Coverage Aspects of Cooperative Multi-hop Line Networks in Composite Fading Environment

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***Abstract*—We consider a cooperative multi-hop line network, where a group of nodes cooperatively transmits the same message to another group of nodes, and model the transmission from one group to another as a discrete-time quasi-stationary Markov process. We derive the transition probability matrix of the Markov chain by considering the wireless channel exhibiting composite shadowing-fading. The sum distribution of the received power by multiple relays is approximated by a single log-normal random variable (RV) by using the moment generating function (MGF)-based technique. This MGF-based technique uses Gauss-Hermite integration to present the sum distribution in closed form. We quantify the signal-to-noise ratio (SNR) margin required to achieve a certain quality of service (QoS) under standard deviation of the shadowing. We also provide the optimal level of cooperation required for obtaining maximum coverage of a line network under a given QoS. Monte Carlo simulations are used to validate the analytical model.**

I. Introduction

Cooperative transmission (CT) is becoming popular in the past several years in both the sensor and the cellular networks because of its high data rate capability and increased reliability. One fast and low overhead CT scheme for physical layer flooding in large networks is Opportunistic Large Array (OLA) [1]. In an OLA transmission, a group of nodes under orthogonal fading channels form a level and transmits the same message to another level of nodes. All nodes that decode the message successfully from the previous level nodes, relay the message together. OLA broadcast are energy-efficient candidate for large dense wireless sensor networks and can increase the reliability and range in mobile networks.

The behavior of dense wireless cooperative networks is studied in [1], assuming an infinite node density per unit area, emitting a constant power. However, this continuum assumption may not be an appropriate candidate for finite density networks. In [2], the authors studied finite extended networks and proposed that as opposed to continuum assumption, an infinite broadcast is not possible in finite density networks. Another study on finite density OLA networks has been done in [3]. The authors derived an analytical model for a finite density cooperative multi- hop network. Under fading channel environment, they provided an upper bound on the network coverage. However, they only considered small-scale fading. A variant of Rayleigh fading model is studied in [4].

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In practical wireless systems, both small-scale as well as large-scale fading known as shadowing are present [5]. According to our literature survey there is no significant work, which has considered composite shadowing-fading for OLA cooperative networks. Therefore, in this paper, we study the performance of cooperative multi-hop linear network under composite shadowing-fading. We model the small-scale fading as Rayleigh distribution and the shadowing as log-normal distribution and the resulting composite distribution is given by the Suzuki distribution [6]. The received signal in a cooperative network at a node is the sum of multiple transmitted signals and each of this signal is affected by small-scale fading as well as shadowing. However, there is no closed-form expression for the probability density function (PDF) of the sum of multiple Suzuki RVs [5]. In literature, different approximation techniques have been proposed to find the sum distribution of multiple Suzuki RVs. In [7], a technique based on an extension of Fenton-Wilkinsons [5] approach is proposed. It is a two step approximation process in which a Suzuki RV is approximated by a log-normal RV in the first step and then by using the Fenton-Wilkinsons method, an approximate sum distribution of the multiple log-normal RVs to a single log-normal RV is achieved. In [8], the sum of Suzuki RVs is approximated by a single Suzuki RV. However, this methods does not consider the problem of addressing the sum by a single log-normal RV. We use the method proposed in [9] to approximate the sum of Suzuki RVs by a single log-normal RV. This method uses moment generating function (MGF) as a tool to approximate the sum distribution and uses the Gauss-Hermite [10] expansion of the MGFs of both log-normal and Suzuki to find the closed-form expression for it. Similarly, to find the sum distribution of the multiple log-normal RVs to a single log-normal RV for shadowing channel the same MGF-based technique is used.

This kind of network can be used as a prelude to study more general random 2D networks operating under composite channel. Typical applications of this linear network include structural health monitoring of buildings and bridges where the nodes are aligned in a regular linear pattern, finding a route path from the source to the destination in mobile ad hoc networks (MANETS), and fault recognition in transmission lines for the future smart grid systems. This topology would also be consistent with a plastic communication cable, where sensor nodes are embedded in a plastic wire and cooperative transmission is performed to transmit the source message from one end of the wire to another. These wires find practical applications in air-industry having lighter weights as compared

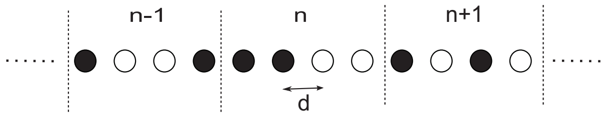

Fig. 1. System model for $M = 4$.

to general copper wires and reduce the unwanted high electric fields in the surroundings [11].

