Equalization for Symmetric Cooperative Relay scheme for Wireless Communications

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***Abstract*— Cooperative relaying methods have attracted a lot of interest in the past several years. A conventional cooperative relaying scheme has a source, a destination and a single relay. This cooperative technique can support one symbol transmission per time slot, called full rate transmission. However, existing full rate cooperative relay approaches have asymmetrical diversity gains for different symbols. Another approach is to use dual relays to overcome these effects. In this paper, we propose the design of equalizers for the cooperative relaying scheme that is assisted with dual relays providing full transmission rate and same macro-diversity to each symbol. The equalization of received symbols is carried out using a minimum mean squared error (MMSE) linear equalizer (LE) and decision feedback equalizer (DFE) which provide inter-symbol-interference (ISI) cancellation capabilities.**

***Index Terms*— Cooperative diversity, relay transmission, MMSE-LE, MMSE-DFE.**

I. Introduction

Cooperative diversity is a novel technique in achieving higher system performance not only in terms of capacity but also in attaining higher diversity gains in wireless systems. Exploiting the broadcast nature of wireless networks, relay nodes help the transmission of same data through different paths, resulting in considerable diversity gain used for robust detection of transmitted signals. Conventional cooperative strategies employ a single node as relay [1-3], which provides performance gain as compared to a direct transmission scenario. The drawback of this system is the asymmetrical transmission of data. By asymmetry, we imply that the diversity gain is not applied to every transmitted symbol thus the performance is only partially improved. Another approach is the use of dual relay system which removes the problem of asymmetricity [4]. The proposed scheme not only provides full transmission rate but also has balanced diversity for each transmitted symbol.

Equalization is of potential importance in any diversity based system. In [4], the dual relay transmission system with a unit memory is converted to a memoryless system where the equalization is carried out with the help of one tap MMSE-LE which also takes knowledge of previously detected symbols. In this paper, we propose MMSE-LE and MMSE-DFE keeping the actual system model. Two approaches are commonly used in cooperative relaying scenarios. The first is known as *amplify-and-forward* transmission (AF), where the relay amplifies the received signal and forwards it to the receiver [5-6]. The second approach is *decode-and-forward* transmission (DF) where relay decodes the incoming signal first and then re-encodes it and sends it to the destination. In this paper, we use the AF mode for dual relay system.

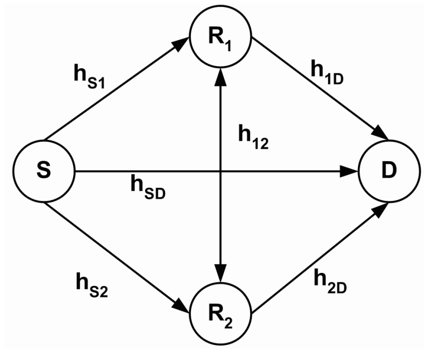

Fig. 1. Dual Relay Transmission system

The paper is organized as follows. Section II presents the basic architecture, functionality, and mathematical model of system with dual relays. Section III comprises of the derivation of MMSE-LE and MMSE-DFE for the dual relay transmission. Finally, in Section IV, we illustrate the simulation results for our proposed cooperative network. The paper then concludes with certain comments in Section V.

II. System Model

Figure 1 shows the symmetrical cooperative relaying scheme consisting of a source, S, destination, D, and a pair of relay stations R_1 and R_2. h_{pq} captures the effect of pathloss, shadowing and frequency non-selective fading from transmitter p to receiver q. The fading is assumed to be independently and identically distributed (i.i.d.) drawn from a set of complex Gaussian elements. This scheme works as follows. The source is always sending data to destination at every time slot. At odd time slot, one relay, for example R_1, is also transmitting what it received at the previous time slot, while the other relay, in this example R_2, is listening to what the source and the relay R_1 are sending. The destination receives the signal transmitted by the source and by one of the relays, in this example R_1, at every odd time slot. At even time slot, the role of the two relays will be inverted and the destination now receives data from source and relay R_2. Thus the received signals at the destination can be expressed as

