The Benefit of Co-Locating Groups of Nodes in Cooperative Line Networks

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***Abstract*—We consider two topologies for the deployment of nodes in a one-dimensional network. The first deployment scenario considers nodes equally spaced on a line, while the second topology has groups of co-located nodes, such that the groups are equally spaced on the line, and such that the two networks have equal average density. In both linear topologies, nodes transmit cooperatively, as opportunistic large arrays, in each hop. The difference is only that in the first topology, the cooperators have disparate path loss, while in the second they do not. We study the multi-hop transmission for both cases, where the one hop distance remains the same for both the topologies. We model both the scenarios with a quasi-stationary Markov chain and show that the co-located groups deployment gives better performance, especially for higher path loss exponents.**

***Index Terms*—Cooperative transmission, stochastic modeling, wireless sensor networks.**

I. INTRODUCTION

MULTI-hop transmission is desirable to keep the cost down in large wireless sensor networks, by employing fewer gateway nodes. However, multi-hop networks suffer from long delays and poor reliability. In order to boost the performance of multi-hop networks, cooperative transmission strategies can be applied to increase the reliability of the system with better latency properties. A promising technique employing multi-hop transmission is the opportunistic large array (OLA) [1], where all nodes that can correctly decode a message immediately retransmit it at the same time, without coordination with other relays, thereby providing transmit diversity. OLA broadcasting is known to have advantages such as range extension [2], fewer hops, and energy efficiency [3]. There are many uncertainties that influence exactly which radios participate in an OLA. Most previous works use the *continuum* assumption, where the number of nodes in a given area goes to infinity keeping the total transmit power per unit area constant [4]. However, practical OLA networks have a finite density of nodes, and the finite density analysis differs considerably with that of asymptotic analysis under the continuum assumption. In [5], the authors theoretically analyzed a one-dimensional network with finite density, where the inter-node distance is kept constant. They derived upper bounds on the coverage of this network under the effects of path loss and Rayleigh fading.

In this paper, we characterize the performance of two different topologies for a line network. These topologies can be considered a precursor to a strip shaped network or a uni-cast cooperative route for the finite density case. Typical applications consistent with strip networks include structural health monitoring of bridges, fault recognition system in transmission lines, and sensors employed in hallways of buildings in a linear fashion. The topology would also be consistent with a plastic communication cable, in which small wireless relays are embedded along a cable made of a non-conducting material [6]. Such *plastic wires* might find applications in areas of high electric fields.

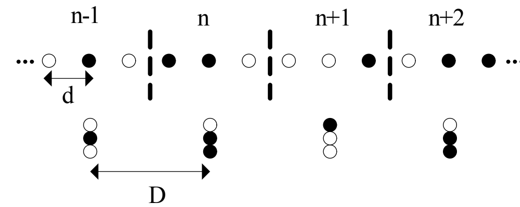

Fig. 1. Equi-distant and co-located topologies in line network.

In some applications, the equi-distant node topology, as in the top part of Fig. 1, might be attractive, owing to the distributed nature of sensors that can monitor a large area at many different locations. However, the cooperating nodes in this topology will necessarily have disparate path loss, leading possibly to a lower effective diversity order. Therefore, we consider a second topology in which each set of cooperating nodes are placed in a co-located group as shown in the bottom of Fig. 1. Because of size and cost constraints, having an antenna array on a single node is not practical. Thus, cooperative diversity, a realization of a multi-antenna array using a cluster of single-antenna devices, can be used as a solution for mitigating fading in wireless multi-hop networks. With a set of transmitters spaced at least a half-wavelength apart, we can establish a cooperative link to the receiver, which receives independent copies of source signals through orthogonal fading channels and tries to decode the message. The diversity in this system can be obtained through space time coding (STC) or using frequency diversity by transmitting the message over orthogonal frequencies. Although a "hop" in our scheme is strictly a multiple-input multiple-output (MIMO) link because it involves simultaneous transmission between one cluster of antennas to another cluster of antennas, this scheme is different from a conventional MIMO scheme as there is no joint detection process from all these receivers, nor can we predict with certainty about the number of transmitters in any particular hop.