After incorporating the wireless channel into the Markov chain model, this paper quantifies the effect of various system parameters such as SNR margin, number of cooperative nodes in a level, standard deviation of shadowing, and path loss exponent on the coverage of the network. We also provide optimal regions of cooperation that can provide maximum coverage under a given quality of service (QoS), SNR margin and shadowing standard deviation constraint. The coverage of a network under three different channel model is compared, i.e., fading, shadowing, and composite shadowing-fading.

The organization of the paper is follows. Section II describes the network layout. In Section III, we model our network as a discrete-time quasi-stationary Markov chain, while in Section IV, we derive the transition probability matrix of the Markov chain. In Section V, we discuss various simulations and analytical results. Section VI concludes the paper and gives directions for the future work.

II. System Description

Consider a 1-dimensional network with infinite number of nodes where the adjacent nodes are at a distance *d* away from each other as shown in Fig. 1. The network is divided into non-overlapping sets of nodes, such that each group or *level* comprises M number of nodes. The M nodes in one level cooperate with each other to forward the same message signal to the M nodes of the next level. However, only those nodes take part in transmission who have decoded the data perfectly from the transmission of previous level nodes. These nodes are called decode-and-forward (DF) nodes. The number of DF nodes in a level is unknown apriori, implying that the network is opportunistic. A node become DF node when the signal-to-noise ratio (SNR) of the received signal, after post-detection combining, is greater than or equal to a modulation dependent threshold, τ. The DF nodes in Fig. 1. are shown by filled circles. We assume same transmit power, P_t, for all the nodes and label the set of indices of DF nodes at time instant or level *n* by $\mathbb{N}_n$. For example, from figure, $\mathbb{N}_{n+1} = \{1, 3\}$, $\mathbb{N}_n = \{1, 2\}$, and $\mathbb{N}_{n-1} = \{1, 4\}$. The received power at a *k*th node at time instant *n* is given by

$$Pr_k(n) = \frac{P_t}{d^\beta} \sum_{m \in \mathbb{N}_{n-1}} \frac{S_{mk}}{(M - m + k)^\beta}, \tag{1}$$

where the summation is over the DF nodes in the previous level $(n-1)$ and β is the path loss exponent with a usual range of 2-4. The composite channel coefficient, S_{mk}, from node *m* in level $(n-1)$ to node *k* in level *n* is modeled as a Suzuki RV, which is a combination of Rayleigh and log-normal RV. We model the multi-path effect as Rayleigh distribution and the shadowing effect as log-normal distribution. The log-normal RV has a mean and a standard deviation, both of which are expressed in decibel (dB). The standard deviation of the shadowing is called the dB spread of the channel and its typical value is between 5-12 dB for wireless channels depending upon the severity of the shadowing. In this work, we assume that nodes in a level have perfect synchronization so that the DF nodes transmit the signal at the same time.

III. Modeling By Markov Chain

We represent the state of each node by a binary indicator RV, $\mathbb{I}$, such that at a time instant *n*, the state of the *k*th node, $\mathbb{I}_k(n) = 1$ represents that node *k* has decoded successfully and $\mathbb{I}_k(n) = 0$ shows that node *k* has not decoded the data correctly. In the same way, the state of each level can be represented as $\mathcal{X}(n) = [\mathbb{I}_1(n), \mathbb{I}_2(n), ..., \mathbb{I}_M(n)]$ where the outcome of $\mathcal{X}(n)$ is an M-bit binary word. Each outcome is a state, and there are 2^M total number of states, starting from 0 to $2^M - 1$ in decimal. If i_n represents the state at time instant n, then from Fig. 1., $i_n = \{1100\}$ in binary, and $i_n = 12$ in decimal. It can be noticed that $\mathcal{X}$ is a memoryless Markov process because the state at any time depends upon the transmissions from the previous level only. Further investigation reveals that the Markov chain, $\mathcal{X}$, can reach an absorbing state at any point in time with some nonzero probability, terminating the process of transmission. At that time the state of Markov chain will be 0 (decimal) and will happen only when all the nodes in a level fail to decode the message perfectly. Thus $\{0\} \cup T$ constitute the state space of the Markov chain $\mathcal{X}$, where T is the finite transient irreducible state space; $T = \{1, 2, ..., 2^{M-1}\}$ and 0 being the absorbing state such that

$$\lim_{n \to \infty} \mathbb{P}\{\mathcal{X}(n) = 0\} \nearrow 1 \text{ a.s.} \tag{2}$$

