Range Extension Using Optimal Node Deployment in Linear Multi-hop Cooperative Networks

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***Abstract*— This paper considers the optimal deployment of nodes in a one-dimensional multi-hop cooperative network for obtaining the maximum network coverage under a quality of service constraint. It is assumed that the total number of nodes to be deployed are fixed and the objective is to arrange the nodes in the form of clusters to obtain maximum coverage such that the end-to-end probability of making successful hops is greater than a threshold. The transmissions that originate from one cluster to another are modeled as discrete-time Markov chain and the transition probability matrix of the chain is derived. By invoking the Perron-Frobenius Theorem and studying its properties, it is shown that a smaller sized cluster of nodes provides the maximum coverage of the network if the path loss exponent is large and vice versa.**

***Index Terms*— Multi-hop networks, cooperative communications, path loss exponent, Markov chain**

I. Introduction

Multi-hop wireless transmission, where radios forward the message of other radios, is becoming popular both in cellular as well as sensor networks. In a conventional multi-hop network, the data is propagated over several multiple radios in a single-input single-output (SISO) fashion, which considerably lowers the end-to-end reliability of data transmission. Cooperative transmissions (CT) has been introduced to increase the reliability of the system, where multiple radios transmit the same information bearing signal to the destination through multiple hops through uncorrelated fading channels. In many wireless networks, multi-hop CT not only increases the reliability of the network but also provides an energy-efficient mechanism for data transmission.

An important application of a cooperative multi-hop network is its ability to extend the range without increasing the transmit power of individual radios. This has been achieved by the implication of increased diversity order of the received signal, which provides a signal-to-noise ratio (SNR) advantage. Many deployment strategies have been proposed to increase the range of a network. For instance, in [1] an experimental CT test-bed, consisting of software defined radios, has been implemented in an indoor environment that shows an increased network coverage as compared to SISO transmissions. Similarly, many approaches as in [2] propose the optimal location of relays to get increased network range.

A simple and fast CT scheme is opportunistic relaying, in which all the nodes that can decode the data at one time, transmit the decoded data to the next layer of nodes and this process continues. This relaying scheme, known as opportunistic large array (OLA) transmission is first studied in [3], however the authors assumed that the number of nodes in an area goes to infinity while keeping the transmit power constant per unit area. This approach may not be very practical for finite density network. A finite density one-dimensional OLA network was studied in [4], where the authors derived the upper bounds on the coverage of the network while keeping equal transmit power of all the nodes and assuming equal distances between them. Random placement of the nodes was studied in [5]. It was then indicated in [6] that a co-located cluster of nodes provides better coverage of the network as compared to spacing them equally on a linear network. However, if the nodes transmit in clusters, where the clusters are equally spaced, it remained unclear that if the total number of nodes to be deployed on the linear network are fixed, what is the best topology, which provides the maximum range extension of that network. In this paper, we have proposed that in a Rayleigh faded environment and with fixed number of nodes, the best topology of nodes (in terms of cluster sizes) that provides maximum coverage depends on the value of the path loss exponent. We have modeled the transmissions from one level to another as discrete-time Markov chain and have shown that the properties of the transition matrix provides useful insight into the coverage of the network.

II. System Model

Consider a linear network topology, as shown in Fig. 1, where the total number of nodes to be deployed are fixed and is denoted by L. We restrict the groups of *candidates* for cooperation in a given hop to have the same number of nodes. A group consists of M co-located nodes, having an inter-group distance D. We assume equal transmit power,

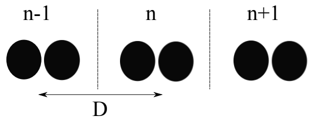

Fig. 1. Deployment of nodes for $M = 2$ and $L = 6$

P_t, of all nodes. Let $\mathbb{N}_n$ with cardinality $\mathcal{K}_n$ denotes the set of indices of those decode-and-forward (DF) nodes that decoded the signal perfectly at the time instant (or hop) n. Thus, the received power at the jth node at time $n+1$ is given as $Pr_j(n+1) = \frac{P_t}{D^\beta}\sum_{m\in\mathbb{N}_n}\mu_{mj}$, where μ_{mj} is the exponential channel gain (corresponding to flat Rayleigh fading) from Node m in the previous level to Node j in the current level and β is the path loss exponent.