To fairly compare the two topologies, we restrict the groups of *candidates* for cooperation in a given hop to have the same number of nodes and have the same centroid, as shown in Fig. 1. Therefore, the only difference between the two topologies

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is that the cooperating nodes in equi-distant topology have disparate path losses, while cooperating nodes in the co-located groups topology do not. Our results will show that the co-located groups topology always performs better, but the magnitude of improvement depends on the system and channel parameters. We model the transmissions in both the cases with a quasi-stationary Markov chain with an absorbing state, where the absorbing state describes all receivers in a hop not being able to decode and the transmissions stop propagating.

II. Network Model

For the equi-distant nodes topology, consider a line of nodes where adjacent nodes are a distance d apart from one another as shown in the top of Fig. 1. The nodes are partitioned, as illustrated by the vertical dashed lines in Fig. 1, into levels or groups of M nodes each. Each group is indexed as $n-1, n, n+1, \ldots$ etc. We assume that the nodes transmit synchronously in OLAs or levels, and that a hop occurs when nodes in one level, e.g., n, transmit a message and at least one node in the next level, e.g., $n+1$, is able to decode the message. Exactly one time slot later, all the nodes that just decoded the message for the first time relay the message. Correct decoding is assumed when a node's received signal-to-noise ratio (SNR), from the previous level only, is greater than or equal to a modulation-dependent threshold, τ. We assume that there is sufficient transmit synchronization between the nodes of a level, such that all the nodes in a level transmit to the next level at the same time. In other words, the transmissions only occur at discrete instants of time n, $n+1$, ... such that the hop number and the time instants can be defined by just one index n.

We assume equal transmit power, P_t, of all nodes. Let $\mathbb{N}_n := \{1, 2, \ldots, \mathcal{K}_n\}$, where $\mathcal{K}_n$ is the cardinality of the set $\mathbb{N}_n$, $\sup_n \mathcal{K}_n \le M$, to be the set of indices of those decode-and-forward (DF) nodes that decoded the signal perfectly at the time instant (or hop) n. For example, from Fig. 1, $\mathbb{N}_n = \{1, 2\}$ and $\mathbb{N}_{n+1} = \{3\}$. The received power at the jth node at the next time instant $n+1$ is given by

$$Pr_j(n+1) = \frac{P_t}{d^\beta} \sum_{m \in \mathbb{N}_n} \frac{\mu_{mj}}{(M-m+j)^\beta}, \tag{1}$$

where μ_{mj} is the exponential channel gain (corresponding to flat Rayleigh fading) from Node m in the previous level to Node j in the current level and β is the path loss exponent.

For the co-located groups topology, a group consists of M co-located nodes, such that the inter-group distance is $D \approx Md$, and hence the inter-node distance in each group can be ignored. Thus, the received power at the jth node at time $n+1$ is given as $Pr_j(n+1) = \frac{P_t}{D^\beta} \sum_{m \in \mathbb{N}_n} \mu_{mj}$.

A. Simplified One-Hop Analysis without Fading

In this subsection, we consider a simplified model that enables the application of Jensen's Inequality, in an effort to provide some insight into the results derived in the next section for the more complex model. Suppose for the nth hop, we are given that exactly two transmitters are active, but we don't know which ones they are. To simplify analysis, let us assume that the transmitters are placed independently in any position within the cluster with equal probability; therefore they could be co-located (with half wavelength spacing). Furthermore, suppose we select a receiver at random from the receiver cluster in this hop. Let ξ_i be the distance between the transmitter i and the receiver. In the equi-distant case, ξ_i is a random variable with $\mathbb{E}\{\xi\} = D$. Next assume there is no fading or $\mu_{mj} = 1$. Then under these assumptions and conditions, the distances are the only random variables and the expected received power at the randomly selected receiver from both transmitters is $\mathbb{E}\{Pr_j(n+1)\} = 2P_t \mathbb{E}\left\{\frac{1}{\xi^\beta}\right\}$. In the co-located topology, the distance is non-random, so under these conditions and assumptions, $\mathbb{E}\{Pr_j(n+1)\} = Pr_j(n+1) = \frac{2P_t}{D^\beta} = \frac{2P_t}{(\mathbb{E}\{\xi\})^\beta}$. Using Jensen's Inequality,