$$y(2k-1) = \sqrt{E_R} h_{2D} \beta_{R_2} x_{R_2}(2k-2) + \sqrt{E_S} h_{SD} x(2k-1) + n_D(2k-1) \quad (1)$$

$$y(2k) = \sqrt{E_R} h_{1D} \beta_{R_1} x_{R_1}(2k-1) + \sqrt{E_S} h_{SD} x(2k) + n_D(2k), \quad (2)$$

where E_s is the transmit power of the source, E_R is the transmit power of relays,[1] and $x_{R_1}(2k-1)$ and $x_{R_2}(2k-2)$ are the signals received by the relay R_1 at time $2k-1$ and by the relay R_2 at time $2k-2$, respectively [4]. The factor β normalizes the received symbol energy at the corresponding relay. Defining the transmitted and received symbol vector as

$$\mathbf{x}_k = \begin{bmatrix} x(2k-1) \\ x(2k) \end{bmatrix}, \quad \text{and} \quad \mathbf{y}_k = \begin{bmatrix} y(2k-1) \\ y(2k) \end{bmatrix},$$

the input-output relation of the dual relay transmission can be expressed in the matrix form as

$$\mathbf{y}_k = \mathbf{P}_k \mathbf{y}_{k-1} + \mathbf{Q}_k \mathbf{x}_k + \mathbf{R}_k \mathbf{x}_{k-1} + \mathbf{n}_k \quad (3)$$

where $\mathbf{P}_k$, $\mathbf{Q}_k$, $\mathbf{R}_k$, and $\mathbf{n}_k$ are given in the following equations. Here we omit the timing indices of β and denote $\beta_{R_i}(2k-1)$ as β_{R_i} for $i = 1, 2$. Thus

$$\mathbf{P}_k = \begin{bmatrix} 0 & \frac{\sqrt{E_R}\beta_{R_2} h_{2D} h_{12}}{h_{1D}} \\ 0 & E_R \beta_{R_1} \beta_{R_2} h_{12}^2 \end{bmatrix},$$

$$\mathbf{Q}_k = \begin{bmatrix} \sqrt{E_S} h_{SD} & 0 \\ \sqrt{E_s}\sqrt{E_R}\beta_{R_1} h_{S1} h_{1D} & \sqrt{E_S} h_{SD} \end{bmatrix},$$

$$\mathbf{R}_k = \begin{bmatrix} 0 & \sqrt{E_S}\sqrt{E_R}\beta_{R_1} h_{2D} \left(h_{S2} - \frac{h_{SD} h_{12}}{h_{1D}} \right) \\ 0 & \sqrt{E_S} E_R \beta_{R_1} \beta_{R_2} h_{12} h_{1D} \left(h_{S2} - \frac{h_{SD} h_{12}}{h_{1D}} \right) \end{bmatrix}$$

and the noise vector is given as

$$\mathbf{n}_k = \mathbf{n}_{D_k} - \mathbf{N}_D \mathbf{n}_{D_{k-1}} + \mathbf{N}_R \mathbf{n}_{R_k} \quad (4)$$

where $\mathbf{N}_D$ and $\mathbf{N}_R$ are given in [4]. $\mathbf{n}_D$ and $\mathbf{n}_R$ are the noise vectors at destination and relays, respectively, with i.i.d. complex Gaussian elements.

III. Equalizer Design

This section presents the derivation of MMSE-LE and MMSE-DFE for the dual relay system.