The state of each node is characterized by a binary indicator function such that for jth node at time n, $\mathbb{I}_j(n) = 1$ represents successful decoding and $\mathbb{I}_j(n) = 0$ represents a failure in decoding. Thus the decision of all nodes in Level n are given as $\mathcal{X}(n) = [\mathbb{I}_1(n), \mathbb{I}_2(n), ..., \mathbb{I}_M(n)]$, which depends only upon the transmission of the previous level, making $\mathcal{X}$ a memoryless Markov process. Because of discrete time slots, $\mathcal{X}$ is a discrete-time Markov chain such that $\mathcal{X} = \{0\} \cup \mathcal{S}$, where $\mathcal{S}$ is a finite transient irreducible state space, corresponding to all the states in which at least one node in the group is able to decode, and 0 is the absorbing state. If we remove the transitions to and from the absorbing state, then the transition probability matrix, **P**, for the Markov chain, $\mathcal{X}$, is right sub-stochastic. Considering these properties, we invoke the Perron-Frobenius theorem [4], which says that there exists a maximum eigenvalue, ρ, and an associated left eigenvector **u** with strictly positive entries such that $\mathbf{uP} = \rho\mathbf{u}$. Since $\forall n$, $\mathbb{P}\{\mathcal{X}(n) = 0\} > 0$, eventual absorption is certain, and the limiting distribution, also called the quasi-stationary distribution, of the Markov chain is given as $\lim_{n\to\infty}\mathbb{P}\{\mathcal{X}(n) = j|T > n\} = u_j, \quad j \in \mathcal{S}$, where $T = \inf\{n \geq 0 : \mathcal{X}(n) = 0\}$ denotes the end of survival time.

For the kth node in the nth level, the conditional probability of being able to decode is given as

$$\mathbb{P}\{\mathbb{I}_k(n) = 1|\zeta\} = \mathbb{P}\{\gamma_k(n) > \tau|\zeta\}, \tag{1}$$

where $\gamma_k(n) = Pr_k(n)/\sigma_k^2$ is the received SNR at kth node, and σ_k^2 is the noise variance of the kth receiver. We denote the event $\{\mathcal{X}(n-1) \in \mathcal{S}\} := \zeta$, implying that the previous state is a transient state. Similarly, the probability of outage or the probability of $\mathbb{I}_k(n) = 0$ is $1 - \mathbb{P}\{\gamma_k(n) > \tau|\zeta\}$, where

$$\mathbb{P}\{\gamma_k(n) > \tau|\zeta\} = \int_\tau^\infty p_{\gamma_k|\zeta}(y)dy, \tag{2}$$

and $p_{\gamma_k|\zeta}(y)$ is the conditional probability density function (PDF) of the received SNR at the kth node conditioned on State $\mathcal{X}(n-1)$.

The received power at a certain node in a group is the sum of the finite powers from the previous-level nodes, where the power received from each transmitting node is exponentially distributed with the same parameter $\tilde{\lambda} = D^\beta\sigma_k^2/P_t$. The resulting PDF of the received power at the kth node in a cluster is Gamma distribution given as

$$p_{\gamma_k|\zeta}(y) = \frac{1}{(\mathcal{K}_n-1)!}\tilde{\lambda}^{\mathcal{K}_n}y^{(\mathcal{K}_n-1)}\exp\left(-\tilde{\lambda}y\right). \tag{3}$$

Evaluating (2) to get the conditional success, we have

$$\mathbb{P}\{\gamma_k(n) > \tau|\zeta\} = \frac{1}{(\mathcal{K}_n-1)!}\Gamma(\mathcal{K}_n, \tilde{\lambda}\tau), \tag{4}$$

where $\Gamma(\mathcal{K}_n, \tilde{\lambda}\tau)$ is the upper incomplete Gamma function. Let $\Phi^{(k)} := \mathbb{P}\{\gamma_k(n) > \tau|\zeta\}$, then after some manipulation, (4) becomes

$$\Phi^{(k)} = \exp\left(-\tilde{\lambda}\tau\right)\sum_{p=0}^{\mathcal{K}_n-1}\frac{\left(\tilde{\lambda}\tau\right)^p}{p!}. \tag{5}$$

Then the one step transition probability for going from State i to j is given as

$$\mathbb{P}_{ij} = \prod_{k\in\mathbb{N}_{n+1}^{(j)}}\left(\Phi^{(k)}\right)\prod_{k\in\overline{\mathbb{N}}_{n+1}^{(j)}}\left(1-\Phi^{(k)}\right), \tag{6}$$

where $\overline{\mathbb{N}}_{n+1}^{(j)}$ is the set of those nodes in the State j, which have not decoded the signal.

III. Results and Network Performance

In this section, we evaluate the performance of the linear network in terms of its coverage under a quality of service (QoS) constraint, η. We define η to be the end-to-end probability of success such that the destination node receives the data with at least η. This QoS is directly related to the one-hop success probability, which is the Perron-Frobenius eigenvalue, ρ, of the Markov chain. The overall success probability in delivering the message to a destination m hops away is given as ρ^m. To fulfill the QoS constraint, we deploy the co-located nodes such that $\rho^m \geq \eta$. In this case, the number of hops, m, depends upon the number of nodes in the cluster, M, and the total number of available nodes, L. For instance, if $L = 24$, then the value that $M \in \{2, 3, 4, 6, 8, 12\}$. If we deploy two nodes per cluster, i.e., $M = 2$, then the total number of hops, m, will be 12. Hence if 90% success probability is required at the network edge, then $\rho \geq \exp\left\{\frac{\log\eta}{m}\right\}$, which turns out to be 0.9913. We also define $\Upsilon = \frac{P_t}{\tau\sigma^2}$ as the normalized SNR and call it SNR margin. The goal is find the maximum one-hop distance D, which guarantees the desired ρ for a given combination of M and L.

