A Full Rate Symmetrical Cooperative Relay Approach for Wireless Systems

Peter S. S. Wang*, Syed Ali Hassan† and Ye (Geoffery) Li†

Abstract—**Cooperative relaying methods have attracted a lot of interest in the past several years. A conventional cooperative relaying scheme has a source, a destination and a single relay. Since it can extend the coverage of wireless systems, this cooperative technique can support one symbol transmission per time slot, called full rate transmission. However, existing full rate cooperative relay approaches are non-symmetrical for different symbols. In this paper, we propose a cooperative relaying scheme that is assisted with dual relays and provides full transmission rate and the same macro-diversity to each symbol.**

I. Introduction

Wireless channels are always proned to multipath fading, which causes the level of received signal to vary with time and location. Different diversity techniques have been developed to enhance the performance of the overall system. The most commonly used diversity techniques are time, frequency and spatial diversity [1,2]. In addition to these techniques, a novel idea to use intermediate nodes as relays has been presented to exploit the inherent broadcast nature of the wireless networks. By transmitting through different fading channels simultaneously, there are chances that one or more channels do not fade deeply at the any time and as a consequence, the effect of multipath fading will be reduced.

A lot of work has been done for systems having a single cooperative node operating as relay as shown in Figure 1 [3-6]. Designing such a cooperative system results in two main advantages: increasing the spatial diversity at the receiver and attaining rate gains [10]. The drawback of this system is the asymmetrical transmission of data. By asymmetry, we imply that the diversity gain is not applied to every transmitted symbol thus the performance is only partially improved. In this paper we propose a novel approach to cooperative diversity employing dual relays to improve system performance. The proposed scheme not only provides full transmission rate but also has balanced diversity for each transmitted symbol. Two approaches are commonly used in cooperative relaying scenarios. The first is known as *amplify-and-forward* transmission (AF), where the relay amplifies the received signal and forwards it to the receiver [8]. The second approach is *decode-and-forward* transmission (DF) where relay decods the incoming signal first and then re-encodes and sends to the destination. In this paper, we use the AF mode and try to improve performance over the conventional cooperative relaying with single relay and direct transmission between a source and destination.

The paper is organized as follows. Section II presents the basic architecture and functionality of systems with direct transmission, cooperative relaying with single relay and then with dual relays. Section III provides a more deep insight into the mathematical modeling of the proposed scheme and MMSE linear equalizer design for the dual relay transmission. Finally, in Section IV, we illustrate the SNR gains for our proposed cooperative network. The paper then concludes with certain comments in Section V.

II. System Models

In this section, we will describe a direct transmission scenario with a single source and destination, a conventional one relay cooperative scheme, and our proposed scheme for dual relays.

A. Direct Transmission

Consider direct transmission of a symbol from a source to destination. The source transmits data at every instant and the received symbol at the destination is given by

$$y(k) = \alpha h x(k) + n(k) \tag{1}$$

where α is the path loss coefficient and h is the channel coefficient assumed to be complex Gaussian with Rayleigh distribution. Noise n is additive white Gaussian noise with zero mean and σ^2 variance. The received signal is passed through a linear equalizer [1-2] to obtain the estimate of the transmitted symbol.

B. One Relay Transmission

A conventional cooperative relaying scheme is shown in Figure 1 consisting of a source, S, destination, D, and a single relay, R. The channel coefficient corresponding to source-destination and source-relay are denoted by h_{SD} and h_{SR}, respectively, while h_{SD} is the equivalent channel corresponding to relay-destination. The functionality of this relay scheme is shown in Table I.

This relay scheme works as follows. At every odd time slot, the source transmits a symbol and both relay and destination receives the transmitted symbol. At even time slot, the relay forwards whatever it receives to the destination,the source transmits another symbol and the received signal at the destination is the superposition of transmitted signals from the source and the relay. This cooperative diversity technique can support one symbol transmission per time slot, i.e., full rate

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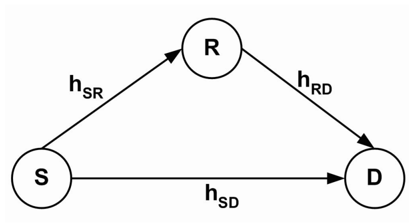

Fig. 1. Conventional one relay system

TABLE I
TRANSMISSION SCHEDULING FOR ONE RELAY CASE

Time index	$2k-1$	$2k$
S	$x(2k-1)$	$x(2k)$
R	*(listening)* $x_R(2k-1) = \alpha h_{SR}x(2k-1)$ + $n_R(2k-1)$	*(sending)* $\beta x_R(2k-1)$
D	$y(2k-1) = h_{SD}x(2k-1)$ + $n_D(2k-1)$	$y(2k) = \beta h_{RD}x_R(2k-1)$ + $h_{SD}x(2k) + n_D(2k)$

transmission. However, it is non-symmetrical transmission for different symbols.