$$\frac{1}{(\mathbb{E}\{\xi\})^\beta} \le \mathbb{E}\left\{\frac{1}{\xi^\beta}\right\}. \tag{2}$$

In words, the average power accumulated at a point from distributed nodes is higher than from a group of nodes located at the centroid position. However, in the presence of fading, the outage probability (i.e., the probability that the power drops below a threshold) is more important than the mean power received. It can be shown (not here, because of space constraints) that with Rayleigh fading, the outage probability, $\mathbb{P}(Pr_j(n+1) \le \tau)$, is lower (i.e., "better") for the co-located case than for the equi-distant case, because small values of $Pr_j(n+1)$ are more likely to happen when there are path loss disparities between the independently faded copies. The analysis in the next sections treats the more general complicated case, which includes Rayleigh fading, does not allow two transmitters in the same place for the equi-distant topology, and for which the number of transmitters is random. This analysis will prove that the co-located case gives a lower outage probability than the equi-distant case.

III. The Statistical Model

The state of each node is characterized by a binary indicator function such that for jth node at time n, $\mathbb{I}_j(n) = 1$ represents successful decoding and $\mathbb{I}_j(n) = 0$ represents a failure in decoding. Thus the decision of all nodes in Level n are given as $\mathcal{X}(n) = [\mathbb{I}_1(n), \mathbb{I}_2(n), \ldots, \mathbb{I}_M(n)]$, which depends only upon the transmission of the previous level, making $\mathcal{X}$ is a memoryless Markov process. Due to discrete time slots, $\mathcal{X}$ is the discrete-time Markov chain such that $\mathcal{X} = \{0\} \cup \mathcal{S}$, where $\mathcal{S}$ is a finite transient irreducible state space, corresponding to all the states in which at least one node in the group is able to decode, and 0 is the absorbing state. If we remove the transitions to and from the absorbing state, then the transition probability matrix, $\mathbf{P}$, for the Markov chain, $\mathcal{X}$, is right sub-stochastic and irreducible with dimension $(2^M - 1) \times (2^M - 1)$. Considering these properties, we invoke the Perron-Frobenius theorem [7], which says that there exists a maximum eigenvalue, ρ, and an associated left eigenvector $\mathbf{u}$ with strictly positive entries such that $\mathbf{uP} = \rho\mathbf{u}$. Since $\forall n$, $\mathbb{P}\{\mathcal{X}(n) = 0\} > 0$, eventual absorption is certain, and the limiting distribution, also called the quasi-stationary distribution, of the Markov chain is given as $\lim_{n\to\infty} \mathbb{P}\{\mathcal{X}(n) = j | T > n\} = u_j, \quad j \in \mathcal{S}$, where $T = \inf\{n \ge 0 : \mathcal{X}(n) = 0\}$ denotes the end of survival time.

