

# A Full Rate Dual Relay Cooperative Approach for Wireless Systems

Syed Ali Hassan, Geoffrey Ye Li, Peter Shu Shaw Wang, and Marilynn Wylie Green

**Abstract:** Cooperative relaying methods have attracted a lot of interest in the past few years. A conventional cooperative relaying scheme has a source, a destination, and a single relay. This cooperative scheme can support one symbol transmission per time slot, and is called full rate transmission. However, existing full rate cooperative relay approaches provide asymmetrical gain for different transmitted symbols. In this paper, we propose a cooperative relaying scheme that is assisted with dual relays and provides full transmission rate with the same macro-diversity to each symbol. We also address equalization for the dual relay transmission system in addition to addressing the issues concerning the improvement of system performance in terms of optimal power allocations.

**Index Terms:** Cooperative diversity, decision-feedback equalizer (DFE), dual relay transmission, minimum mean-squared error (MMSE), MMSE linear equalizer (LE), power allocation.

## I. INTRODUCTION

Cooperative diversity is an attractive technique in achieving higher system performance in terms of capacity and diversity gains in wireless systems. Exploiting the broadcast nature of wireless networks, relay nodes help the transmission of data through different channels, resulting in considerable improvement in system performance. Conventional cooperative strategies employ a single node as relay [1]–[5], which provides performance gain as compared to a direct transmission scenario. The relay station is usually half-duplex, that is, it can not transmit and receive data in the same time slot. As a result, the diversity gain is asymmetrical, that is, the diversity gain is not applied to every transmitted symbol and thus the performance is only partially improved. In this work, we use dual relays to achieve symmetrical diversity gain and recover the multiplexing loss.

A lot of work has been done on systems having a single cooperative node operating as relay as shown in Fig. 1 [1]–[5]. Some researchers focus on the receiver design to mitigate the effects of inter-symbol interference (ISI) and reduce the bit-error rate (BER) of transmission [6] while others focus on channel capacity and outage behaviors [3], [7]. From [8], designing a cooperative system may result in two potential advantages: Increasing

spatial diversity at the receiver and attaining high transmission rate. To overcome the asymmetrical nature of a single-relay network, there are several approaches, e.g., employing space-time block coding (STBC) at the relays and source [9]. Another emerging approach is the use of multiple relays in which more than one relay station help the source in transmission of data. The technique commonly known as relay selection in [10] and [11] provides better performance. More recently, a multiple relay approach with feedback is proposed in [12]. In this paper, we propose a novel and simple approach to cooperative diversity employing dual relays to improve system performance. The proposed scheme not only provides full transmission rate, keeping spectral efficiency, but also has balanced diversity for each transmitted symbol without using any STBC or feedback.

Power allocation also plays an important role in cooperative relaying [13]–[14]. We have shown that adjusting power to terminals according to signal-to-noise ratio (SNR) results in considerable improvement in system performance.

Two approaches are commonly used in cooperative relaying scenarios. The first is known as amplify-and-forward (AF) transmission, where the relay amplifies the received signal and forwards it to the receiver [8]. The second approach is decode-and-forward (DF) transmission where relay decodes the incoming signal first and then re-encodes and sends to the receiver. In this paper, we use the AF mode and try to improve performance over the conventional cooperative relaying with single relay and direct transmission between a source and destination.

The rest of the paper is organized as follows. Section II presents the basic architecture and function of systems with direct transmission, cooperative relaying with single relay and then with dual relays. Section III comprises the derivation of minimum mean-squared error (MMSE) linear equalizer (LE) and MMSE decision-feedback equalizer (DFE) for the dual relay transmission system. Finally, in Section IV, we illustrate the SNR gains for our proposed cooperative network and demonstrate the effects of power allocation. The paper then concludes with our comments in Section V.

## II. SYSTEM MODELS

In this section, we describe a direct transmission scheme with a single source and destination, a conventional one relay cooperative scheme, and our proposed scheme for dual relays.

### A. Direct Transmission

Consider direct transmission of a symbol from a source to destination. The source transmits a symbol at each time slot and the received symbol at the destination is given by

$$y_k = \sqrt{E_S} h x_k + n_k \quad (1)$$

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Table 1. Transmission scheduling for one relay case.

| Time index  | $2k - 1$   | $2k$  |
|-------------|--|---|
| Source      | $x(2k - 1)$  | $x(2k)$   |
| Relay       | (Listening)<br>$x_R(2k - 1) = \sqrt{E_S}h_{SR}x(2k - 1) + n_R(2k - 1)$ | (Sending)<br>$\sqrt{E_R}\beta x_R(2k - 1)$                                    |
| Destination | $y(2k - 1) = \sqrt{E_S}h_{SD}x(2k - 1) + n_D(2k - 1)$                  | $y(2k) = \sqrt{E_R}\beta h_{RD}x_R(2k - 1) + \sqrt{E_S}h_{SD}x(2k) + n_D(2k)$ |

