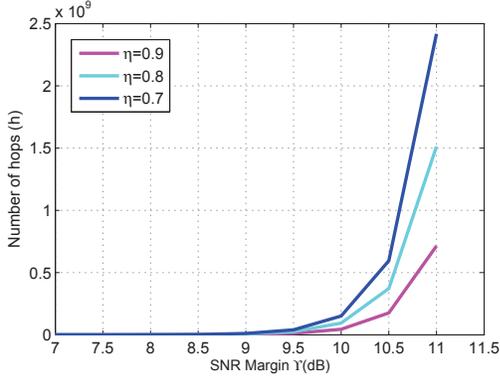


Fig. 6: Number of hops for various value of  $\eta$

there is a trade off, as an increase in the vertical dimension require a large number of nodes to cover the same region as compared to topology of nodes with larger horizontal stretch. To achieve high reliability, one can go with the arrangement in CASE D, whereas, to improve the latency or propagation delay, the better choice is CASE A.

## VI. CONCLUSION AND FUTURE WORK

In this paper, we have derived an analytical probability model for a 2-D network, where nodes cooperate with each other for transmission from one level to another, which is modeled as a quasi-stationary Markov chain. Corresponding to that, a transition matrix is derived by considering the channel as joint lognormal-Rice fading. An MGF-based technique along with Gauss-Hermite integration is used to find this joint distribution, where the sum of these lognormal-Rice RVs is approximated by a single lognormal RV. The effect on network's coverage area for some specific SNR margin required to achieve a certain QoS is quantified as a function of various parameters. For future this work can be extended for the 2-D random model, in which nodes are placed at random location.

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