The factor β in Table I normalizes the received symbol energy at the relay and is given as

$$\beta = \sqrt{\frac{1}{\alpha|h_{SR}|^2 + \sigma_R^2}} \tag{2}$$

where σ_R^2 is the noise variance at the relay. In summary, the effective input-output relation for a single relay cooperative scheme in AF mode is given by

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n} \tag{3}$$

where **y** ,**x** and **n** are the received data vector, transmitted data vector, and the noise vector, respectively, defined as

$$\mathbf{y} = [y(2k-1) \;\; y(2k)]^T$$

$$\mathbf{x} = [x(2k-1) \;\; x(2k)]^T$$

$$\mathbf{n} = [n_D(2k-1) \;\; \beta h_{RD}n_R(2k-1) + n_D(2k)]^T$$

and the equivalent channel matrix, **H** can be expressed as

$$\mathbf{H} = \begin{bmatrix} h_{SD} & 0 \\ \alpha\beta h_{SR}h_{SD} & h_{SD} \end{bmatrix} \tag{4}$$

The non-diagonal representation of the equivalent channel matrix explains the non-symmetric behavior of the network for symbols at different time slots.

C. Dual Relay Transmission

In order to provide a balanced diversity gain to all received symbols, we propose a symmetrical cooperative relaying scheme, which overcomes the non-symmetrical transmission for different symbols in the conventional cooperative relaying. As shown in Figure 2, the symmetrical cooperative relaying scheme consists of the source, S, destination, D, and a pair of relay stations R_1 and R_2. It can provide full data transmission rate and balanced gain to each transmitted symbol. As before, h_{pq} is the channel coefficient from transmitter p to receiver q and α's are the pathloss coefficients.

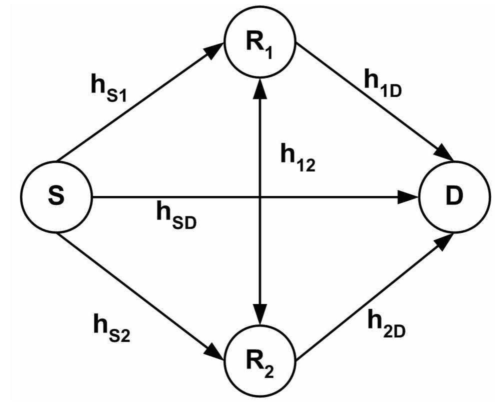

Fig. 2. System model with dual relays

The transmitted and received signals of this cooperative relaying scheme are described in Table II. The source is always sending the data to destination at every time slot. At odd time slot, one relay, for example R_1, is also transmitting what it received at the previous time slot, while the other relay, in this example R_2, is listening to what the source and the relay R_1 are sending. The destination receives the signal transmitted by the source and by one of the relays, in this example R_1 at every odd time slot. At even time slot, the role of the two relays will be inverted and the destination now receives data from source and relay R_2.

Through out the paper, we assume Rayleigh fading channel, no channel knowledge at the transmitters, perfect channel state information at the receivers and perfect synchronization.

III. MATHEMATICAL MODELING OF THE SYSTEM

This section is devoted to express the proposed system in terms of analytical expressions and the derivation of MMSE linear equalizer (LE), which is different from the MMSE-LE for traditional communication systems.

