SNR Estimation for a Non-Coherent Binary Frequency Shift Keying Receiver

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***Abstract*—This paper deals with the problem of estimating the average signal-to-noise ratio (SNR) for a communication system employing non-coherent binary frequency shift keying (NCBFSK) over fading channels with white Gaussian noise (AWGN). The maximum likelihood (ML) estimator and one using data statistics have been derived and simulated for various scenarios including data-aided (DA), non-data aided (NDA) and joint estimation using both the data and pilot sequences. We also derive the Cramer-Rao bound (CRB) for the estimators. The results show that for a particular region of interest (e.g. high SNR or low SNR) and depending upon the availability of pilot sequence, a particular SNR estimation scheme is suitable.**

I. INTRODUCTION

Estimates of signal-to-noise ratio (SNR) are used in many wireless receiver functions, including signal detection, power control algorithms and turbo decoding etc. The motivation for the study reported here is that SNR estimation is a way for a receiver to determine if it is near the edge of the decoding range of its source, and therefore, in a preferred location to participate in a cooperative transmission [1-2]. Furthermore, if the radios are energy constrained, e.g., if they are in a sensor network, non-coherent demodulation may be desired to reduce circuit consumption of energy. Therefore, in this paper, we consider the estimation of average SNR in an FSK non-coherent demodulator, over a Rayleigh fading channel.

Several authors have attacked the problem of estimating SNR for binary phase shift keying (BPSK) and frequency shift keying (FSK). For example, [3] compares a variety of techniques for SNR estimation in AWGN for M-PSK signals. Many approaches also include the channel effects such as multipath fading and address the issues of SNR estimation for fading channels for BPSK e.g., in [5-7]. FSK enables a simple receiver design that employs envelope detection [8]. In [9], the authors have estimated the SNR for non-coherent binary FSK (NCBFSK) receiver, assuming unity noise power spectral density. However, in implementations, noise power must also be estimated. In this paper, we derive two types of estimators of SNR, a maximum likelihood estimator (MLE) and an estimator that uses block statistics, such that neither of them assume knowledge of noise power. [9] derives the SNR estimate for pilot and data symbols separately. We provide ML versions of partially data-aided (PDA), non-data aided (NDA), joint PDA-NDA, and fully data-aided (FDA) estimators for SNR. The PDA approach uses only the training sequence for estimation while the NDA approach does blind estimation using the entire sequence. The joint PDA-NDA uses all the information, operating blindly on the non-training part of the sequence. The FDA estimator uses the detected data as training sequence for SNR estimation and is reasonable in a multi-hop broadcast application, where every node must decode the entire message; the detected data are all assumed to be correct in the paper regardless of the value of SNR.

The rest of the paper is organized as follows. In the next section, we describe the system model and the notations used for the BFSK case. Section III deals with the derivations of the SNR estimators, which includes three sub-cases for MLE and also the estimator using data statistics. Then we will derive the CRB and in Section V, we will discuss the simulation results for various estimators and overall estimator performance in terms of mean-squared error and CRB. The paper then concludes in Section VI.

II. SYSTEM MODEL

Consider a communication system employing BFSK modulation. From [9], the received signal, for the BFSK case, is given as

$$\mathbf{v}_i = \mathbf{s}_i \alpha_i + \mathbf{n}_i \tag{1}$$

where each of $\mathbf{v}_i$, $\mathbf{s}_i$, and $\mathbf{n}_i$ are real vectors with a dimension of 2×1, and $\mathbf{s}_i$ is given as $[1 \quad 0]^T$ or $[0 \quad 1]^T$ with equal probability. For the sake of simplicity, we assume that the constellation symbol energy is unity so that the total power of the signal is given as $E\left\{\alpha^2\right\} = S/2$, where α is a zero mean fading coefficient drawn from a Gaussian distribution. Similarly, the noise is also a Gaussian random variable with zero mean and variance $N/2$, thus the SNR is given by $\gamma = S/N$. $\mathbf{s}_i$, α_j, and $\mathbf{n}_k$ are assumed to be independent of each other for any i, j and k. The all-real signal model follows from the temporary assumption of a coherent receiver. Our interest is to find the estimate of SNR using the observed data $\left[\mathbf{v}_1^T \quad \mathbf{v}_2^T \quad \cdots \quad \mathbf{v}_k^T\right]^T$. For the estimation schemes considered, we assume that that there are g pilot symbols and l data

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symbols so that the total packet length is $k = g+l$. Throughout the paper, we assume perfect timing recovery at the receiver.