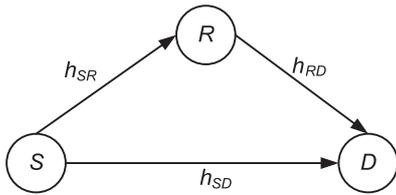


Fig. 1. One relay transmission system.

where  $E_S$  is the transmit power of source,  $h$  captures the effect of pathloss, shadowing, and frequency non-selective fading of wireless channels, and  $n_k$  is additive white Gaussian noise with zero mean and variance  $\sigma^2$ . The received signal is passed through a linear equalizer [15], [16] to detect the transmitted symbol.

### B. One Relay Transmission

A conventional cooperative relaying scheme is shown in Fig. 1 consisting of a source,  $S$ , destination,  $D$ , and a single relay,  $R$ . As before,  $h_{pq}$  denotes the gain of wireless channel from transmitter  $p$  to receiver  $q$ , which is assumed to be independent and identically distributed (i.i.d.) complex Gaussian. The transmitted and received signals of this scheme are shown in Table 1.

This relay scheme works as follows. At every odd time slot, the source transmits a symbol and both relay and destination receives the transmitted symbol. At even time slot, the relay forwards whatever it receives to the destination, the source transmits another symbol and the received signal at the destination is the superposition of transmitted signals from the source and the relay. This cooperative diversity technique can support one symbol transmission per time slot, i.e., full rate transmission. However, it is asymmetrical transmission for different symbols because diversity gain is applied only to the alternating transmitted symbols.

In Table 1,  $E_S$  and  $E_R$  denote the transmit powers of the source and the relay, respectively. The factor  $\beta$  normalizes the received symbol energy at the relay and is given as

$$\beta = \sqrt{\frac{1}{E_S|h_{SR}|^2 + \sigma_R^2}} \quad (2)$$

where  $\sigma_R^2$  is the noise variance at the relay. In summary, the effective input-output relation for a single relay cooperative

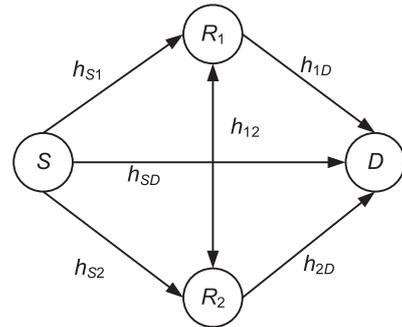


Fig. 2. Dual relay transmission system.

scheme in AF mode can be expressed as

$$\mathbf{y}_k = \mathbf{H}\mathbf{x}_k + \mathbf{n}_k \quad (3)$$

where  $\mathbf{y}_k$ ,  $\mathbf{x}_k$ , and  $\mathbf{n}_k$  are the received signal vector, transmitted signal vector, and the noise vector, respectively, defined as

$$\mathbf{y}_k = \begin{bmatrix} y(2k - 1) \\ y(2k) \end{bmatrix}, \quad \mathbf{x}_k = \begin{bmatrix} x(2k - 1) \\ x(2k) \end{bmatrix}, \quad (4)$$

$$\mathbf{n}_k = \begin{bmatrix} n_D(2k - 1) \\ \sqrt{E_R}\beta h_{RD}n_R(2k - 1) + n_D(2k) \end{bmatrix}. \quad (5)$$

$n_D$  and  $n_R$  denote the effects of receiver noise and other forms of interference at destination and relay, respectively. The equivalent channel matrix,  $\mathbf{H}$ , can be expressed as

$$\mathbf{H} = \begin{bmatrix} \sqrt{E_S}h_{SD} & 0 \\ \sqrt{E_S}\sqrt{E_R}\beta h_{SR}h_{SD} & \sqrt{E_S}h_{SD} \end{bmatrix}. \quad (6)$$

The expression of the channel matrix in (6) indicates the non-symmetric behavior of the system for symbols at different time slots.

### C. Dual Relay Transmission

In order to provide a balanced diversity gain to all transmitted symbols, we propose a symmetrical cooperative relaying scheme, which overcomes the asymmetrical transmission for different symbols in the conventional cooperative relaying. As shown in Fig. 2, the symmetrical relaying scheme consists of a source,  $S$ , a destination,  $D$ , and a pair of relay stations,  $R_1$  and  $R_2$ . As before,  $h_{pq}$  is the channel coefficient from transmitter  $p$  to receiver  $q$ .