A. Input-Output Relationship

From Table II, the received signals at the destination can be expressed as

$$\begin{aligned} y(2k-1) =& \sqrt{E_R}h_{2D}\alpha_{2D}\beta_{R_2}(2k-2)x_{R_2}(2k-2) + \\ & \sqrt{E_s}h_{SD}x(2k-1) + n_D(2k-1), \end{aligned} \tag{5}$$

$$\begin{aligned} y(2k) =& \sqrt{E_R}h_{1D}\alpha_{1D}\beta_{R_2}(2k-1)x_{R_1}(2k-1) + \\ & \sqrt{E_s}.h_{SD}x(2k) + n_D(2k), \end{aligned} \tag{6}$$

where E_s is the transmit power of the source and E_R is the transmit power of relays. We assume the transmit power of relays to be equal. $x_{R_1}(2k-1)$ and $x_{R_2}(2k-2)$ are the signals received by the relay R_1 at time $2k-1$ and by the relay R_2

TABLE II
TRANSMISSION SCHEDULING FOR DUAL RELAY CASE

Time index	$2k-1$	$2k$
Source	$x(2k-1)$	$x(2k)$
*Relay*1	*(listening)* $x_{R_1}(2k-1) = \alpha_{S1}\sqrt{E_s}h_{S1}x(2k-1)$ $+\alpha_{12}\sqrt{E_R}\beta_{R_2}(2k-2)h_{12}x_{R_2}(2k-2)+n_{R_1}(2k-1)$	*(sending)* $x_{R_1}(2k-1)$
*Relay*2	*(sending)* $x_{R_2}(2k-2)$	*(listening)* $x_{R_2}(2k) = \alpha_{S2}\sqrt{E_s}h_{S2}x(2k)$ $+\alpha_{12}\sqrt{E_R}\beta_{R_1}(2k-1)h_{12}x_{R_1}(2k-1)+n_{R_2}(2k)$
Destination	$y(2k-1) = \sqrt{E_s}h_{SD}x(2k-1)$ $+\alpha_{2D}\sqrt{E_R}\beta_{R_2}(2k-2)h_{2D}x_{R_2}(2k-2)+n_D(2k-1)$	$y(2k) = \sqrt{E_s}h_{SD}x(2k)$ $+\alpha_{1D}\sqrt{E_R}\beta_{R_1}(2k-1)h_{1D}x_{R_1}(2k-1)+n_D(2k)$

at time $2k-2$ respectively. Those signals are defined as

$$\begin{aligned} x_{R_2}(2k-2) =& \sqrt{E_s}\alpha_{S2}h_{S2}x(2k-2)+ \\ & \sqrt{E_R}h_{12}\alpha_{12}x_{R_1}(2k-3).\beta_{R_1}(2k-3)+ \\ & n_{R2}(2k-2), \end{aligned} \tag{7}$$

$$\begin{aligned} x_{R_1}(2k-1) =& \alpha_{S1}\sqrt{E_s}h_{S1}x(2k-1)+ \\ & \sqrt{E_R}h_{12}\alpha_{12}x_{R_2}(2k-2)\beta_{R_2}(2k-2)+ \\ & n_{R1}(2k-1), \end{aligned} \tag{8}$$

The factor α_{1D} comprises of effects of pathloss and shadowing, and is normalized with respect to the distance between source and destination i.e. d_{SD}. The quantities α_{2D}, α_{S2} and α_{12} are also similar coefficients between relay R_2-destination, source-relayR_2 and relays R_1-R_2 respectively.

The factor $\beta_{R_1}(2k-1)$ normalizes the received symbol energy at the relay R_1 and is given as

$$\beta_{R_1}(2k-1) = \frac{1}{\sqrt{E\left\{|x_{R_1}(2k-1)|^2\right\}}} \tag{9}$$

where

$$\begin{aligned} E\left\{|x_{R_1}(2k-1)|^2\right\} =& \alpha_{S1}^2E_s|h_{S1}|^2+\sigma_{R_1}^2 \\ & +\alpha_{12}^2E_R|h_{12}|^2\left[\beta_{R_2}(2k-2)\right]^2 \end{aligned} \tag{10}$$

and $\sigma_{R_1}^2 = E|n_{R_1}|^2$. Similarly

$$\beta_{R_2}(2k-2) = \frac{1}{\sqrt{E\left\{|x_{R_2}(2k-2)|^2\right\}}} \tag{11}$$