III. Estimation Techniques

As mentioned previously, we will derive ML estimation for three cases, namely PDA, NDA and joint PDA-NDA. Another approach is using the statistics of observable data, which we call Estimation using Data Statistics (EDS).

A. Partially Data Aided MLE

Without the loss of generality, the g pilot symbols are set to $[1 \quad 0]^T$. The probability density functions (PDFs) of the received symbols $\mathbf{v}_i = [x_i \quad y_i]^T$, are given as

$$p_x(x_i) = \frac{1}{\sqrt{\pi}\sqrt{S+N}} \exp\left(-\frac{x_i^2}{S+N}\right) \tag{2}$$

and

$$p_y(y_i) = \frac{1}{\sqrt{\pi}\sqrt{N}} \exp\left(-\frac{y_i^2}{N}\right). \tag{3}$$

The joint pdf of $\mathbf{v}$ is given as

$$p_v(\mathbf{v}_i) = \frac{1}{\pi\sqrt{S+N}\sqrt{N}} \exp\left(-\frac{x_i^2}{S+N} - \frac{y_i^2}{N}\right). \tag{4}$$

Thus the log-likelihood distribution of g received symbols is given as

$$\begin{aligned}\Lambda_{PDA} = &- g\ln\pi - \frac{g}{2}\ln(S+N) - \frac{g}{2}\ln N \\ &- \frac{1}{S+N}\left(\sum_{i=1}^{g} x_i^2\right) - \frac{1}{N}\left(\sum_{i=1}^{g} y_i^2\right).\end{aligned} \tag{5}$$

To find the MLE of SNR, $\hat{\gamma}$, we use the property that the ML estimate of the ratio of two parameters (S and N here), is the ratio of the individual ML estimates of the two parameters [10]. Thus using this property, the MLE of SNR can be written as

$$\hat{\gamma} = \frac{\hat{S}_{ML}}{\hat{N}_{ML}} \tag{6}$$

Thus by differentiating (5) with respect to S and N, and setting the derivatives equal to zero results in

$$\hat{S} = \frac{2}{g}\left(\sum_{i=1}^{g} x_i^2\right) - \frac{2}{g}\left(\sum_{i=1}^{g} y_i^2\right), \tag{7}$$

and

$$\hat{N} = \frac{2}{g}\left(\sum_{i=1}^{g} y_i^2\right). \tag{8}$$

Thus the MLE of the SNR using the pilot sequence is

$$\hat{\gamma}_{PDA} = \frac{\sum_g x_i^2 - \sum_g y_i^2}{\sum_g y_i^2}. \tag{9}$$

B. Non Data-Aided MLE

Assuming equal prior probabilities of transmitted symbols, the PDF of the received symbols is given as

$$\begin{aligned}p_v(\mathbf{v}) = &\frac{1}{2\pi\sqrt{S+N}\sqrt{N}} \\ &\left[\exp\left(-\frac{x_i^2}{S+N} - \frac{y_i^2}{N}\right) + \exp\left(-\frac{x_i^2}{N} - \frac{y_i^2}{S+N}\right)\right].\end{aligned} \tag{10}$$

Using the fact that

$$\exp(x_1) + \exp(x_2) = 2\exp\left(\frac{x_1+x_2}{2}\right)\cosh\left(\frac{x_1-x_2}{2}\right),$$

where cosh(.) is the hyperbolic cosine function, the log-likelihood function for k received symbols, is given as

$$\begin{aligned}\Lambda_{NDA} = &- \frac{k}{2}\ln N - \frac{k}{2}\ln(S+N) \\ &- \frac{1}{2}\sum_{i=1}^{k}\left(x_i^2 + y_i^2\right)\left[\frac{1}{N} + \frac{1}{S+N}\right] \\ &- k\ln\pi + \sum_{i=1}^{k}\ln\cosh\left(\frac{x_i^2 - y_i^2}{2}\frac{S}{N(S+N)}\right).\end{aligned} \tag{11}$$