Table 2. Transmission scheduling for dual relay case.

| Time index  | $2k - 1$  | $2k$  |
|-------------|---|---|
| Source      | $x(2k - 1)$   | $x(2k)$   |
| Relay 1     | (Listening)<br>$x_{R_1}(2k - 1) = \sqrt{E_S}h_{S1}x(2k - 1)$<br>$+\sqrt{E_R}h_{12}\beta_{R_2}x_{R_2}(2k - 2) + n_{R_1}(2k - 1)$ | (Sending)<br>$x_{R_1}(2k - 1)$  |
| Relay 2     | (Sending)<br>$x_{R_2}(2k - 2)$  | (Listening)<br>$x_{R_2}(2k) = \sqrt{E_S}h_{S2}x(2k)$<br>$+\sqrt{E_R}h_{12}\beta_{R_1}x_{R_1}(2k - 1) + n_{R_2}(2k)$ |
| Destination | $y(2k - 1) = \sqrt{E_S}h_{SD}x(2k - 1)$<br>$+\sqrt{E_R}h_{2D}\beta_{R_2}x_{R_2}(2k - 2) + n_D(2k - 1)$                          | $y(2k) = \sqrt{E_S}h_{SD}x(2k)$<br>$+\sqrt{E_R}h_{1D}\beta_{R_1}x_{R_1}(2k - 1) + n_D(2k)$                          |

The transmitted and received signals of this cooperative relaying scheme are described in Table 2. The source is always sending the data to the destination at every time slot. During an odd time slot, one relay, for example  $R_1$ , is also transmitting what it received during the previous time slot, while the other relay, in this example  $R_2$ , is listening to what the source and the relay  $R_1$  are sending. The destination receives the signal transmitted by the source and by one of the relays, in this example  $R_1$  at every odd time slot. During an even time slot, the role of the two relays will be inverted and the destination will now receive data from the source and relay  $R_2$ . Thus, the system can provide a full data transmission rate and balanced gain to each transmitted symbol. Throughout the paper, we assume a Rayleigh fading channel, perfect channel state information (CSI) available at the receivers, and perfect synchronization. We also assume perfect CSI at the transmitters for power allocation.

From Table 2, the received signals at the destination can be expressed as

$$y(2k - 1) = \sqrt{E_R}h_{2D}\beta_{R_2}x_{R_2}(2k - 2) + \sqrt{E_S}h_{SD}x(2k - 1) + n_D(2k - 1), \quad (7)$$

$$y(2k) = \sqrt{E_R}h_{1D}\beta_{R_1}x_{R_1}(2k - 1) + \sqrt{E_S}h_{SD}x(2k) + n_D(2k) \quad (8)$$

where  $E_S$  is the transmit power of the source,  $E_R$  is the transmit power of relays, and  $x_{R_1}(2k - 1)$  and  $x_{R_2}(2k - 2)$  are the signals received by the relay  $R_1$  at time  $2k - 1$  and by the relay  $R_2$  at time  $2k - 2$ , respectively. They can be expressed as

$$x_{R_2}(2k - 2) = \sqrt{E_R}h_{12}\beta_{R_1}x_{R_1}(2k - 3) \quad (9)$$

$$+\sqrt{E_S}h_{S2}x(2k - 2) + n_{R_2}(2k - 2),$$

$$x_{R_1}(2k - 1) = \sqrt{E_R}h_{12}\beta_{R_2}x_{R_2}(2k - 2) \quad (10)$$

$$+\sqrt{E_S}h_{S1}x(2k - 1) + n_{R_1}(2k - 1).$$

The factor  $\beta_{R_1}$  normalizes the received symbol energy at the relay  $R_1$  and is given as

$$\begin{aligned} \beta_{R_1} &= \frac{1}{\sqrt{E\{|x_{R_1}(2k - 1)|^2\}}} \\ &= \frac{1}{\sqrt{E_S|h_{S1}|^2 + E_R|h_{12}|^2 + \sigma_{R_1}^2}} \end{aligned} \quad (11)$$

where  $\sigma_{R_1}^2 = E\{|n_{R_1}|^2\}$ . Similarly,

$$\beta_{R_2} = \frac{1}{\sqrt{E_S|h_{S2}|^2 + E_R|h_{12}|^2 + \sigma_{R_2}^2}}. \quad (12)$$