Defining the transmitted and received symbol vector as

$$\mathbf{y}_k = \begin{bmatrix} y(2k-1) \\ y(2k) \end{bmatrix}, \quad \mathbf{x}_k = \begin{bmatrix} x(2k-1) \\ x(2k) \end{bmatrix},$$

the input-output relation of the dual relay transmission can be expressed in the matrix form as

$$\mathbf{y}_k = \mathbf{P}\mathbf{y}_{k-1}+\mathbf{Q}\mathbf{x}_k+\mathbf{R}\mathbf{x}_{k-1}+\mathbf{n}_k \tag{12}$$

where **P**, **Q**, **R**, and $\mathbf{n}_k$ and are given in the following equations. Here we omit the timing indices of the factors β and denote $\beta_{R_i}(2k-1)$ as β_{R_i} for $i = 1,2$. Thus

$$\mathbf{P} = \begin{bmatrix} 0 & \frac{\sqrt{E_R}\alpha_{1D}\beta_{R_2}h_{2D}h_{12}}{h_{1D}} \\ 0 & E_R\alpha_{12}^2\beta_{R_1}\beta_{R_2}h_{12}^2 \end{bmatrix},$$

$$\mathbf{Q} = \begin{bmatrix} \sqrt{E_s}h_{SD} & 0 \\ \sqrt{E_s}\sqrt{E_R}\alpha_{1D}\alpha_{S1}\beta_{R_1}h_{S1}h_{1D} & \sqrt{E_s}h_{SD} \end{bmatrix},$$

$$\mathbf{R} = \begin{bmatrix} 0 & r_{12} \\ 0 & r_{22} \end{bmatrix}$$

with

$$r_{12} = \sqrt{E_s}\sqrt{E_R}\alpha_{2D}\beta_{R_1}h_{2D}\left(\alpha_{S2}h_{S2}-\frac{\alpha_{12}h_{SD}h_{12}}{\alpha_{1D}h_{1D}}\right)$$

$$r_{22} = \sqrt{E_s}E_R\alpha_{12}\beta_{R_1}\beta_{R_2}h_{12}h_{1D} \left(\alpha_{1D}\alpha_{S2}h_{S2}-\frac{\alpha_{12}h_{SD}h_{12}}{h_{1D}}\right)$$

and noise is given as

$$\mathbf{n}_k = \mathbf{n}_{D_k}-\mathbf{N}_D\mathbf{n}_{D_{k-1}}+\mathbf{N}_R\mathbf{n}_{R_k} \tag{13}$$

where $\mathbf{N}_D = \begin{bmatrix} 0 & \frac{\sqrt{E_s}\alpha_{12}\alpha_{2D}\beta_{R_2}h_{2D}h_{12}}{\alpha_{1D}h_{1D}} \\ 0 & E_R\alpha_{12}^2\beta_{R_1}\beta_{R_2}h_{12}^2 \end{bmatrix}\mathbf{n}_{D_{k-1}}$,

$$\mathbf{N}_R = \begin{bmatrix} \sqrt{E_R}\alpha_{2D}\beta_{R_2}h_{2D} & 0 \\ E_R\alpha_{2D}\alpha_{1D}\beta_{R_1}\beta_{R_2}h_{1D}h_{12} & \sqrt{E_R}\alpha_{1D}\beta_{R_1}h_{1D} \end{bmatrix},$$

$$\mathbf{n}_{D_k} = \begin{bmatrix} n_D(2k-1) \\ n_D(2k) \end{bmatrix}, \quad \mathbf{n}_{R_k} = \begin{bmatrix} n_R(2k-1) \\ n_R(2k) \end{bmatrix},$$

where $\mathbf{n}_D$ and $\mathbf{n}_R$ are noise coefficients at destination and relays, respectively and defined as complex Gaussian variables.

B. MMSE-LE Design

Concerning the determination of the MMSE for the linear equalizer, we consider the scheme as shown in Figure 3

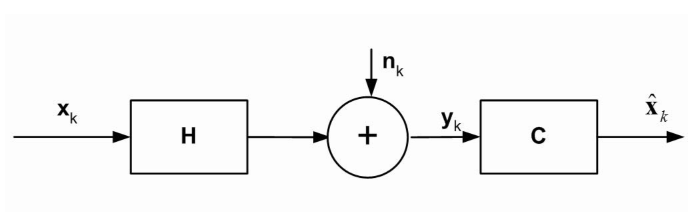

Fig. 3. Transmission model incorporating equalizer

In the block diagram, $\mathbf{x}_k$ and $\mathbf{y}_k$ denotes the 2×1 input and output signal vector, respectively, and $\mathbf{H}$ is the impulse response of the channel. $\mathbf{C}$ is the equalizer response and $\hat{\mathbf{x}}_k$ is the estimated symbol vector. Omitting the timing index k, the MMSE is defined as