The partial derivatives of (11) with respect to S and N are given in (12) and (13) (at the bottom of next page), respectively, where $\psi = \frac{x_i^2 - y_i^2}{2}\frac{S}{N(S+N)}$. An exact solution to (12) and (13) is difficult to obtain because of the non-linearity of the $\tanh(.)$ function. It can be observed that $\tanh(x) \approx +1$ when $x > 0$ and $\tanh(x) \approx -1$ when $x < 0$. Thus using the high SNR approximation $\frac{S}{S+N} \approx 1$ for $S >> 1$, hyperbolic tangent function can be approximated to a signum function. Putting (12) and (13) equal to zero, we get

$$\hat{S} = \frac{1}{k}\left[\sum_{i}\left(x_i^2 + y_i^2\right) + \sum_{i=1}^{k}\left|x_i^2 - y_i^2\right|\right] - \hat{N}, \tag{14}$$

and

$$\hat{N} = \frac{1}{k}\left[\sum_{i=1}^{k}\left(x_i^2 + y_i^2\right) - \sum_{i=1}^{k}\left|x_i^2 - y_i^2\right|\right]. \tag{15}$$

Thus the estimate of SNR is given as

$$\hat{\gamma}_{NDA} = \frac{2\sum_{i=1}^{k}\left|x_i^2 - y_i^2\right|}{\sum_{i=1}^{k}\left(x_i^2 + y_i^2\right) - \sum_{i=1}^{k}\left|x_i^2 - y_i^2\right|}. \tag{16}$$

C. Joint Estimation Using Pilot and Data Symbols

Consider g pilot symbols and l data symbols, so that the total packet is of length $k = g + l$. Assuming independent received symbols, the joint PDF is the product of PDFs resulting from pilot and data symbols. Thus the log-likelihood function from the joint PDF is given as

$$\Lambda_{joint} = -\frac{k}{2}\ln\left[N\left(S+N\right)\right] - \frac{1}{S+N}\sum_{i=1}^{g} x_i^2 - \frac{1}{N}\sum_{i=1}^{g} y_i^2 - k\ln\pi - \frac{1}{2}\sum_{i=g+1}^{g+l}\left(x_i^2+y_i^2\right)\left(\frac{S+2N}{N(S+N)}\right) + \sum_{i=g+1}^{g+l}\ln\cosh\left(\frac{x_i^2-y_i^2}{2}\frac{S}{N(S+N)}\right). \quad (17)$$

Using similar approximations as done in the previous section and taking partial derivatives with respect to S and N and setting them equal to zero result in the estimate of SNR as

$$\hat{\gamma}_{joint} = \frac{2\left(\sum_{i=g+1}^{g+l}|x_i^2-y_i^2| + \sum_{i=1}^{g} x_i^2 - y_i^2\right)}{\sum_{i=g+1}^{g+l}\left(x_i^2+y_i^2-\left(|x_i^2-y_i^2|\right)\right) + 2\sum_{i=1}^{g} y_i^2}. \quad (18)$$

D. EDS Approach

Extending the BPSK approach used in [5] for BFSK, we define a 2×2 matrix $\mathbf{Z}$ which is determined using the observed data statistics, given as

$$\mathbf{Z} = \left(E|\mathbf{v}_i|\right)\left(E|\mathbf{v}_i|\right)^T\left(E\left\{\mathbf{v}_i\mathbf{v}_i^T\right\}\right)^{-1}, \quad (19)$$

where $E|\mathbf{v}| = [E|x| \quad E|y|]^T$. Since $\mathbf{v}_i = [x_i \quad y_i]^T$, where x and y are mutually uncorrelated, thus the autocorrelation of $\mathbf{v}$ is a diagonal matrix given as

$$E\left\{\mathbf{v}_i\mathbf{v}_i^T\right\} = \begin{bmatrix} \frac{1}{4}(S+2N) & 0 \\ 0 & \frac{1}{4}(S+2N) \end{bmatrix}. \quad (20)$$

The method proceeds from the observation that if we have a variable X which follows a Normal distribution with zero mean and variance σ^2, the absolute value $|X|$ follows a half-Normal distribution which has a mean $\sqrt{2/\pi}\sigma$. Using this fact and that $|x|$ and $|y|$ are identically distributed, we have

$$E|\zeta| = \frac{1}{2}\sqrt{\frac{1}{\pi}}\sqrt{S+N} + \frac{1}{2}\sqrt{\frac{1}{\pi}}\sqrt{N} = \frac{1}{2}\sqrt{\frac{1}{\pi}}\left[\sqrt{S+N}+\sqrt{N}\right], \qquad \zeta\in\{x,y\} \quad (21)$$

and

$$\left(E|\zeta|\right)^2 = \frac{N}{4\pi}\left[\gamma+2+2\sqrt{1+\gamma}\right]. \quad (22)$$