[1]assuming the transmit power of relays to be equal

A. MMSE-LE

The observed vector $\mathbf{y}_k$ is passed through a LE and the detected symbol vector is given as

$$\hat{\mathbf{x}_k} = \sum_{n=-N}^{N} \mathbf{C}_n \mathbf{y}_{k-n} \quad (5)$$

where

$$\mathbf{C}_n = \begin{bmatrix} c_{n0} & c_{n1} \\ c_{n2} & c_{n3} \end{bmatrix} \quad (6)$$

Thus (5) can be written in matrix notation as

$$\hat{\mathbf{x}_k} = \mathbf{C}^H \mathbf{y} \quad (7)$$

where $\mathbf{C}$ is $(4N+2) \times 2$ matrix and $\mathbf{y}$ is $(4N+2) \times 1$ column vector. Introducing the error autocorrelation matrix $\mathbf{R}_{ee} = E\left\{\mathbf{e}_k \mathbf{e}_k^H\right\}$, the MSE is given as

$$\Im = E\{\| \hat{\mathbf{x}_k} - \mathbf{x}_k \|^2\} \quad (8)$$

The coefficient matrix of the equalizer turns out to be the solution of Wiener filter [7] given as

$$\mathbf{C} = \mathbf{R}_y^{-1} \mathbf{R}_{yx} \quad (9)$$

and the resulting MMSE becomes

$$\Im_{min} = tr\left[\mathbf{R}_x - \mathbf{R}_{yx}^H \mathbf{R}_y^{-1} \mathbf{R}_{yx}\right] \quad (10)$$

The dimensions of the auto-correlation matrix $\mathbf{R}_y$ and cross-correlation matrix $\mathbf{R}_{yx}$ are $(4N+2) \times (4N+2)$ and $(4N+2) \times 2$, respectively. The cross-correlation matrix is given as

$$\mathbf{R}_{yx} = \mathbf{E}\left[\mathbf{y}\mathbf{x}_k^H\right] = \begin{bmatrix} \mathbf{r}_{yx}(-N) \\ \vdots \\ \mathbf{r}_{yx}(N) \end{bmatrix} \quad (11)$$

where each element $\mathbf{r}_{yx}(n)$ is a 2×2 matrix for $-N \leq n \leq N$, and is given as

$$\mathbf{r}_{yx}(n) = \begin{cases} \mathbf{P}^{-n-1}(\mathbf{PQ} + \mathbf{R}) & -N \leq n \leq -1 \\ \mathbf{Q} & n = 0 \\ \mathbf{0} & \text{otherwise} \end{cases} \quad (12)$$

It can be noticed from (11) and (12) that the cross-correlation matrix becomes zero for the causal part of the FIR equalizer. Thus for dual relay system, an acausal filter with taps from $n = -N$ to $n = 0$ will be sufficient. The autocorrelation of the received signal is given as

$$\mathbf{R}_y = E\left\{\mathbf{y}\mathbf{y}^H\right\} = \begin{bmatrix} \mathbf{r}_y(-N,-N) & \cdots & \mathbf{r}_y(-N,N) \\ \vdots & \ddots & \vdots \\ \mathbf{r}_y(N,-N) & \cdots & \mathbf{r}_y(N,N) \end{bmatrix} \quad (13)$$

where each element $\mathbf{r}_y(n,m)$ is a 2×2 matrix for $-N \leq n, m \leq N$, and is given as in (14). The cross-correlations matrices $\mathbf{L}$, $\mathbf{S}$, $\mathbf{M}$, and $\mathbf{T}$ are again 2×2 given as

$$\mathbf{L}(n,m) = \begin{cases} \mathbf{Q} & m = n \\ \mathbf{P}^{m-n-1}(\mathbf{PQ} + \mathbf{R}) & m > n \\ \mathbf{0} & otherwise \end{cases}$$