Defining the transmitted and received symbol vectors as

$$\mathbf{x}_k = \begin{bmatrix} x(2k - 1) \\ x(2k) \end{bmatrix} \quad \text{and} \quad \mathbf{y}_k = \begin{bmatrix} y(2k - 1) \\ y(2k) \end{bmatrix}$$

the input-output relation of the dual relay transmission system can be expressed in the matrix form as

$$\mathbf{y}_k = \mathbf{P}\mathbf{y}_{k-1} + \mathbf{Q}\mathbf{x}_k + \mathbf{R}\mathbf{x}_{k-1} + \mathbf{n}_k \quad (13)$$

where  $\mathbf{P}$ ,  $\mathbf{Q}$ ,  $\mathbf{R}$ , and  $\mathbf{n}_k$  are given in the following equations:

$$\mathbf{P} = \begin{bmatrix} 0 & \frac{\sqrt{E_R}\beta_{R_2}h_{2D}h_{12}}{h_{1D}} \\ 0 & E_R\beta_{R_1}\beta_{R_2}h_{12}^2 \end{bmatrix},$$

$$\mathbf{Q} = \begin{bmatrix} \sqrt{E_S}h_{SD} & 0 \\ \sqrt{E_S}\sqrt{E_R}\beta_{R_1}h_{S1}h_{1D} & \sqrt{E_S}h_{SD} \end{bmatrix},$$

$$\mathbf{R} = \begin{bmatrix} 0 & \sqrt{E_S}\sqrt{E_R}\beta_{R_1}h_{2D}\left(h_{S2} - \frac{h_{SD}h_{12}}{h_{1D}}\right) \\ 0 & \sqrt{E_S}E_R\beta_{R_1}\beta_{R_2}h_{12}h_{1D}\left(h_{S2} - \frac{h_{SD}h_{12}}{h_{1D}}\right) \end{bmatrix},$$

and the noise vector is given as

$$\mathbf{n}_k = \mathbf{n}_{Dk} - \mathbf{N}_D\mathbf{n}_{Dk-1} + \mathbf{N}_R\mathbf{n}_{Rk} \quad (14)$$

where

$$\mathbf{N}_D = \begin{bmatrix} 0 & \frac{\sqrt{E_R}\beta_{R_2}h_{2D}h_{12}}{h_{1D}} \\ 0 & E_R\beta_{R_1}\beta_{R_2}h_{12}^2 \end{bmatrix},$$

$$\mathbf{N}_R = \begin{bmatrix} \sqrt{E_R}\beta_{R_2}h_{2D} & 0 \\ E_R\beta_{R_1}\beta_{R_2}h_{1D}h_{12} & \sqrt{E_R}\beta_{R_1}h_{1D} \end{bmatrix},$$

$$\mathbf{n}_{Dk} = \begin{bmatrix} n_D(2k - 1) \\ n_D(2k) \end{bmatrix}, \quad \mathbf{n}_{Rk} = \begin{bmatrix} n_R(2k - 1) \\ n_R(2k) \end{bmatrix}.$$

$\mathbf{n}_D$  and  $\mathbf{n}_R$  are the noise vectors at destination and relays, respectively, with i.i.d. complex Gaussian elements.

### III. RECEIVER DESIGN

This section focuses on equalization for the proposed system. The section starts with the design of a MMSE LE followed by MMSE DFE.

#### A. MMSE LE

The received signal vector  $\mathbf{y}_k$  is passed through a linear equalizer and the detected symbol vector is given as

$$\hat{\mathbf{x}}_k = \sum_{n=-N}^N \mathbf{C}_n \mathbf{y}_{k-n} \quad (15)$$

where

$$\mathbf{C}_n = \begin{bmatrix} c_{n0} & c_{n1} \\ c_{n2} & c_{n3} \end{bmatrix}. \quad (16)$$

Thus, (15) can be written in matrix notation as

$$\hat{\mathbf{x}}_k = \mathbf{C}^H \mathbf{y} \quad (17)$$

where  $\mathbf{C}$  is  $(4N + 2) \times 2$  matrix and  $\mathbf{y}$  is  $(4N + 2) \times 1$  column vector. Introducing the error autocorrelation matrix  $\mathbf{R}_{ee} = E \{ \mathbf{e}_k \mathbf{e}_k^H \}$ , the MSE is given as

$$\mathfrak{S} = E \{ |\hat{\mathbf{x}}_k - \mathbf{x}_k|^2 \}. \quad (18)$$

The coefficient matrix of the equalizer turns out to be the solution of Wiener filter [16] given as

$$\mathbf{C} = \mathbf{R}_y^{-1} \mathbf{R}_{yx}, \quad (19)$$

and the resulting MMSE becomes

$$\mathfrak{S}_{\min} = tr [\mathbf{R}_x - \mathbf{R}_{yx}^H \mathbf{R}_y^{-1} \mathbf{R}_{yx}]. \quad (20)$$