Thus $\mathbf{Z}$ turns out to be a matrix with all elements being equal and is expressed as $z\mathbf{1}_2$, where $\mathbf{1}_2$ is a 2×2 matrix of all ones and z is given as

$$z = \frac{\left(E|\zeta|\right)^2}{E\left\{\zeta^2\right\}} \qquad \zeta\in\{x,y\}. \quad (23)$$

After putting in the values from (20) and (22), z is given as

$$z = \frac{\gamma+2+2\sqrt{1+\gamma}}{\pi(2+\gamma)}. \quad (24)$$

In practice, we replace the expectations in (19) with the corresponding block averages to compute the SNR estimate. Figure 1 shows the relationship between computed z for various values of SNR. Since the actual curve is highly steep in the higher SNR region, it is difficult to obtain a curve fitting for computing the estimates of SNR. Thus using high SNR approximation $\frac{\gamma+1}{\gamma+2}\approx 1$, the approximate relationship between z and $\hat{\gamma}$ is given as

$$\hat{\gamma} = \frac{4}{\left(\pi z-1\right)^2} - 1 \quad (25)$$

The figure thus suggests, that the EDS approach will suffer from penalties in both the high and low SNR regimes. In the high SNR regime, the steepness of the curve will make the estimate very sensitive to errors in z, while at low SNR, the approximation error will have a bad effect on the estimation. It should also be noted that the estimate of SNR resulting from EDS approach is similar as that of MLE, if the estimation is done using pilot sequence alone. Thus the EDS approach

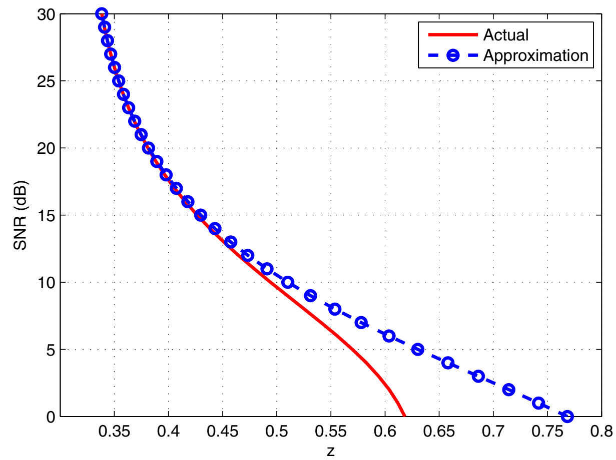

Fig. 1. Approximation of SNR in EDS approach

$$\frac{\partial\Lambda_{NDA}}{\partial S} = -\frac{k}{2}\left[\frac{1}{(S+N)}\right] + \frac{1}{2(S+N)^2}\sum_{i=1}^{k}\left(x_i^2+y_i^2\right) + \sum_{i=1}^{k}\tanh\left(\psi\right)\left(\frac{x_i^2-y_i^2}{2}\right)\left[\frac{1}{(S+N)^2}\right] \quad (12)$$

$$\frac{\partial\Lambda_{NDA}}{\partial N} = -\frac{k}{2}\left[\frac{S+2N}{N(S+N)}\right] + \sum_{i=1}^{k}\left(x_i^2+y_i^2\right)\left[\frac{1}{2N^2} + \frac{1}{2(S+N)^2}\right] - \sum_{i=1}^{k}\tanh\left(\psi\right)\left(\frac{x_i^2-y_i^2}{2}\right)\left[\frac{S(S+2N)}{N^2(S+N)^2}\right] \quad (13)$$

mentioned here will be treated as non-data aided scheme for SNR estimation.

IV. Cramer-Rao Lower Bounds

In this section, we will derive the CRLB for the FDA SNR estimator. Although we could have different CRBs for all the different cases discussed in the previous section, the one that would really serve as a benchmark on the variance of all estimators is from the FDA case, which is same as PDA but uses all the information in the packet as a training sequence. Considering the unknown parameters as a vector i.e., $\theta = [S \quad N]^T$, the CRB for the SNR is given as [11]

$$CRB = \frac{\partial \mathbf{g}(\theta)}{\partial \theta} \mathbf{I}^{-1}(\theta) \frac{\partial \mathbf{g}(\theta)^T}{\partial \theta}, \tag{26}$$

where $\mathbf{g}(\theta) = \frac{S}{N}$, the Jacobian of $\mathbf{g}(\theta)$ is given as

$$\mathbf{J_g}(\theta) = \begin{bmatrix} \frac{1}{N} & \frac{-S}{N^2} \end{bmatrix}, \tag{27}$$

and $\mathbf{I}(\theta)$ is the Fisher information matrix (FIM) given as

$$[\mathbf{I}(\theta)]_{ij} = -E\left[\frac{\partial^2 \Lambda}{\partial \theta_i \theta_j}\right]. \tag{28}$$

