A Dual Relay Transmission System for Wireless Communications: Power Allocations and Channel Capacity

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Abstract

Cooperative relaying methods have attracted a lot of interest in the past several years. A conventional cooperative relaying scheme has a source, destination and a single relay. To overcome the effects of asymmetrical nature of one relay network, we have proposed a dual relay system that achieves significant system performance in terms of bit-error rate, signal-to-noise ratio and channel capacity over the conventional one relay and direct transmission systems. This paper presents the protocol of the dual relay system and addresses the issues of optimal power allocations at the terminals and the increase of system performance in terms of channel capacity.

1. Introduction

Cooperative diversity is an attractive technique in achieving higher system performance in terms of capacity and diversity gains in wireless systems. Exploiting the broadcast nature of wireless networks, relay nodes help the transmission of data through different channels, resulting in considerable improvement in system performance. Conventional cooperative strategies employ a single node as relay [1-6], which provides performance gain as compared to a direct transmission scenario. The relay station is usually half-duplex, that is, it can not transmit and receive data in the same time slot. Due to half-duplex characteristic of the relay, the diversity gain is asymmetrical which results in a loss of spectral efficiency. By asymmetry, we imply that the diversity gain is not applied to every transmitted symbol and thus the performance is only partially improved. Another approach to cater for this problem is the use of dual relays in the communication model which provides balanced diversity gain to each transmitted symbol [7]. In this work, we demonstrate that the use dual relays achieve higher channel capacity and the performance is improved using optimal power allocations at the terminals.

Power allocations also play an important role in cooperative relaying [8-9]. We have shown that adjusting power to terminals according to signal-to-noise ratio (SNR), results in considerable improvement in system performance. Analyzing the system as a whole, we also measure the channel capacity as compared to conventional and direct transmission system and show the increase in capacity for dual relay transmission system (DRTS). Two approaches are commonly used in cooperative relaying scenarios. The first is known as *amplify-and-forward* transmission (AF), where the relay amplifies the received signal and forwards it to the receiver [1]. The second approach is *decode-and-forward* transmission (DF) where relay decodes the incoming signal first and then re-encodes and sends to the destination. In this paper, we use the AF mode and try to improve performance over the conventional cooperative relaying with single relay and direct transmission between a source and destination.

The paper is organized as follows. Section 2 presents the basic architecture, functionality and mathematical model of the dual relay transmission system. Sections 3 and 4 provide detailed analysis for the power allocations at the terminals and the increase in channel capacity for DRTS. The paper then concludes with certain comments in Section 5.

2. System Model

Figure 1 shows the symmetrical cooperative relaying scheme consisting of a source, S, destination, D, and a pair of relay stations R_1 and R_2. h_{pq} captures the effect of pathloss, shadowing and frequency non-selective fading from transmitter p to receiver q. The fading is assumed to be independently and identically distributed (i.i.d.) drawn from a set of complex Gaussian elements. This scheme works as follows. The source is always sending data to destination at every time slot. At odd time slot, one relay, for

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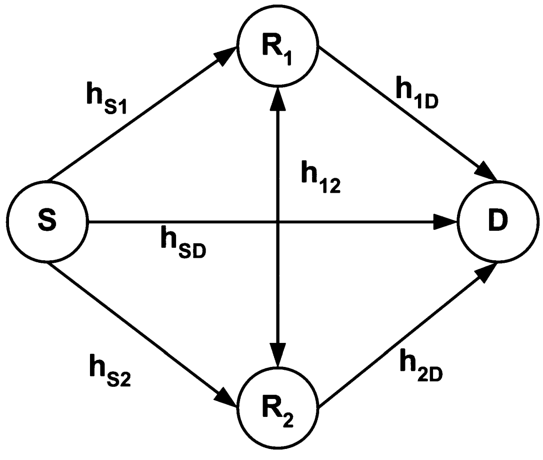