The Markov chain, $\mathcal{X}$, can be completely characterized by finding the transition probability matrix, $\mathbf{P}$, corresponding to $\mathcal{X}$. If we remove the transitions to and from the absorbing states the resulting $\mathbf{P}$ is square, irreducible and right sub-stochastic with a dimension of $(2^M - 1) \times (2^M - 1)$.

By the theory of Markov chain, a distribution $\mathbf{u} = (u_i, i \in T)$ is called ρ-invariant distribution if $\mathbf{u}$ is the left eigenvector of this particular transition matrix, $\mathbf{P}$, which corresponds to ρ, where ρ is the maximum eigenvalue of $\mathbf{P}$, i.e., $\mathbf{uP} = \rho\mathbf{u}$. In the meantime $\forall n, \mathbb{P}\{\mathcal{X}(n) = 0\} > 0$, therefore ultimate killing is certain. However, we are interested in finding the distribution of the transient states, just before the absorbing state is reached. This limiting distribution is known as the quasi-stationary distribution of the Markov chain [14], and is independent of the initial conditions of the process. The ρ-invariant distribution for one-step transition probability matrix of the Markov chain on T give us this unique distribution. To find the quasi-stationary distribution, we first calculate the *maximum* eigenvector, $\hat{\mathbf{u}}$, of $\mathbf{P}$. Defining $\mathbf{u} = \hat{\mathbf{u}} / \sum_{i=1}^{2^M-1} \hat{\mathbf{u}}_i$, as a normalized version of $\hat{\mathbf{u}}$ that sums to one, gives the quasi-stationary distribution of $\mathcal{X}$. Hence the unconditional probability of being in state *j* at time instant *n* is given as

$$\mathbb{P}\{\mathcal{X}(n) = j\} = \rho^n u_j, \; j \in T, n \geq 0. \tag{3}$$

We also let $\Phi = \inf\{n \geq 0 : \mathcal{X}(n) = 0\}$ denotes the end of survival time, i.e., the time at which the killing occurs. It follows then

$$\mathbb{P}\{\Phi > n + n_0 | \Phi > n\} = \rho^{n_0}, \tag{4}$$

while the quasi-stationary distribution of the Markov chain is given as

$$\lim_{n\to\infty} \mathbb{P}\{\mathcal{X}(n)=j|\Phi>n\}=u_j,\ j\in T. \tag{5}$$

IV. Formulation of Transition Probability Matrix

In this section, we find the transition probability matrix of the Markov chain, which provides the quasi-stationary distribution of the chain.

A. Log-normal Approximation of the Sum of multiple Suzuki RVs for Composite Channel

Now we find the state transition probability matrix, $\mathbf{P}$, of the network, by considering the channel as composite fading channel. Let *i* and *j* represent two states of the system at time instant $(n-1)$ and n, respectively, such that *i*, *j* $\in\{1,2,...,2^M-1\}$, where *i* and *j* are the decimal equivalent of the binary word formed by the set of binary indicator RVs. The received SNR at time instant n on the *k*th node is given as $\gamma_k(n)=Pr_k(n)/\sigma^2_{noise}$, where σ^2_{noise} is the noise variance at the *k*th receiver, and Pr is the received power as given in (1). For all the nodes in a level, we assume identical noise variance. Now for a node *k*, the conditional probability of being able to decode successfully at time *n* is given as

$$\begin{aligned}&\mathbb{P}\{\text{node } k \text{ of level } n \text{ will decode } |\varphi\}=\\&\quad\mathbb{P}\{\mathbb{I}_k(n)=1|\varphi\}=\mathbb{P}\{\gamma_k(n)>\tau|\varphi\}, \end{aligned}\tag{6}$$

where the event φ is defined as $\varphi=\{\mathcal{X}(n-1)\in T\}$, indicating that the previous state is a transient state. Similarly, the probability of outage or the probability of $\mathbb{I}_k(n)=0$ is given as $1-\mathbb{P}\{\gamma_k(n)>\tau|\varphi\}$, where