A. The Transition Probability Matrix

For the kth node in the nth level, the conditional probability of being able to decode is given as

$$\mathbb{P}\left\{\mathbb{I}_k(n) = 1|\zeta\right\} = \mathbb{P}\left\{\gamma_k(n) > \tau|\zeta\right\}, \quad (3)$$

where $\gamma_k(n) = Pr_k(n)/\sigma_k^2$ is the received SNR at kth node, and σ_k^2 is the noise variance of the kth receiver. We denote the event $\{\mathcal{X}(n-1) \in \mathcal{S}\} := \zeta$, implying that the previous state is a transient state. Similarly, the probability of outage or the probability of $\mathbb{I}_k(n) = 0$ is $1 - \mathbb{P}\left\{\gamma_k(n) > \tau|\zeta\right\}$, where

$$\mathbb{P}\left\{\gamma_k(n) > \tau|\zeta\right\} = \int_{\tau}^{\infty} p_{\gamma_k|\zeta}(y)dy, \quad (4)$$

and $p_{\gamma_k|\zeta}(y)$ is the conditional probability density function (PDF) of the received SNR at the kth node conditioned on State $\mathcal{X}(n-1)$. Here, we divide our analysis into two subsections, one for each of the topologies.

1) Transition Matrix for the Equi-Distant Topology: The received power at a certain node is the sum of the finite powers from the previous-level nodes, each of which is exponentially distributed with parameter λ_k, $k = 1, 2, ..., \mathcal{K}_n$. Thus the PDF of received power in this case is given by hypoexponential distribution [8], which is valid only if λ_k $\forall k$ are different. For M nodes in a level, consider the index sets corresponding to the ith state at time n as
$\mathbb{N}_n^{(i)} = \{1, 2, ..., \mathcal{K}_n\}$ and $\overline{\mathbb{N}}_n^{(i)} = \{1, 2, ..., M\} \setminus \mathbb{N}_n^{(i)}$,
to be the sets of those nodes which are 1 and 0, respectively. Then the one step transition probability from States i to j is given as

$$\mathbb{P}_{ij} = \prod_{k \in \mathbb{N}_{n+1}^{(j)}} \left(\psi_m^{(k)}\right) \prod_{k \in \overline{\mathbb{N}}_{n+1}^{(j)}} \left(1 - \psi_m^{(k)}\right), \quad (5)$$

where

$$\psi_m^{(k)} = \sum_{m \in \mathbb{N}_n(i)} C_m^{(k)} \exp\left(-\lambda_m^{(k)}\tau\right),$$

$$\lambda_m^{(k)} = \frac{d^{\beta}(M-m+k)^{\beta}\sigma_m^2}{P_t}, \quad \text{and} \quad C_m^{(k)} = \prod_{\zeta \neq m} \frac{\lambda_\zeta^{(k)}}{\lambda_\zeta^{(k)} - \lambda_m^{(k)}}.$$

2) Transition Matrix for Co-Located Groups Topology: In this case, the received power at a certain node in a group is the sum of the finite powers from the previous-level nodes, where the power received from each transmitting node is exponentially distributed with the same parameter $\tilde{\lambda} = D^{\beta}\sigma_k^2/P_t$. Since all the nodes are co-located, and there are no disparate path losses that affect the parameter of the exponential distribution, the PDF of the received power at the kth node in a cluster is Gamma distribution [8] given as

$$p_{\gamma_k|\zeta}(y) = \frac{1}{(\mathcal{K}_n - 1)!}\tilde{\lambda}^{\mathcal{K}_n} y^{(\mathcal{K}_n - 1)} \exp\left(-\tilde{\lambda}y\right). \quad (6)$$

Evaluating (4) to get the conditional success of the kth node, we have

$$\mathbb{P}\left\{\gamma_k(n) > \tau|\zeta\right\} = \frac{1}{(\mathcal{K}_n - 1)!}\Gamma(\mathcal{K}_n, \tilde{\lambda}\tau), \quad (7)$$

where $\Gamma(\mathcal{K}_n, \tilde{\lambda}\tau)$ is the upper incomplete Gamma function. Let $\Phi^{(k)} := \mathbb{P}\left\{\gamma_k(n) > \tau|\zeta\right\}$, then after some manipulation, (7) becomes

$$\Phi^{(k)} = \exp\left(-\tilde{\lambda}\tau\right) \sum_{p=0}^{\mathcal{K}_n - 1} \frac{\left(\tilde{\lambda}\tau\right)^p}{p!}. \quad (8)$$

Then the one step transition probability for going from State i to j is same as given in (5) with $\psi_m^{(k)}$ replaced with $\Phi^{(k)}$.