Figure 1. Dual Relay Transmission system

example R_1, is also transmitting what it received at the previous time slot, while the other relay, in this example R_2, is listening to what the source and the relay R_1 are sending. The destination receives the signal transmitted by the source and by one of the relays, in this example R_1, at every odd time slot. At even time slot, the role of the two relays will be inverted and the destination now receives data from source and relay R_2. Throughout the paper, we assume Rayleigh fading channel, perfect channel state information at the receivers, and perfect synchronization. The received signals at the destination can be expressed as

$$\begin{aligned} y(2k-1) =& \sqrt{E_R} h_{2D} \beta_{R_2} x_{R_2}(2k-2) + \\ & \sqrt{E_S} h_{SD} x(2k-1) + n_D(2k-1) \qquad (1) \\ y(2k) =& \sqrt{E_R} h_{1D} \beta_{R_1} x_{R_1}(2k-1) + \\ & \sqrt{E_S} h_{SD} x(2k) + n_D(2k), \qquad (2) \end{aligned}$$

where E_s is the transmit power of the source, E_R is the transmit power of relays,[1] and $x_{R_1}(2k-1)$ and $x_{R_2}(2k-2)$ are the signals received by the relay R_1 at time $2k-1$ and by the relay R_2 at time $2k-2$, respectively [7]. The factor β normalizes the received symbol energy at the corresponding relay. Defining the transmitted and received symbol vector as

$$\mathbf{x}_k = \begin{bmatrix} x(2k-1) \\ x(2k) \end{bmatrix}, \quad \text{and} \quad \mathbf{y}_k = \begin{bmatrix} y(2k-1) \\ y(2k) \end{bmatrix},$$

the input-output relation of DRTS can be expressed in the matrix form as

$$\mathbf{y}_k = \mathbf{P}\mathbf{y}_{k-1} + \mathbf{Q}\mathbf{x}_k + \mathbf{R}\mathbf{x}_{k-1} + \mathbf{n}_k \qquad (3)$$

[1]assuming the transmit power of relays to be equal

where $\mathbf{P}$, $\mathbf{Q}$, $\mathbf{R}$ and $\mathbf{n}_k$ and are given in the following equations.

$$\mathbf{P} = \begin{bmatrix} 0 & \frac{\sqrt{E_R}\beta_{R_2} h_{2D} h_{12}}{h_{1D}} \\ 0 & E_R \beta_{R_1} \beta_{R_2} h_{12}^2 \end{bmatrix},$$

$$\mathbf{Q} = \begin{bmatrix} \sqrt{E_S} h_{SD} & 0 \\ \sqrt{E_S}\sqrt{E_R}\beta_{R_1} h_{S1} h_{1D} & \sqrt{E_S} h_{SD} \end{bmatrix},$$

$$\mathbf{R} = \begin{bmatrix} 0 & \sqrt{E_S}\sqrt{E_R}\beta_{R_1} h_{2D} \left(h_{S2} - \frac{h_{SD} h_{12}}{h_{1D}} \right) \\ 0 & \sqrt{E_S} E_R \beta_{R_1} \beta_{R_2} h_{12} h_{1D} \left(h_{S2} - \frac{h_{SD} h_{12}}{h_{1D}} \right) \end{bmatrix},$$

and the noise vector is given as

$$\mathbf{n}_k = \mathbf{n}_{D_k} - \mathbf{N}_D \mathbf{n}_{D_{k-1}} + \mathbf{N}_R \mathbf{n}_{R_k}, \qquad (4)$$

where

$$\mathbf{N}_D = \begin{bmatrix} 0 & \frac{\sqrt{E_R}\beta_{R_2} h_{2D} h_{12}}{h_{1D}} \\ 0 & E_R \beta_{R_1} \beta_{R_2} h_{12}^2 \end{bmatrix},$$

$$\mathbf{N}_R = \begin{bmatrix} \sqrt{E_R}\beta_{R_2} h_{2D} & 0 \\ E_R \beta_{R_1} \beta_{R_2} h_{1D} h_{12} & \sqrt{E_R}\beta_{R_1} h_{1D} \end{bmatrix},$$

$$\mathbf{n}_{D_k} = \begin{bmatrix} n_D(2k-1) \\ n_D(2k) \end{bmatrix}, \quad \mathbf{n}_{R_k} = \begin{bmatrix} n_R(2k-1) \\ n_R(2k) \end{bmatrix}.$$

$\mathbf{n}_D$ and $\mathbf{n}_R$ are the noise vectors at destination and relays, respectively, with i.i.d complex Gaussian elements.