The dimensions of the auto-correlation matrix  $\mathbf{R}_y$  and cross-correlation matrix  $\mathbf{R}_{yx}$  are  $(4N+2) \times (4N+2)$  and  $(4N+2) \times 2$ , respectively. The cross-correlation matrix is given as

$$\mathbf{R}_{yx} = E \{ [\mathbf{y} \mathbf{x}_k^H] \} = \begin{bmatrix} \mathbf{r}_{yx}(-N) \\ \vdots \\ \mathbf{r}_{yx}(N) \end{bmatrix} \quad (21)$$

where each element  $\mathbf{r}_{yx}(n)$  is a  $2 \times 2$  matrix for  $-N \leq n \leq N$  and is given as

$$\mathbf{r}_{yx}(n) = \begin{cases} \mathbf{P}^{-n-1} (\mathbf{P}\mathbf{Q} + \mathbf{R}), & -N \leq n \leq -1, \\ \mathbf{Q}, & n = 0, \\ \mathbf{0}, & \text{otherwise.} \end{cases} \quad (22)$$

The autocorrelation of the received signal is given as

$$\mathbf{R}_y = E \{ \mathbf{y} \mathbf{y}^H \} = \begin{bmatrix} \mathbf{r}_y(-N, -N) & \cdots & \mathbf{r}_y(-N, N) \\ \vdots & \ddots & \vdots \\ \mathbf{r}_y(N, -N) & \cdots & \mathbf{r}_y(N, N) \end{bmatrix} \quad (23)$$

where each element  $\mathbf{r}_y(n, m)$  is a  $2 \times 2$  matrix for  $-N \leq n, m \leq N$ , and is given as in (24). The cross-correlations matrices  $\mathbf{L}$ ,  $\mathbf{S}$ ,  $\mathbf{M}$ , and  $\mathbf{T}$  are again  $2 \times 2$  given as

$$\mathbf{L}(n, m) = \begin{cases} \mathbf{Q}, & m = n, \\ \mathbf{P}^{m-n-1} (\mathbf{P}\mathbf{Q} + \mathbf{R}), & m > n, \\ 0, & \text{otherwise,} \end{cases}$$

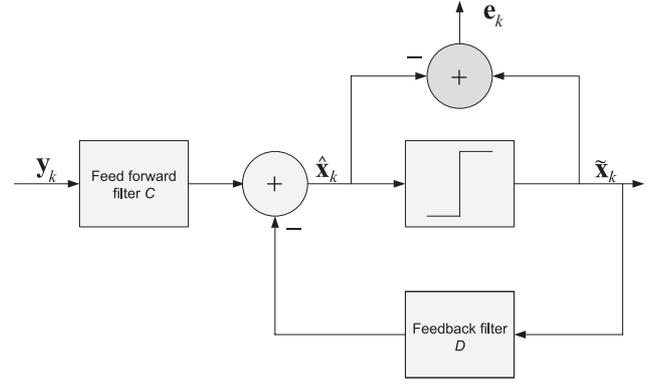


Fig. 3. Block diagram of decision feedback equalizer.

$$\mathbf{S}(n, m) = \begin{cases} \mathbf{Q}, & m = n + 1, \\ \mathbf{P}^{m-n-2} (\mathbf{P}\mathbf{Q} + \mathbf{R}), & m > n + 1, \\ 0, & \text{otherwise.} \end{cases}$$

$\mathbf{M}$  and  $\mathbf{T}$  are the Hermitian equivalent of  $\mathbf{L}$  and  $\mathbf{S}$ , respectively, for  $n$  replaced with  $m$ . The autocorrelations of the transmitted signal,  $\mathbf{R}_x$ ,  $\tilde{\mathbf{R}}_x$ , and  $\hat{\mathbf{R}}_x$ , are identity matrices for  $m = n$ ,  $m = n - 1$ , and  $m = n + 1$ , respectively. The noise variance is given as

$$\sigma_{m-n}^2 = \begin{cases} \sigma_R^2 \mathbf{N}_R \mathbf{N}_R^H + \sigma_D^2 (\mathbf{I} - \mathbf{N}_D \mathbf{N}_D^H), & m = n, \\ 0, & \text{otherwise} \end{cases} \quad (25)$$

where  $\sigma_R^2$  and  $\sigma_D^2$  are the variances of noise at relays and destination, respectively.