$$\mathbb{P}\{\gamma_k(n)\geq\tau|\varphi\}=\int_\tau^\infty p_{\gamma_{k|\varphi}}(y)dy, \tag{7}$$

where $p_{\gamma_{k|\varphi}}$ is the conditional PDF of the received SNR at the *k*th node conditioned on the state $\mathcal{X}(n-1)$. It can be observed that the received SNR at a certain node is the sum of the finite SNRs from the previous level nodes (assuming maximal ratio combining for coherent modulation scheme), each of which follows Suzuki distribution. However, the sum distribution of Suzuki RVs does not exist in closed-form [5]. Therefore, to find the sum distribution of Suzuki RVs, we use the moment generating function (MGF)-based method as proposed in [9]. The MGF of a RV Y is given by

$$\Psi_Y(s)=\int_0^\infty \exp(-sy)p_Y(y)dy. \tag{8}$$

MGF exhibits two important properties; first, MGF is the weighted integral of the PDF with adjustable parameter s and second, the MGF of the sum of independent RVs can be expressed as the product of the MGFs of individual RVs as given by

$$\Psi_{(\sum_{k=1}^N Y_k)}(s)=\prod_{k=1}^N \Psi_{Y k}(s). \tag{9}$$

We approximate the sum of N Suzuki RVs $(S_1,S_2,...,S_N)$ by a single log-normal RV $Y=10^{0.1X}$, where X is a Gaussian RV. This MGF-based approximation method requires that both the MGF of Suzuki and log-normal RV need to be in closed-form. However, the MGF of both the Suzuki and log-normal RV do not exist in closed-form and can be numerically computed using the Gauss-Hermite quadrature integration [10]. In Gauss-Hermite quadrature integration, the integral is evaluated by an approximate sum where each component of the summation depends upon a specific weight. Specifically, the MGF of kth Suzuki RV by Gauss-Hermite integration after discarding the remainder terms can be written as

$$\widehat{\Psi}_{S_k}(s;\mu_k,\sigma_k)=\sum_{c=1}^C \frac{w_c/\sqrt{\pi}}{1+s\exp\left(\frac{\sqrt{2}\sigma_k a_c+\mu_k}{\xi}\right)}, \tag{10}$$

where C is the Hermite integration order and a large value of C corresponds to higher accuracy, w_c is the weight corresponding to the abscissas, a_c, and ξ is a constant; $\xi=10/\ln 10$. The values of w_c and a_c for C up to 20 are available in tabular form in [13]. The μ_k and σ_k are the mean and standard deviation of the kth Suzuki RV. Similarly, by using the Gauss-Hermite integration, the MGF of the log-normal RV $Y=10^{0.1X}$ is given as

$$\widehat{\Psi}_Y(s;\mu_X,\sigma_X)=\sum_{c=1}^C \frac{w_c}{\sqrt{\pi}}\exp\left[-s\exp\left(\frac{\sqrt{2}\sigma_X a_c+\mu_X}{\xi}\right)\right], \tag{11}$$

where μ_X and σ_X are the mean and standard deviation of the Gaussian RV X. The task is to find the μ_X and σ_X of X as a function of the mean and standard deviation of the individual RVs $(S_1,S_2,...,S_N)$. The μ_X and σ_X can be found by solving the following two equations

$$\widehat{\Psi}_Y(s_i;\mu_X,\sigma_X)=\prod_{k=1}^N \widehat{\Psi}_{S_k}(s_i;\mu_k,\sigma_k),\ \text{at } i=1 \text{ and } 2. \tag{12}$$

By using (10) and (11), Equation (12) becomes

$$\begin{aligned}&\sum_{c=1}^C \frac{w_c}{\sqrt{\pi}}\exp\left[-s_i\exp\left(\frac{\sqrt{2}\sigma_X a_c+\mu_X}{\xi}\right)\right]=\prod_{k=1}^N\\&\left(\sum_{c=1}^C \frac{w_c/\sqrt{\pi}}{1+s_i\exp\left(\frac{\sqrt{2}\sigma_k a_c+\mu_k}{\xi}\right)}\right),\ \text{at } i=1 \text{ and } 2,\end{aligned} \tag{13}$$