3. Power Allocations

An important issue in AF relaying is that during the transmission of data from relay, the amplifying factor and the power gain of relay enhances not only the signal but also the noise. Thus power control of relays especially at low SNR becomes critical. Another factor is the channel between the two relays. An interesting case arises because of the inter-relay channel conditions, i.e., at hight inter-relay channel gain, the noise gets enhanced in addition to the signal. Thus it is necessary to control the power according to the channel to get an optimal performance. Figure 2 shows the effect of power allocations for the dual relay system for both high and low inter-relay channel gains. Power is distributed to the source and relays so that the following equality holds

$$E_S + E_R = E_0 \qquad (6)$$

where E_0 is the total transmit power of the system. For *without PA* case, equal power is allocated to both the source and relays. It can be seen that the system performs better with PA at the terminals in each case. It can further be noticed that high inter-relay channel gain necessarily requires power control to limit the noise propagating back and forth

$$
\begin{aligned}
\theta =& \frac{1}{\det(\sigma^2)} \left[g\left(\Gamma_{12}, \Gamma_{S1}\right) g\left(\Gamma_{12}, \Gamma_{S2}\right) \left(1 - \frac{\Gamma_{1D}}{\Gamma_{12}}\right) \phi + \Gamma_{SD} \left(\Gamma_{SD} + \psi\right) \right] \\
\phi =& E_S 2 \Re \left\{ \frac{h_{SD} h_{S1}^{*} h_{2D}^{*}}{h_{12}^{*}} \right\} \sigma_D^2 \\
\psi =& 1 + g\left(\Gamma_{12}, \Gamma_{S1}\right) g\left(\Gamma_{12}, \Gamma_{S2}\right) \frac{\Gamma_{2D}}{\Gamma_{12}} \left(1 - \frac{\Gamma_{1D}}{\Gamma_{12}}\right) + g\left(\Gamma_{S1}, \Gamma_{12}\right) \left(1 + g\left(\Gamma_{12}, \Gamma_{S2}\right) \frac{\Gamma_{2D}}{\Gamma_{12}}\right) \\
\det(\sigma^2) =& \sigma_D^4 \left[1 - g\left(\Gamma_{12}, \Gamma_{S1}\right) g\left(\Gamma_{12}, \Gamma_{S2}\right) \left(\Gamma_{12} - \Gamma_{1D}\right) \left(\Gamma_{12} + \Gamma_{2D}\right) - g\left(\Gamma_{12}, \Gamma_{S2}\right) \Gamma_{2D} \left(\frac{1}{\Gamma_{1D}} - \frac{1}{\Gamma_{12}}\right) \right. \\
& \left. + \frac{\Gamma_{1D}}{\Gamma_{12}} g\left(\Gamma_{12}, \Gamma_{S1}\right) \right]
\end{aligned}
\tag{5}
$$

between the relays. During optimal PA, it has been observed that at low SNR, more power should be allocated to source and vice versa. Thus it can be seen that the network performs better and allocating more power to relays at high SNR results in a considerable improvement.

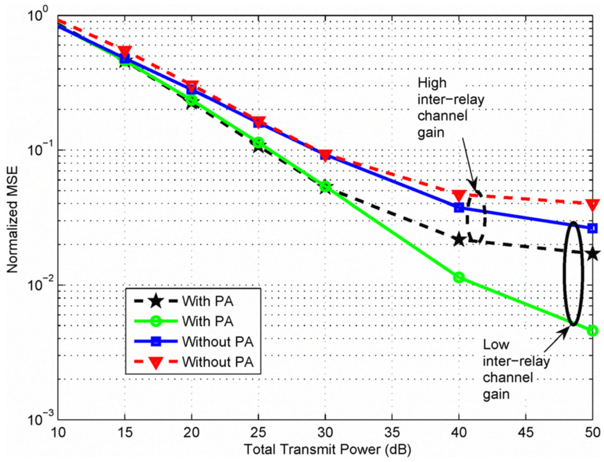

Figure 2. Effect of Power Allocation

4. Channel Capacity

Cooperative communications not only improves system performance in terms of BER reduction but also provides capacity increase over the traditional communication systems. This section focuses on the capacity issues for DRTS besides one relay and direct transmission scenarios. For a direct transmission case, the received symbol at the destination is given by

$$
y_k = \sqrt{E_S} h_{SD} x_k + n_k \tag{7}
$$

where h_{SD} captures the effect of pathloss, shadowing and frequency non-selective fading of wireless channel between