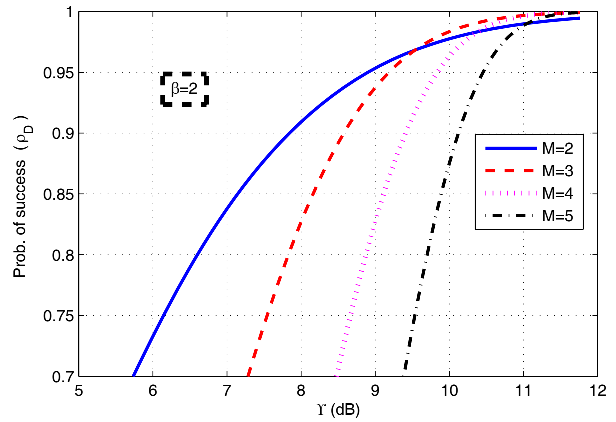

Fig. 2. Behavior of eigenvalues in the co-located topology.

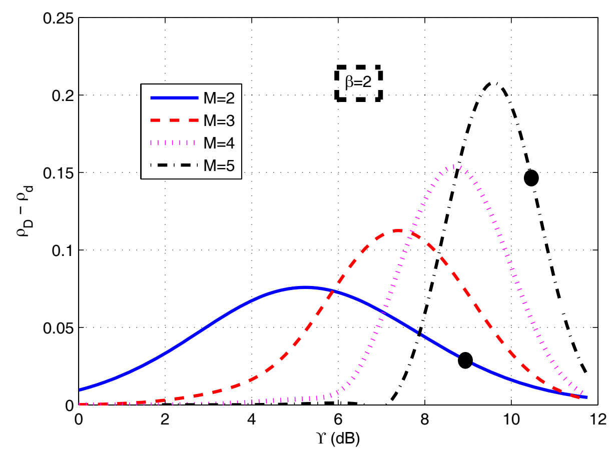

Fig. 3. Eigenvalue differences between two topologies; $\beta = 2$.

IV. Results and Performance Analysis

In this section, we show the relative performance of the two topologies in terms of the one-step success probability of making a successful hop, which indicates that at least one node in the forward level has decoded the message successfully. This success probability, which is the Perron-Frobenius eigenvalue of the matrix, depends upon many parameters such as transmit power, inter-node or inter-group distance, path loss exponent, etc. Thus infinite solutions exist of the quasi-stationary distribution. To reduce the design space, we define $\Upsilon = \frac{P_t}{\tau\sigma^2}$ as the normalized SNR with respect to the threshold τ and call this the SNR margin. Note that in the simulation results, we have used $d = 1$, which implies that Υ, in the equi-distant topology, can be thought of as the SNR margin from a single transmitter d distance away.

We denote the one step success probability for the equi-distant topology as ρ_d and for the co-located groups topology

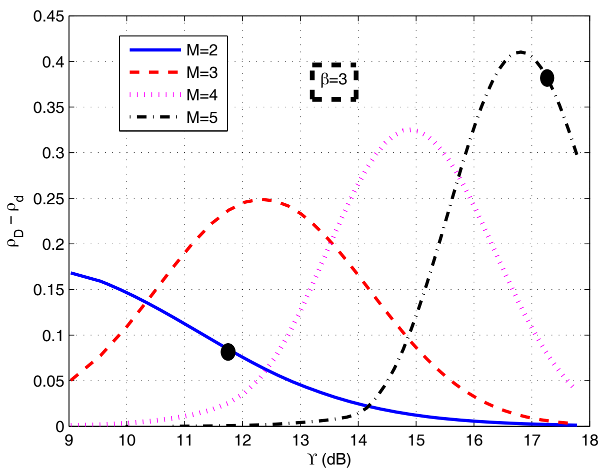

Fig. 4. Eigenvalue differences between two topologies; $\beta = 3$.