$$\Im = \mathbf{E}\{\| \hat{\mathbf{x}} - \mathbf{x} \|^2\} \tag{14}$$

$$\Im = \mathbf{E}\{(\hat{\mathbf{x}} - \mathbf{x})\,(\hat{\mathbf{x}} - \mathbf{x})^H\} \qquad \text{with } \hat{\mathbf{x}} = \mathbf{C}\,\mathbf{y} \tag{15}$$

$$\Im = \mathbf{C}\,\mathbf{R_y}\,\mathbf{C}^H - \mathbf{C}\,\mathbf{E}\{\mathbf{y}\mathbf{x}^H\} - \mathbf{E}\{\mathbf{x}\mathbf{y}^H\}\,\mathbf{C}^H + \mathbf{R_x} \tag{16}$$

$\mathbf{R_x}$ and $\mathbf{R_y}$ are the autocorrelation function of transmitted and received signals, respectively, where $\mathbf{R_x} = \mathbf{I}$ and $\mathbf{R_y}$ is given as

$$\begin{aligned} \mathbf{R_y} &= \mathbf{E}\{\mathbf{y}\mathbf{y}^H\} \quad \text{with} \quad \mathbf{y} = \mathbf{H}\cdot\mathbf{x} + \mathbf{n} \\ &\qquad \text{as } \mathbf{x} \text{ and } \mathbf{n} \text{ are uncorrelated} \\ &= \mathbf{H}\,\mathbf{E}\{\mathbf{x}\mathbf{x}^H\}\,\mathbf{H}^H + \mathbf{R_n} \\ \mathbf{E}\{\mathbf{y}\mathbf{x}^H\} &= \mathbf{H}\,\mathbf{R_x} \\ \mathbf{E}\{\mathbf{x}\mathbf{y}^H\} &= \mathbf{R_x}\mathbf{H}^H \end{aligned}$$

$\mathbf{R_n} = \mathbf{E}\{\mathbf{n}\mathbf{n}^H\}$ is the autocorrelation function of noise and is given as

$$\mathbf{R_n} = \sigma_R^2 \mathbf{N}_R \mathbf{N}_R^H + \sigma_D^2 \left(\mathbf{I} - \mathbf{N}_D \mathbf{N}_D^H\right)$$

Thus MMSE becomes

$$\begin{aligned} \Im = &\left(\mathbf{C} - \mathbf{R_x}\mathbf{H}^H\mathbf{R_y}^{-1}\right)\,\mathbf{R_y}\,\left(\mathbf{C} - \mathbf{R_x}\,\mathbf{H}^H\mathbf{R_y}^{-1}\right)^H \\ &+ \mathbf{R_x}\,\left(\mathbf{I} - \mathbf{H}^H\mathbf{R_y}^{-1}\mathbf{H}\mathbf{R_x}\right) \end{aligned} \tag{17}$$

Our goal is to minimize the above error equation. Thus finding the minima of error using stochastic gradient technique, $\mathbf{C}$ is given as

$$\mathbf{C} - \mathbf{R_x}\mathbf{H}^H\mathbf{R_y}^{-1} = 0 \tag{18}$$

which corresponds to

$$\mathbf{C} = \mathbf{R_x}\mathbf{H}^H\mathbf{R_y}^{-1} \tag{19}$$

Thus plugging equation 19 in equation 17, the following expression for $\Im$ turns out:

$$\Im = \mathbf{R_x}\left(\mathbf{I} - \mathbf{H}^H\mathbf{R_y}^{-1}\mathbf{H}\mathbf{R_x}\right) \tag{20}$$

Using matrix inversion lemma, the MMSE-LE can be found as

$$\Im = \mathbf{R_x} - \left(\mathbf{R_x}^{-1} + \mathbf{H}^H\mathbf{R_n}^{-1}\mathbf{H}\right)^{-1}\mathbf{H}^H\mathbf{R_n}^{-1}\mathbf{H}\mathbf{R_x} \tag{21}$$

As can be noticed from input output relationship of equation (12) that we have to retrieve four unknowns with only two equations. Thus the system is ill-conditioned. That is why, we assume that we know $\mathbf{x}_{k-1}$ for the first time. Consequently the equation we are considering now is given as

$$\mathbf{z}_k = \mathbf{y}_k - \mathbf{P}\mathbf{y}_{k-1} + \mathbf{R}\mathbf{x}_{k-1} = \mathbf{Q}\mathbf{x}_k + \mathbf{n}_k \tag{22}$$