where, as already stated μ_X and σ_X are the unknown. The right hand side of (13) consists entirely of known quantities and is evaluated twice at s_1 and s_2. By evaluating at $s_1=0.2$ we can find μ_X, while using $s_2=1.0$ gives σ_X. The values of s_1 and s_2 have been found by solving an optimization problem as listed in [9]. It can be noted that (13) is a non linear equation and can only be solved numerically. We used `fsolve` function in MATLAB to solve it. Once the values of μ_X and σ_X have been calculated, the description of the sum distribution can be completely specified, i.e., the sum of Suzuki RVs has been approximated by a log-normal RV with calculated μ_X and σ_X. Hence the conditional probability that the received SNR $(Y^{(k)}=10^{0.1X^{(k)}})$ at the kth node is greater than or equal to τ in (7) becomes

$$\begin{aligned}&\mathbb{P}\left\{Y^{(k)}\geq\tau|\varphi\right\}=\mathbb{P}\left(10^{0.1X^{(k)}}\geq\tau\right)=\\&\mathbb{P}\left(X^{(k)}\geq 10\log\tau\right)=Q\left(\frac{10\log\tau-\mu_X{}^{(k)}}{\sigma_X{}^{(k)}}\right),\end{aligned} \tag{14}$$

where Q-function denotes the tail probability; $Q(x) = \frac{1}{2\pi}\int_x^{\infty} e^{-t^2/2}dt$. Thus the success probability of a node depends upon the threshold τ, μ_X, and σ_X; while the μ_X and σ_X further depend on the number N of Suzuki RVs (the number of DF nodes) and the μ and σ of each Suzuki RV as given in (13). Equation (14) provides us the success probability of a single node to decode. For M nodes in a level, consider $\mathbb{N}_n^{(j)}$ and $\overline{\mathbb{N}}_n^{(j)}$ as the set of indices of those nodes, which are 1 and 0, respectively, at time instant n in state j, then the one-step transition probability of going from state i to state j is given by

$$\mathbb{P}_{ij} = \prod_{k\in\mathbb{N}_n^{(j)}} \left\{ Q\left(\frac{10\log\tau - {\mu_X}^{(k)}}{{\sigma_X}^{(k)}}\right)\right\} \times \prod_{k\in\overline{\mathbb{N}}_n^{(j)}} \left\{ 1 - Q\left(\frac{10\log\tau - {\mu_X}^{(k)}}{{\sigma_X}^{(k)}}\right)\right\}. \quad (15)$$

The one step transition probability is the product of the success probabilities of those nodes, which decode successfully, times the product of the outage probabilities of those nodes, which do not decode successfully. Equation (15) gives one entry of the matrix $\mathbf{P}$. Similarly, we can find the transition probability matrix $\mathbf{P}$ by finding the transition probabilities for all the transient state space.

B. Log-normal Approximation of the Sum of multiple Log-normal RVs for Shadowing Channel

In the previous subsection, we have found the transition probability matrix $\mathbf{P}$ for the composite channel model, but here we are considering the channel as a shadowing channel. Shadowing is modeled as a log-normal RV and again there is no closed-form expression of the PDF of the sum of log-normal RVs [5]. There are different approximation techniques to find the sum distribution for log-normal RVs such as Fenton-Wilkinson [12], Shwarts and Yeh [16], and Farley [16], but each of these technique have their respective disadvantages as mentioned in [15]. Therefore, we use the same MGF-based method to find the sum distribution of the log-normal RVs [9]. This method approximates the sum of log-normal RVs to a single log-normal RV. Approximating the sum of N log-normal RVs $(\bar{S_1}, \bar{S_2}, ..., \bar{S_N})$ to a single log-normal RV $\bar{Y} = 10^{0.1\bar{X}}$, (12) becomes

$$\widehat{\Psi}_{\bar{Y}}(s_i; \mu_{\bar{X}}, \sigma_{\bar{X}}) = \prod_{k=1}^{N} \widehat{\Psi}_{\bar{S}_k}(s_i; \bar{\mu}_k, \bar{\sigma}_k), \text{ at } i = 1 \text{ and } 2, \quad (16)$$

where $\widehat{\Psi}_{\bar{Y}}(s_i; \mu_{\bar{X}}, \sigma_{\bar{X}})$ is the MGF of the log-normal RV $\bar{Y}$, and $\widehat{\Psi}_{\bar{S}_k}(s_i; \bar{\mu}_k, \bar{\sigma}_k)$ is the MGF of *k*th log-normal RV $\bar{S}_k$. By putting the MGFs obtained by Gauss-Hermite integration of both log-normal $\bar{Y}$ and $\bar{S}_k$, (16) becomes