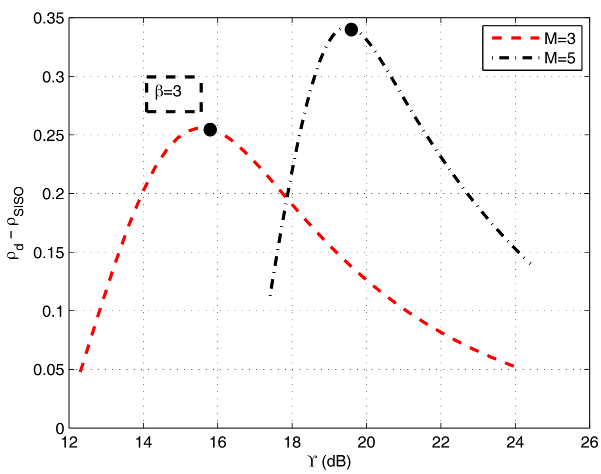

Fig. 5. Eigenvalue differences between equi-distant and SISO topologies.

as ρ_D. Fig. 2 shows the behavior of ρ_D as a function of Υ for a path loss exponent of 2. It can be observed that for a specific cluster size, the success probability increases monotonically with the increase in SNR margin. It can be further noticed that if we increase the cluster size, an additional SNR margin is required to get the same success probability as that of a smaller sized cluster. This is because by increasing the cluster size, the inter-group distance also increases, which requires more SNR margin to get the same quality of service.

Fig. 3 shows the difference between the success probabilities of co-located and equi-distant topologies for the path loss exponent of 2. We observe that the maximum difference increases as we increase M. However, this difference dominates at some specific SNR margin values. For instance, if we require 95% success probability for $M = 2$ in a co-located case, then from Fig. 2 we require $\Upsilon = 8.9dB$. However, from Fig. 3, we notice that at this SNR margin, the equi-distant topology also performs almost the same since $\rho_D - \rho_d \approx 0.027$ as indicated by the black circle. For the same packet delivery ratio for $M = 5$, the co-located case requires $\Upsilon = 10.45dB$, however the difference in success probabilities for the two cases is more significant at 0.1485 at this SNR margin value. At very high SNR margin, e.g., 12dB, the performance of both the topologies is again the same, because the path loss effects are diminished with high transmit power and the partition constraint. For the larger path loss exponent of $\beta = 3$, Fig. 4 shows $\rho_D - \rho_d$, where the black circles show the 95% success probability for the co-located topology. We observe a larger difference between the two topologies, especially for the rightmost dot, which indicates that for $M = 5$, the co-located case has 0.95 probability of success, while the equally spaced case has only 0.57 probability of success. We attribute this difference to the large differences in path loss among the (up to) 5 equally spaced transmitters.

The topologies that we discussed above are both cooperative. However, it is also interesting to note the performance of one of the cooperative topologies, for instance, the equi-distant topology as compared to single-input single-output (SISO) topology. In particular, we compare the results with the configuration in which a single node is located in the centroid of each group and transmits with its power adjusted according to the expected number of successful nodes in the other configurations of equi-distant topology. Fig. 5 shows $\rho_d - \rho_{SISO}$, where ρ_{SISO} is the one-step success probability for the SISO topology. It can be seen that the cooperative topology performs better on all the values of SNR margin and the performance margin for increasing the number of nodes in equi-distant topology, (which is same as increasing the distance in SISO topology with increased average power) is larger. The black circles represent 95% success probability for the equi-distant topology.

V. Conclusion

We considered two different topologies for deploying a one-dimensional sensor network where the nodes can be equi-distant from one another or they can be combined to form co-located groups. We derived the stochastic models for the transmissions for both topologies and showed that the co-located topology always outperforms the equi-distant topology, and the performance difference is larger for larger path loss exponents.

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