Thus assuming the above linear model, **H=Q** and the equalizer from equation (19) is given as

$$\mathbf{C} = \mathbf{Q}^H\left(\mathbf{Q}\mathbf{Q}^H + \mathbf{R}_n\right)^{-1} \tag{23}$$

Based on the above mathematical analysis, the designed receiver is used to detect the transmitted symbols where dual relay transmission provides a balanced diversity gain to each symbol. Therefore, the system performance should be the best as compared to conventional cooperative relaying.

IV. Simulation Parameters and Results

This section gives the simulation results of different transmission scenarios that we have discussed in the previous sections. In all cases, the transmitted signal is a random bipolar sequence modulated through quadrature phase-shift keying (QPSK) modulation. Symbols are transmitted through different channels that are all considered to be Rayleigh fading with zero mean and unit variance. For all cases, there is an average power constrained on the transmit power. We assume the transmit power to be fixed and the noise variance is changed for relays and destination to give a defined SNR value. This is because we assume that we do not have any channel state information at the transmitter so we can not apply waterfilling criteria to vary the transmit power according to the fading coefficients. The definition of noise in direct transmission is straight forward and has been identified in Table I for one relay transmission. For the dual relay transmission the noise $\mathbf{n}_k$ is defined as in equation (13).

However, we can address the issue whether the noise, $\mathbf{n}_k$, is still a Gaussian variable. $\mathbf{n}_k$ is a function of $\mathbf{n}_D$ and $\mathbf{n}_R$ that are Gaussian variables but it is not a linear combination of these coefficients. However considering the central limit theorem which states that sum of two random variables with arbitrary probability density functions (pdfs) results in another random variable whose pdf tends to be a Gaussian one. Thus $\mathbf{n}_k$ here can be assumed to be Gaussian.

Figure 4 shows the simulation results for the above stated simulation parameters. There are three SNR vs BER curves for three different relay schemes for a Rayleigh fading channel. We use 20,000 realization of data with 10 iterations to achieve a reliable result. It can be clearly seen that the performance is the best in the case of two relays and is considerably the worst in the case of point-to-point transmission. The case of one relay lies in between the two curves, as expected.

V. Conclusions and Recommendations

We have proposed a novel cooperative relaying scheme with dual relays and full rate transmission system. Based on the analysis and simulation results, this new scheme provides balanced diversity gain to each received symbol and thus enhance system performance over the conventional one relay scheme and point-to-point communication. If there are not enough dedicated relay stations between the source and destination nodes, we may utilize a relay-enabled mobile station or a base station acting as a relay station. We are working on the same scenario with optimum power allocation in the source and its associated relay transmissions which may increase the performance further.

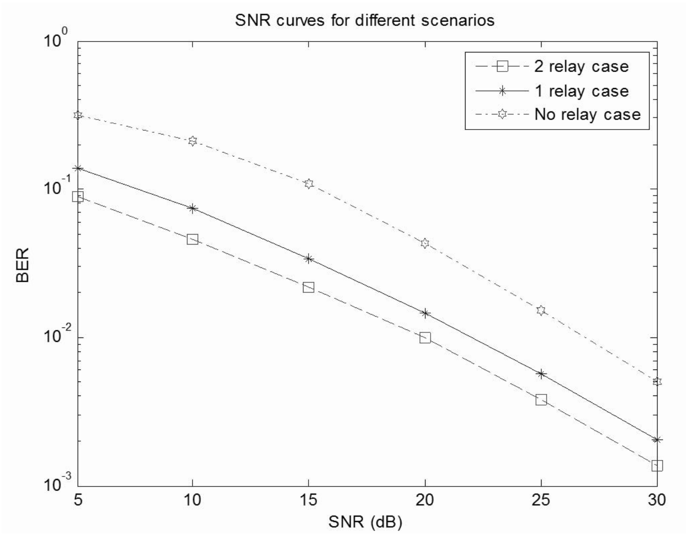

Fig. 4. Performance comparison of three scenarios (1) direct transmission without any relay, (2) the cooperative relaying scheme with one relay, (3) the cooperative relaying scheme with dual relays.

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