Fig. 2 shows the relationship between the maximum achievable distance and the path loss exponents for the case described above at $\Upsilon = 15dB$. It can be observed the maximum achievable distance decreases with an increase in the

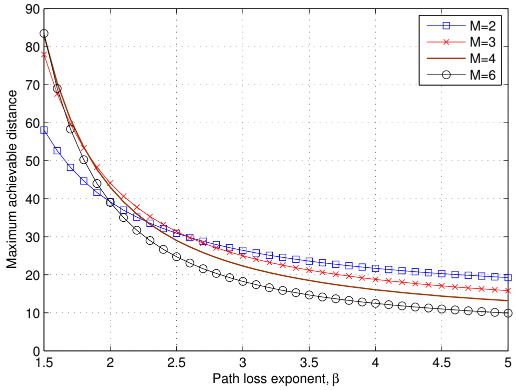

Fig. 2. Relationship between maximum achievable distance and path loss exponent for various cluster sizes

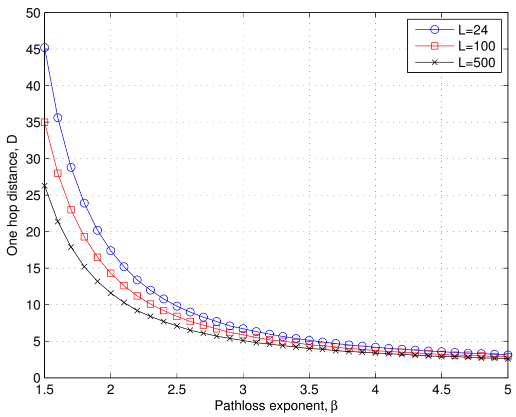

Fig. 3. Effect of network length on maximum one hop distance

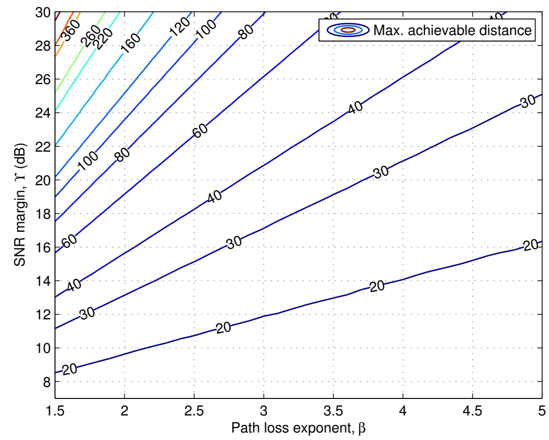

Fig. 4. Contours of maximum achievable distance as a function of path loss exponent and SNR margin

the path loss exponent. However, an interesting behavior is seen by deploying different cluster sizes. In other words, a particular diversity order provides the optimal performance in terms of the network coverage for a particular value of path loss exponent. It can be seen that a small sized cluster is suitable at a relatively larger value of path loss exponent and vice versa. Hence, a suitable combination of M and β governs the maximum range extension in linear multi-hop networks. Similar behavior is observed at various values of SNR margins.

Fig. 3 represents the maximum one-hop distance for $M = 2$ case, where the total number of nodes can be $L = \{24, 100, 500\}$. In all the cases, $\eta = 0.9$ while $\Upsilon = 10$dB, however, the number of hops, m, increases as L increases. It can be observed that as the network grows in length, the one-hop distance decreases regardless of the value of the path-loss exponent. Similar results are obtained for different cluster sizes but are not shown to avoid repetition.

It is sometimes desired to achieve a particular distance for a particular β at the expense of an SNR margin. Fig. 4 shows the contours of the network coverage distance as a function of SNR margin and the path loss exponent for $M = 2$ and $L = 24$ case. It can be seen that various combinations of the SNR margin and path loss exponent gives the desired maximum achievable distance.

IV. Conclusions

In this paper, we have studied the performance of a multi-hop cooperative linear network where the nodes can be deployed as clusters and the size of the cluster can be varied. The transmission from one cluster of nodes to another is modeled as a memoryless Markov chain and the transition matrix of the chain has been derived. It is concluded that the maximum network coverage depends on the path loss exponent and the cluster size under an end-to-end probability of success constraint. A smaller sized cluster provides better network coverage for a large path loss exponent and vice versa.

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