$$\sum_{c=1}^{C} \frac{w_c}{\sqrt{\pi}} \exp\left[-s_i \exp\left(\frac{\sqrt{2}\bar{\sigma}_X a_c + \bar{\mu}_X}{\xi}\right)\right] = \prod_{k=1}^{N} \left(\sum_{c=1}^{C} \frac{w_c}{\sqrt{\pi}} \exp\left[-s_i \exp\left(\frac{\sqrt{2}\bar{\sigma}_k a_c + \bar{\mu}_k}{\xi}\right)\right]\right), \text{ at } i = 1 \text{ and } 2,$$

where the right hand side again consists of known quantities and the unknown $\bar{\mu}_X$ and $\bar{\sigma}_X$ are found by numerically solving

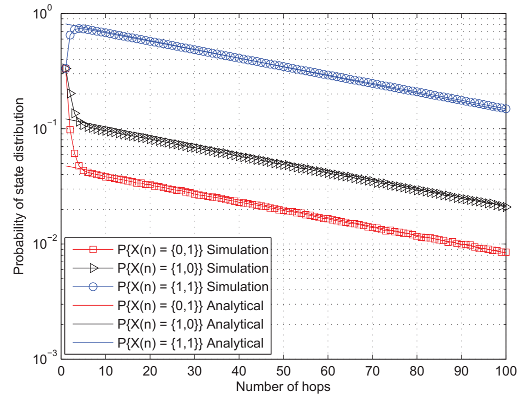

Fig. 2. Comparison of analytical and simulation model for $M = 2$.

it for $s_1 = 0.2$ and $s_2 = 1.0$, respectively. Once the $\bar{\mu}_X$ and $\bar{\sigma}_X$ are obtained, Equations (14) and (15) can be used to find the node success probability and one-step transition probability under shadowing channel model, respectively. In a similar way we can find other entries of the matrix $\mathbf{P}$. It should be pointed out here that shadowing generally impacts the system performance in a correlated manner [17]. However, in order to compare the three channel models, we assume independent fading and shadowing. The case of correlated shadowing is left as a future work.

V. Results and System Performance

In this section, we present our simulation and analytical results. We obtain all the results by considering the composite channel model unless otherwise stated. In order to simulate the composite envelope, we generate the fading and shadowing processes separately and then multiply them together, while keeping unit mean for the fading envelope. For simulation purposes, we first assume an initial distribution of the first hop and then calculate the received power at a node in the next hop. The indicator function, $\mathbb{I}$, is set to 1 only if the received power is greater than the threshold τ. Same procedure is repeated for all the nodes in the current hop, which forms the current state and the process continues until an absorbing (all-zero) state is encountered. Fig. 2. shows the probability of state distribution of the Markov chain at different hops, for $\sigma = 10$ dB and $M = 2$. For $M = 2$, the total number of transient states is 3, namely $\{0,1\}$, $\{1,0\}$, and $\{1,1\}$, and this figure shows the probability of being in these transient states at various hops by using both the simulation results and the analytical model. The simulation results are obtained by averaging over one million simulation experiments, whereas the analytical curves are obtained by using (3). It is clear from the figure that both the analytical and the simulation results are quite close to each other, which confirms the accuracy of the proposed analytical model. It can be noticed for the simulation results that the initial distribution of all the three states have nearly equal probability of occurrence, i.e., $1/3$. However, as the hop count increases, the distribution approaches the quasi-stationary distribution obtained from (3). The probability of being in a transient state decreases as the hop count increases, which shows that eventually the transmission will stop propagating, which is in accordance with Equation (2). It

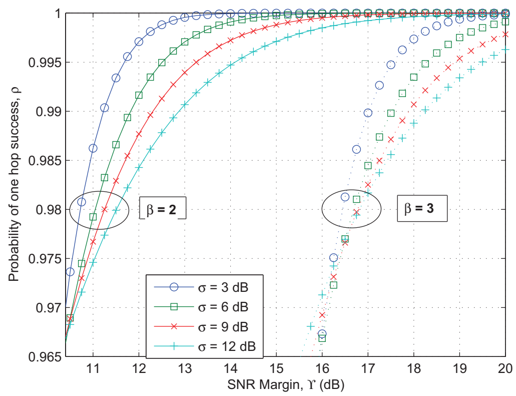

Fig. 3. Probability of one-hop success vs. SNR margin for $M = 3$.