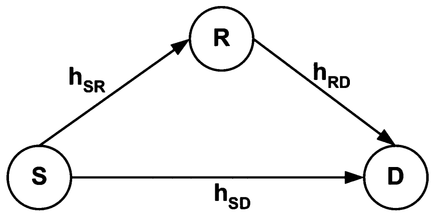

Figure 3. A single relay cooperative system

the source and destination. In this case, the average mutual information of the input and output, in other words capacity, for i.i.d complex Gaussian inputs, is given by [10]

$$
C_1 = \log(1 + \Gamma_{SD}) \tag{8}
$$

where $\Gamma_{SD} = \frac{E_S |h_{SD}|^2}{\sigma^2}$ is the signal to noise ratio and σ^2 is the variance of noise. For the single relay case shown in Figure 3, the input output relationship is given as [7]

$$
\mathbf{y}_k = \mathbf{H}\mathbf{x}_k + \mathbf{n}_k \tag{9}
$$

where the equivalent channel matrix, $\mathbf{H}$ can be expressed as

$$
\mathbf{H} = \begin{bmatrix} \sqrt{E_S} h_{SD} & 0 \\ \sqrt{E_S}\sqrt{E_R} \beta h_{SR} h_{SD} & \sqrt{E_S} h_{SD} \end{bmatrix}. \tag{10}
$$

Defining the SNR for one relay case as,

$$
\Gamma_{pq} = \frac{E_p |h_{pq}|^2}{\sigma_q^2} \tag{11}
$$

from transmitter p to receiver q, the channel capacity is given as [1]

$$
\begin{aligned}
C_2 =& \log[1 + \Gamma_{SD}\{1 + f\left(\Gamma_{SR}, \Gamma_{RD}\right) + \\
& g\left(\Gamma_{SR}, \Gamma_{RD}\right) + \Gamma_{SD} f\left(\Gamma_{SR}, \Gamma_{SD}\right)\}]
\end{aligned}
\tag{12}
$$

where

$$
f(x, y) = \frac{1 + x}{1 + x + y} \quad \text{and} \quad g(x, y) = \frac{x}{1 + x + y}
$$

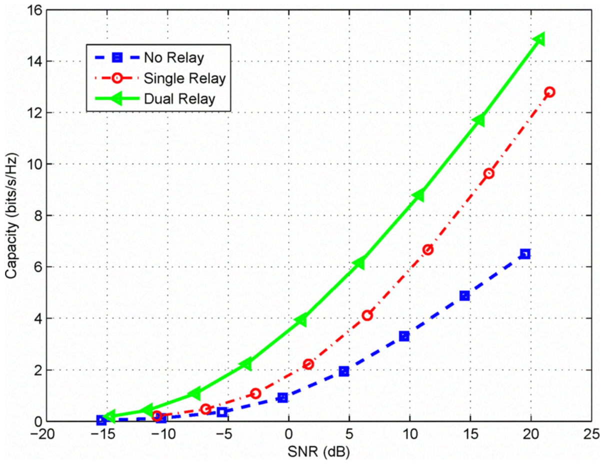

Figure 4. Capacity behavior for the three schemes

For dual relay transmission system, defining the SNR for each link as in (11), the channel capacity is given as

$$C_3 = \log\left(1 + \Gamma_{SD} + \theta\right) \tag{13}$$

where θ is defined in (5) with its other parameters.

Figure 4 shows the comparison of C_1, C_2, and C_3 with respect to SNR. Dual relay system shows maximum channel capacity as compared to the other systems over a specific SNR. Thus dual relay system can provide high rate data transmission with lower probability of error under same conditions.

5. Conclusion

We have proposed a novel cooperative relaying scheme with dual relays and full rate transmission capabilities. Based on the analysis and simulation results, this new scheme provides balanced diversity gain to each transmitted symbol and thus enhance system performance and channel capacity over the conventional one relay scheme and point-to-point communication. Since the AF mode is used, power allocations at the relays is necessary for robust transmission and to control the propagating noise from one terminal to other. In short, AF mode performs better at high SNR because of low noise propagation. Hence it can be argued that DF mode at low SNR would work better as compared to the AF mode and vice versa. An adaptive AF-DF mode can be utilized for the dual relay transmission for further system performance.

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