$$\mathbf{r}_y(n,m) = \mathbf{P}\mathbf{r}_{y(k-1)}\mathbf{P}^H + \mathbf{Q}\underbrace{E\left\{\mathbf{x}_{k-n}\mathbf{x}_{k-m-1}^H\right\}}_{\tilde{\mathbf{R}}_x}\mathbf{R}^H + \mathbf{P}\underbrace{E\left\{\mathbf{y}_{k-n-1}\mathbf{x}_{k-m-1}^H\right\}}_{\mathbf{L}}\mathbf{R}^H + \mathbf{P}\underbrace{E\left\{\mathbf{y}_{k-n-1}\mathbf{x}_{k-m}^H\right\}}_{\mathbf{S}}\mathbf{Q}^H + \mathbf{Q}\mathbf{R}_x\mathbf{Q}^H$$
$$+\mathbf{R}\underbrace{E\left\{\mathbf{x}_{k-n-1}\mathbf{y}_{k-m-1}^H\right\}}_{\mathbf{M}}\mathbf{P}^H + \mathbf{R}\mathbf{R}_x\mathbf{R}^H + \mathbf{Q}\underbrace{E\left\{\mathbf{x}_{k-n}\mathbf{y}_{k-m-1}^H\right\}}_{\mathbf{T}}\mathbf{P}^H + \mathbf{R}\underbrace{E\left\{\mathbf{x}_{k-n-1}\mathbf{x}_{k-m}^H\right\}}_{\hat{\mathbf{R}}_x}\mathbf{Q}^H + \sigma_{m-n}^2\mathbf{I} \quad (14)$$

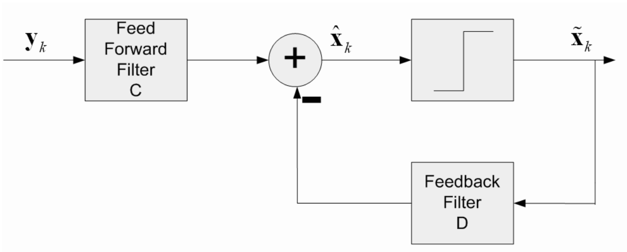

Fig. 2. Block diagram of Decision Feedback Equalizer

$$\mathbf{S}(n,m) = \begin{cases} \mathbf{Q} & m = n+1 \\ \mathbf{P}^{m-n-2}(\mathbf{PQ}+\mathbf{R}) & m > n+1 \\ \mathbf{0} & otherwise \end{cases}$$

$\mathbf{M}$ and $\mathbf{T}$ are the Hermitian equivalent of $\mathbf{L}$ and $\mathbf{S}$, respectively, for n replaced with m. The autocorrelations of the transmitted signal, $\mathbf{R}_x$, $\tilde{\mathbf{R}}_x$, and $\hat{\mathbf{R}}_x$, are identity matrices for $m = n$, $m = n - 1$, and $m = n+1$, respectively. The noise variance is given as

$$\sigma_{m-n}^2 = \begin{cases} \sigma_R^2\mathbf{N}_R\mathbf{N}_R^H + \sigma_D^2\left(\mathbf{I} - \mathbf{N}_D\mathbf{N}_D^H\right) & m = n \\ \mathbf{0} & otherwise \end{cases} \quad (15)$$

where σ_R^2 and σ_D^2 are the variances of noise at relays and destination, respectively.

B. MMSE-DFE

This subsection describes the design of a MMSE-DFE for the dual relay system. The block diagram of the system is shown in Figure 2 where the MSE is defined as

$$\Im = \mathbf{E}\left\{|\tilde{\mathbf{x}}_k - \hat{\mathbf{x}}_k|^2\right\} \quad (16)$$

where $\tilde{\mathbf{x}}_k$ are the detected symbols which are used for the interference cancellation caused from the previous symbols. Thus feedback filter (FBF) D helps mitigating the ISI caused from post-cursor samples. Let the length of FIR filters for feed-forward filter (FFF) and FBF are N_1 and N_2 respectively, the MSE is given as

$$\Im = \mathbf{E}\left\{\left|\tilde{\mathbf{x}}_k - \left[\sum_{n=-N_1}^{0}\mathbf{C}_n\mathbf{y}_{k-n} - \sum_{m=1}^{N_2}\mathbf{D}_m\tilde{\mathbf{x}}_{k-m}\right]\right|^2\right\} \quad (17)$$