#### B. MMSE DFE

This subsection describes the design of a MMSE DFE for the dual relay system. The block diagram of the system is shown in Fig. 3. From Fig. 3, the MSE is defined as

$$\mathfrak{S} = E \{ |\tilde{\mathbf{x}}_k - \hat{\mathbf{x}}_k|^2 \} \quad (26)$$

where  $\tilde{\mathbf{x}}_k$  are the detected symbols which are used for the interference cancellation caused from the previous symbols. Thus, feedback filter (FBF),  $D$ , helps mitigate the ISI caused from post-cursor samples. Let the length of FIR filters for feed-forward filter (FFF) and FBF are  $N_1$  and  $N_2$ , respectively, the MSE is given as

$$\mathfrak{S} = E \left\{ \left| \tilde{\mathbf{x}}_k - \left[ \sum_{n=-N_1}^0 \mathbf{C}_n \mathbf{y}_{k-n} - \sum_{m=1}^{N_2} \mathbf{D}_m \tilde{\mathbf{x}}_{k-m} \right] \right|^2 \right\}. \quad (27)$$

In matrices form

$$\mathfrak{S} = E \left\{ \left( \tilde{\mathbf{x}}_k - \mathbf{C}^H \mathbf{Y} + \mathbf{D}^H \tilde{\mathbf{x}}_p \right) \left( \tilde{\mathbf{x}}_k - \mathbf{C}^H \mathbf{y} + \mathbf{D}^H \tilde{\mathbf{x}}_p \right)^H \right\}. \quad (28)$$

Here  $\mathbf{C}$  and  $\mathbf{D}$  are coefficient matrices with dimensions  $(2N_1 \times 2)$  and  $(2N_2 \times 2)$ , respectively, and  $\mathbf{y}$ ,  $\tilde{\mathbf{x}}_p$ , and  $\tilde{\mathbf{x}}_k$  are column vectors with dimensions  $(2N_1 \times 1)$ ,  $(2N_2 \times 1)$ , and  $(2 \times 1)$ , respectively. The vector,  $\tilde{\mathbf{x}}_p$ , contains the previous detected symbols which are assumed to be correct. The responses of the FFF

$$\begin{aligned} \mathbf{r}_y(n, m) = & \mathbf{P}\mathbf{r}_y(k-1)\mathbf{P}^H + \underbrace{\mathbf{Q}E\{\mathbf{x}_{k-n}\mathbf{x}_{k-m-1}^H\}}_{\tilde{\mathbf{R}}_x}\mathbf{R}^H + \underbrace{\mathbf{P}E\{\mathbf{y}_{k-n-1}\mathbf{x}_{k-m-1}^H\}}_{\mathbf{L}}\mathbf{R}^H + \underbrace{\mathbf{P}E\{\mathbf{y}_{k-n-1}\mathbf{x}_{k-m}^H\}}_{\mathbf{S}}\mathbf{Q}^H + \mathbf{Q}\mathbf{R}_x\mathbf{Q}^H \\ & + \underbrace{\mathbf{R}E\{\mathbf{x}_{k-n-1}\mathbf{y}_{k-m-1}^H\}}_{\mathbf{M}}\mathbf{P}^H + \mathbf{R}\mathbf{R}_x\mathbf{R}^H + \underbrace{\mathbf{Q}E\{\mathbf{x}_{k-n}\mathbf{y}_{k-m-1}^H\}}_{\mathbf{T}}\mathbf{P}^H + \underbrace{\mathbf{R}E\{\mathbf{x}_{k-n-1}\mathbf{x}_{k-m}^H\}}_{\tilde{\mathbf{R}}_x}\mathbf{Q}^H + \sigma_{m-n}^2\mathbf{I}. \end{aligned} \quad (24)$$

and FBF are given as

$$\mathbf{C} = (\mathbf{R}_y - \Phi\Phi^H)^{-1}\mathbf{R}_{yx}, \quad (29)$$

$$\mathbf{D} = \Phi^H\mathbf{C} \quad (30)$$

where  $\mathbf{R}_y$  and  $\mathbf{R}_{yx}$  are given in (24) and (21), respectively. Assuming correct decisions in DFE and thus denoting  $\tilde{\mathbf{x}}_p$  as  $\mathbf{x}$ ,  $\Phi$  is given as  $\Phi = \mathbf{E}[\mathbf{y}\mathbf{x}]$ . The dimensions of  $\Phi$  are same as  $\mathbf{R}_y$ , i.e.,  $(4N+2) \times (4N+2)$  and has the same form as in (23) where each element  $\phi \in \Phi$  is a  $2 \times 2$  matrix given as

$$\begin{aligned} \phi(n, m) = & \mathbf{E}[\mathbf{y}_{k-n}\mathbf{x}_{k-m}^H] \\ = & \underbrace{\mathbf{P}\mathbf{E}\{\mathbf{y}_{k-n-1}\mathbf{x}_{k-m}^H\}}_{\tilde{\mathbf{S}}}\mathbf{Q}^H + \underbrace{\mathbf{R}\mathbf{E}\{\mathbf{x}_{k-n-1}\mathbf{x}_{k-m}^H\}}_{\tilde{\mathbf{R}}_x}. \end{aligned} \quad (31)$$