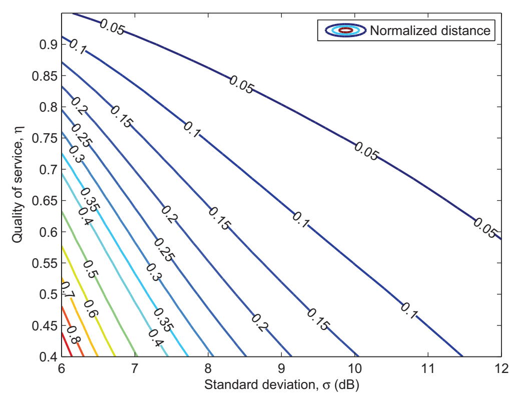

Fig. 4. Contour of normalized distance as a function of η and σ; $\Upsilon = 15$ db and $M = 3$.

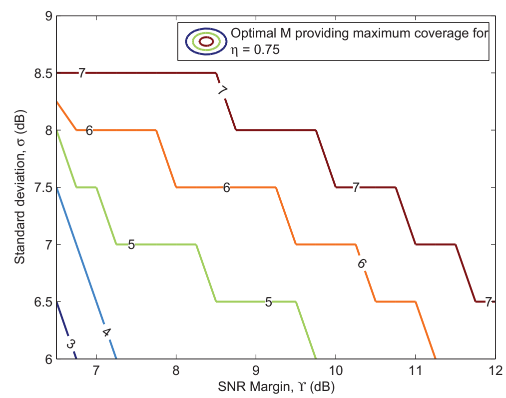

Fig. 5. Optimal M for maximum coverage at $\eta = 0.75$.

can be further observed that the slopes of the curves for all the three transient state remain the same, which means that the probability of being in either of these states remain unchanged regardless of the number of hops. Similar results are obtained for other M, which are not shown here to avoid repetitions.

Before discussing further results, we define a few terms such as probability of one-hop success, ρ and the SNR margin, Υ. The probability of one-hop success, ρ, is the probability that at least one node in a level decodes successfully or, in other words, the probability that the Markov chain $\mathcal{X}(n)$ is in a transient state T, i.e., $\mathbb{P}\{\mathcal{X}(n) \in T\}$. This probability of one-hop success is given by the Perron-Frobenius eigenvalue of $\mathbf{P}$ [14]. The SNR margin, Υ, is the normalized received SNR at a node, which is a distance d away from it transmitter such as

$$\Upsilon = \frac{P_t}{d^\beta \tau}. \tag{17}$$

We assume unit P_t and unit d for all the results, while β and τ have their usual meaning of path loss exponent and the modulation dependent threshold, respectively. The value of β is 2 unless otherwise stated. Thus, if SNR margin is 10 dB, then $P_t = 1$, $d = 1$, $\beta = 2$, and $\tau = 0.1$. In the same way we change the SNR margin by changing τ. For Fig. 3. and onwards all the results are obtained by the analytical model.

Fig. 3. shows the probability of one-hop success ρ, versus SNR margin Υ for different values of shadowing standard deviation σ and keeping $M = 3$ fixed. It can be noticed that for a specific σ, as the SNR margin is increased, the probability of one-hop success also increases. However, by increasing the σ of the log-normal shadowing, the probability of one-hop success drops at a specific value of SNR margin, which shows the effect of the severity of shadowing on the network performance. It can also be noticed that if we increase the path loss exponent, β, from 2 to 3 then an additional SNR margin of 6 dB (approximately) is required to achieve the same ρ. It can further be noticed that all the curves for different σ converge at higher Υ, which show that by increasing Υ we can overcome the losses incurred due to composite fading and path loss.

For the deployment point of view, it is desired to optimize certain parameters like transmit power of the nodes or the distance between them to achieve certain quality of service (QoS), η. The QoS is defined as the probability that a message is delivered to a certain distance without being entered into an absorbing state. The ideal value of QoS is 1. We can use (4) to find an upper bound on the number of hops, n_0, one can go with a given η, i.e., $\rho^{n_0} \geq \eta$, which gives

$$n_0 \leq \frac{\ln \eta}{\ln \rho}, \tag{18}$$

while by multiplying the number of hops n_0 by M gives the maximum distance than can be reached with a certain QoS η. Fig. 4. shows the contours of the network coverage in term of normalized distance as a function of η and σ at a specific SNR margin of $\Upsilon = 15$ dB and $M = 3$. The normalization is done for a better representation of the figure. It can be noticed that a particular distance can be reached by different combination of η and σ. The increase in η or σ drops the coverage of the network. At $\sigma = 6$ dB the normalized distance of 0.1 can be reached with $\eta \approx 0.90$, however if σ is increased to 11 dB, then the same normalized distance can only be reached by $\eta \approx 0.45$ as shown in Fig. 4. This loss of QoS shows the effect of increasing the severity of shadowing on the network.