In matrices form

$$\Im = \mathbf{E}\left\{\left(\tilde{\mathbf{x}}_k - \mathbf{C}^H\mathbf{Y} + \mathbf{D}^H\tilde{\mathbf{x}}_p\right)\left(\tilde{\mathbf{x}}_k - \mathbf{C}^H\mathbf{y} + \mathbf{D}^H\tilde{\mathbf{x}}_p\right)^H\right\} \quad (18)$$

Here $\mathbf{C}$ and $\mathbf{D}$ are coefficient matrices with dimensions $(2N_1\times 2)$ and $(2N_2\times 2)$, respectively, and $\mathbf{y}$, $\tilde{\mathbf{x}}_p$, and $\tilde{\mathbf{x}}_k$ are column vectors with dimensions $(2N_1\times 1)$, $(2N_2\times 1)$, and (2×1) respectively. The vector $\tilde{\mathbf{x}}_p$ contains the previous detected symbols which are assumed to be correct. The response of the FFF and FBF are given as

$$\mathbf{C} = \left(\mathbf{R}_y - \mathbf{\Phi}\mathbf{\Phi}^H\right)^{-1}\mathbf{R}_{yx} \quad (19)$$
$$\mathbf{D} = \mathbf{\Phi}^H\mathbf{C} \quad (20)$$

where $\mathbf{R}_y$ and $\mathbf{R}_{yx}$ are given in (14) and (11), respectively. Assuming correct decisions in DFE and thus denoting $\tilde{\mathbf{x}}_p$ as $\mathbf{x}$, $\mathbf{\Phi}$ is given as $\mathbf{\Phi} = E[\mathbf{yx}]$. The dimensions of $\mathbf{\Phi}$ are same as $\mathbf{R}_y$, i.e., $(4N+2)\times(4N+2)$ and has the same form as in (13) where each element $\phi \in \mathbf{\Phi}$ is a 2×2 matrix given as

$$\begin{aligned}\phi(n,m) &= \mathbf{E}\left[\mathbf{y}_{k-n}\mathbf{x}_{k-m}^H\right] \\ &= \mathbf{P}\underbrace{\mathbf{E}\left\{\mathbf{y}_{k-n-1}\mathbf{x}_{k-m}^H\right\}}_{\tilde{\mathbf{S}}} + \mathbf{R}\underbrace{\mathbf{E}\left\{\mathbf{x}_{k-n-1}\mathbf{x}_{k-m}^H\right\}}_{\tilde{\tilde{\mathbf{R}}}_x}\end{aligned} \quad (21)$$

The indices n and m are given as $-N_1 \le n \le 0$ and $1 \le m \le N_2$. $\tilde{\mathbf{S}}$ and $\tilde{\tilde{\mathbf{R}}}_x$ are variants of $\mathbf{S}$ and $\tilde{\mathbf{R}}_x$ as defined in the previous subsection. MMSE in this case is given as

$$\Im_{min} = tr\left[\mathbf{R}_x - \mathbf{R}_{yx}^H\left(\mathbf{R}_y - \mathbf{\Phi}\mathbf{\Phi}^H\right)^{-1}\mathbf{R}_{yx}\right] \quad (22)$$

IV. Simulation Results

This section presents the performance improvement of the dual relay system over the conventional one relay and direct transmission scenarios. In all cases, the transmitted signal is a random bipolar sequence modulated through quadrature phase-shift keying (QPSK) modulation. Symbols are transmitted through different channels that are all considered to be Rayleigh fading with zero mean and unit variance. For all cases, there is an average total transmit power constraint. Figure 3 shows the performance of the dual relay system with MMSE-LE. It can be seen that a single tap equalizer gives the worst performance because the system is with unit memory. Increasing taps to two and further, improves the performance because of ISI cancellation. Figure 4 shows the comparison of three scenarios and it can be noticed that the diversity gain