The indices  $n$  and  $m$  are given as  $-N_1 \leq n \leq 0$  and  $1 \leq m \leq N_2$ .  $\tilde{\mathbf{S}}$  and  $\tilde{\mathbf{R}}_x$  are variants of  $\mathbf{S}$  and  $\mathbf{R}_x$  as defined in the previous subsection. MMSE in this case is given as

$$\mathfrak{S}_{\min} = \text{tr} \left[ \mathbf{R}_x - \mathbf{R}_{yx}^H (\mathbf{R}_y - \Phi\Phi^H)^{-1} \mathbf{R}_{yx} \right]. \quad (32)$$

#### IV. SIMULATION RESULTS

This section presents the performance of the cooperative dual relay system over conventional one relay and direct transmission scenarios. We will also describe the effects of power allocation between source and relay nodes and the channel capacity of the proposed system. In all cases, the transmitted signal is a random bipolar sequence modulated through quadrature phase-shift keying (QPSK). Symbols are transmitted through different channels that are all considered to be Rayleigh fading with zero mean and unit variance. For all cases, there is an average total transmit power constraint.

##### A. Comparison of BER

Fig. 4 compares the BERs for all cases discussed in Section II. The MMSE LE is used for the dual relay system. It can be seen that the dual relay system performs best as compared to the other two scenarios with the same simulation parameters i.e. terminals power, channel coefficients, and noise coefficients etc. Fig. 5 compares LE and DFE for equal number of taps in both equalizers. It can be seen that the normalized MSE is further reduced in case of DFE because of the cancellation of post-cursor interference. At low SNR, LE performs better because of error propagation in DFE but at high SNR we achieve substantial performance improvement for DFE in terms of MSE.

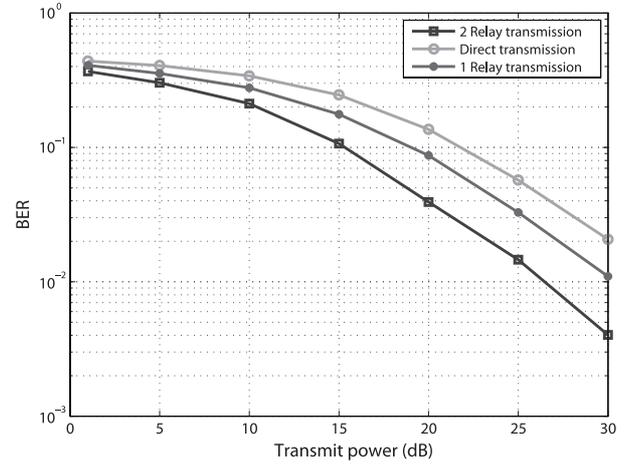


Fig. 4. Comparison of three scenarios.

##### B. Power Allocations

An important issue in AF relaying is that during the transmission of data from relay, the amplifying factor and the power gain of relay enhances not only the signal but also the noise. Thus, power control of relays especially at low SNR becomes critical. Another factor is the channel between the two relays. An interesting case arises because of the inter-relay channel conditions, i.e., at high inter-relay channel gain, the noise gets enhanced in addition to the signal. Thus, it is necessary to control the power according to the channel to get an optimal performance. Fig. 6 shows the effect of power allocations for the dual relay system for both high and low inter-relay channel gains. Power is distributed to the source and relays so that the following equality holds

$$E_0 = \underbrace{\alpha E_0}_{E_S} + \underbrace{(1-\alpha)E_0}_{E_R} \quad \alpha \in (0, 1) \quad (33)$$

where  $E_0$  is the total transmit power of the system. For *without PA* case, equal power is allocated to both the source and relays, i.e.,  $\alpha = 1/2$ . It can be seen that the system performs better with PA at the terminals in each case. It can further be noticed that high inter-relay channel gain necessarily requires power control to limit the noise propagating back and forth between the relays. During optimal power allocation, it has been observed that at low SNR, more power should be allocated to source and vice versa as shown in Fig. 7. Notice that for smaller values of  $E_0$  (upper two curves), the MSE drops down as the ratio of source to relay power increases, i.e., more power is allocated to the source as compared to relays. Similarly for high values of  $E_0$ , this phenomenon is reversed. Thus, it can be seen

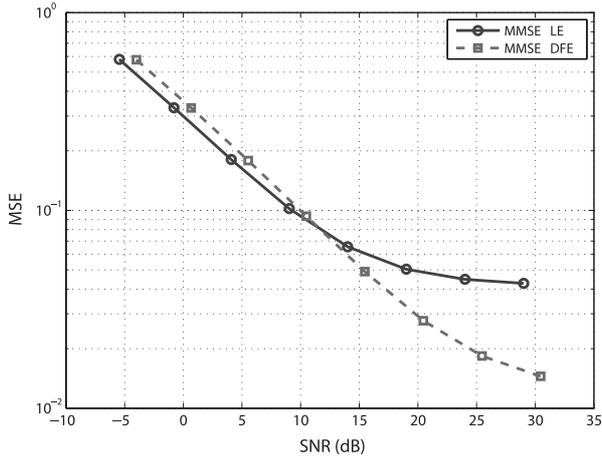


Fig. 5. Comparison of MMSE LE and MMSE DFE.