Given a value of the SNR margin and the environment statistics in term of σ, an important question to be asked is, "what level of cooperation is optimal to have the maximum

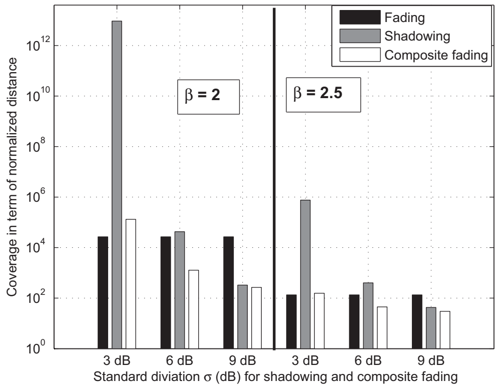

Fig. 6. Coverage of the network under three different channel models.

coverage of the network?" In other words we are interested in finding the optimal value of M that yields maximum coverage for given channel conditions. Fig. 5. shows the contours of the optimal M, for $\eta = 0.75$, under various SNR margins and standard deviation of the shadowing. This result is obtained by first calculating the number of hops n_0 using (18) and then multiplying the number of hops n_0 by M to find the coverage. It can be noticed that when both σ and Υ are small, then a lower M will provide maximum coverage and vice versa. It can be seen that at $\sigma = 7$ dB and $\Upsilon = 8$ dB , the maximum coverage is obtained by selecting $M = 5$, however if σ is unchanged and the Υ is increased to 10 dB, then the maximum coverage is obtained by $M = 6$.

Fig. 6. shows the coverage of the network for three different channel model, i.e., fading, shadowing and composite shadowing-fading. The coverage is shown in term of normalized distance at $M = 3$, $\Upsilon = 15$ dB, and $\eta = 0.90$. Three coverage behaviors are shown for shadowing and composite channel models at three different σ's. Since, fading is independent of σ, the result of fading channel is repeated with different results of shadowing and composite channel. The coverage of fading channel model has been obtained from [3]. It can be noticed that fading channel has the worst coverage when compared to shadowing and composite channels at $\sigma = 3$ dB. Shadow-only channel model provides the best coverage because of small $\sigma = 3$, introducing a macro-diversity effect. At $\sigma = 6$ dB, both the coverage of fading and shadowing are comparable while composite channel gives the worst performance. The coverage at $\sigma = 9$ dB is worst for composite shadowing-fading while fading only channel provides the best performance. It can be seen that increasing the β from 2 to 2.5 does not change the trend of the coverage for the three channel models, however, the coverage of each channel model drops by increasing the path loss exponent. It can be inferred from the figure that within normal range of σ, i.e., 6-12 dB composite channel provides the lowest coverage as compared to fading and the shadowing channel model. Hence it is recommended to consider both the small-scale fading as well as shadowing while quantifying the performance of wireless networks.

VI. Conclusion

A stochastic model for a cooperative multi-hop line network is presented. The transmission from one level to another is modeled as a quasi-stationary Markov process. The transition probability matrix of the Markov chain is derived by considering the wireless channel as a composite shadowing-fading channel. Shadowing is modeled as a log-normal RV while multipath fading is modeled as a Rayleigh RV. A multiplicative model is used to find the mixture distribution, which becomes a Suzuki distribution. The sum of the multiple Suzuki RVs is approximated by an MGF-based technique, which uses the Gauss-Hermite integration to find the closed-form expression for the MGF of Suzuki and log-normal RVs. The SNR margin required to achieve a certain quality of service under various system parameters has been quantified. The optimal number of nodes in a level to give maximum coverage for a given SNR margin and shadowing standard deviation has been quantified for a given QoS. A possible future work is to expand the current model to a two dimensional grid and incorporate the interference in the model if multiple flows are considered.

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