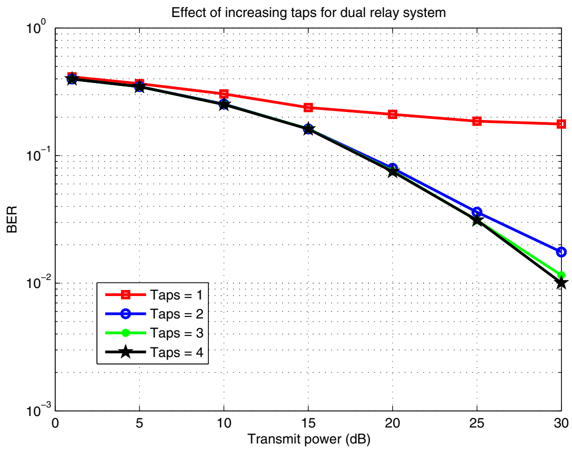

Fig. 3. MMSE-LE performance

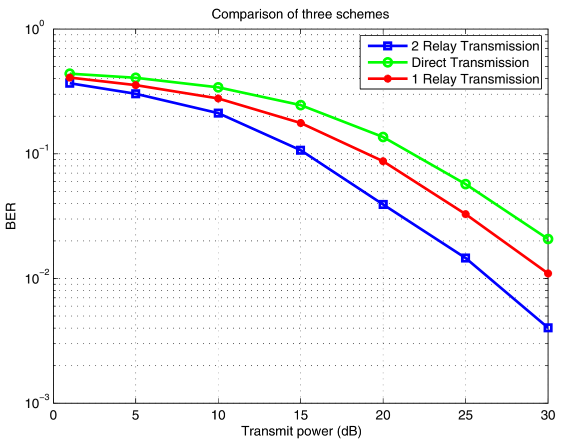

Fig. 4. Comparison of three scenarios

appears to be dominant in dual relay case (with MMSE-LE) as compared to other two transmission scenarios. Figure 5 shows the comparison of LE and DFE. It can be seen that the error is further reduced in case of DFE because of the cancellation of post-cursor interference. At low SNR and same number of taps, LE performs better because of error propagation in DFE but at high SNR we achieve substantial performance improvement for DFE in terms of MSE.

V. Conclusion

The paper presents the design of equalizers for the dual relay transmission scheme for cooperative diversity. It has been observed that DFE performs better than LE under same conditions and the performance is improved over other conventional systems. An important issue in AF relaying is that when the data is transmitted from relay, the amplifying factor and the power gain of relay enhances not only the signal but also the noise. So an important task is to control the power of relays especially at low SNR because at low SNR, noise power is large which gets enhanced by the relays. Thus it can be argued that optimal power allocations for the dual relay transmission system will further enhance the system performance.

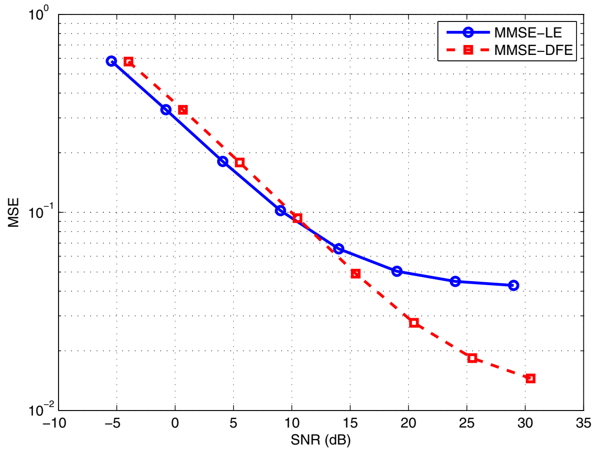

Fig. 5. Comparison of MMSE-LE and MMSE-DFE

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