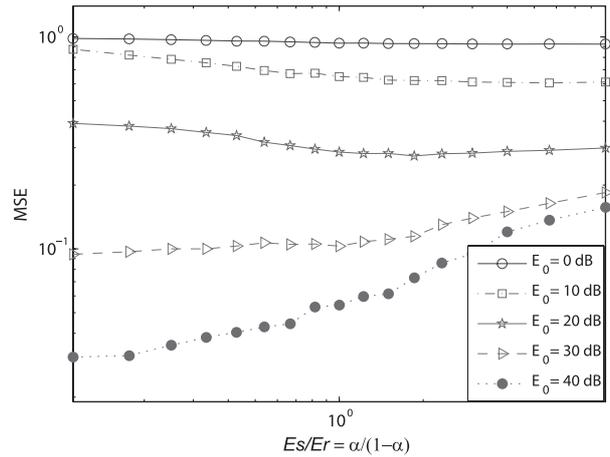


Fig. 7. Effect of source to relay power ratio on MSE.

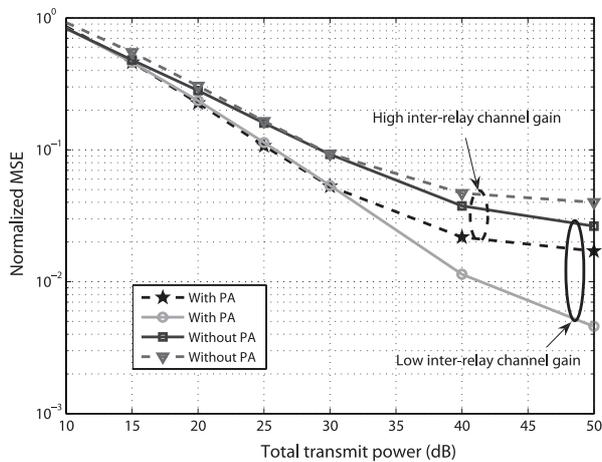


Fig. 6. Effect of power allocation.

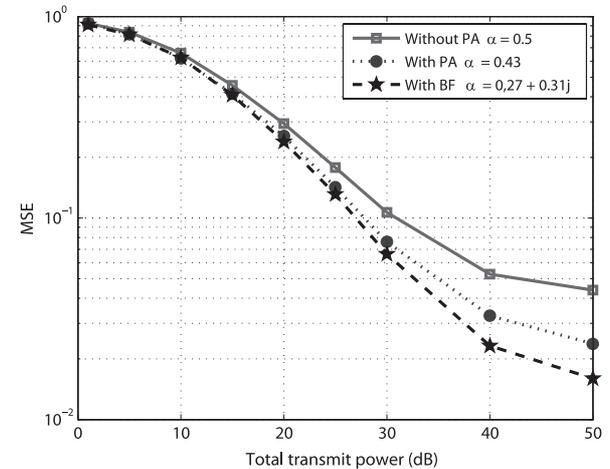


Fig. 8. Effect of beamforming on MSE.

that the network performs better and allocating more power to relays at high SNR results in a considerable improvement.

To further improve the performance of the system, we can replace the constant magnitude  $\alpha$ , in (33), with a complex number, which in general can be done since we assume perfect CSI at the transmitters. By using this so-called beamforming (BF) phenomenon, we chose the value of  $\alpha$ , which gives the least MSE thus reducing the error rate of the system as can be seen from Fig. 8.

**V. CONCLUSIONS AND RECOMMENDATIONS**

We have proposed a novel cooperative relaying scheme with dual relays and full rate transmission capabilities. Based on the analysis and simulation results, this new scheme provides balanced diversity gain to each transmitted symbol and thus enhance system performance over the conventional one relay scheme and point-to-point communication. Since the AF mode is used, power allocations at the relays is necessary for robust transmission and to control the propagating noise from one terminal to other. In short, AF mode performs better at high SNR

because of low noise propagation and less complexity as compared to the DF mode of operation. Hence, it can be argued that DF mode at low SNR would work better as compared to the AF mode and vice versa. An adaptive AF-DF mode can be utilized for the dual relay transmission for further system performance.

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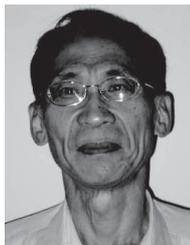
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