The FIM for the FDA estimator is given as

$$\mathbf{I}(\theta) = \begin{bmatrix} \frac{k}{2(S+N)^2} & \frac{k}{2(S+N)^2} \\ \frac{k}{2(S+N)^2} & \frac{k}{2(S+N)^2} + \frac{k}{2N^2} \end{bmatrix}, \tag{29}$$

which gives the CRB from (26) as

$$CRB_{FDA} = \frac{4}{k}(1+\gamma)^2. \tag{30}$$

This bound has been plotted in Figures 2 and 3, which are further discussed below.

V. Simulation Results

In this section, we examine the normalized mean squared error (normalized with respect to the square of the true value of SNR) of the estimator using simulations for different cases. Figures 2 and 3 show the NMSE averaged over 10000 trials for different schemes discussed and for different packet lengths. It is reasonable if we discuss two cases separately, i.e., packets with short and long lengths, respectively. For the following discussions, we are only considering the estimators with no decision feedback (i.e., non-FDA cases). The FDA scenario will be discussed afterwards.

A. Short Length Packet

Consider a packet with 8 pilot symbols and 28 data symbols. The NMSE is plotted in Figure 2. It can be seen that the best performance in terms of NMSE is given by the estimator which utilizes both the data and pilot sequence together, i.e., the joint PDA-NDA estimator. The EDS approach does not perform well and it cannot give any estimation beyond a very small range due to the limitations of the availability of data (the approximation error of the ensemble averages with time averages for small data set is large). Thus for a short length packet and with the availability of the pilot, the joint data estimation performs best and if the pilot is not available, then the NDA MLE also gives better performance.

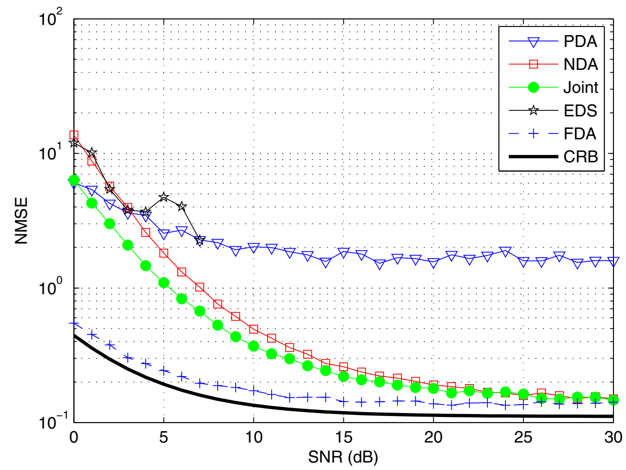

Fig. 2. NMSE for g=8 and l=28

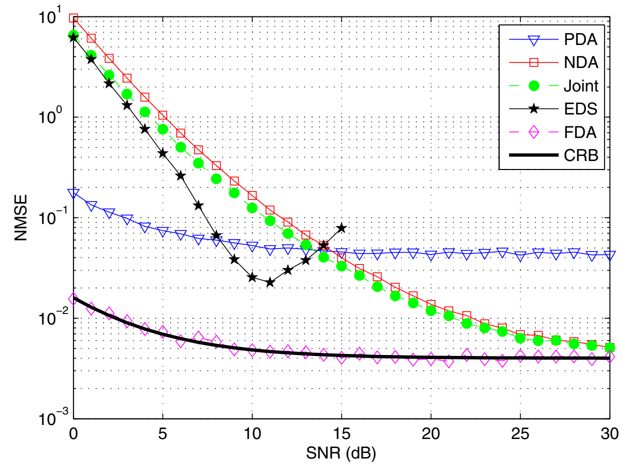

Fig. 3. NMSE for g=100 and l = 900

B. Long Length Packet

Here we consider a packet with long data sequence containing 100 symbols as pilot and 900 symbols being the actual data. Thus it can seen from Figure 3, that the PDA estimation performs the best as its NMSE is small and almost constant over the entire SNR range. MLE for the other two cases suffers from the approximation errors in the low SNR regime. The EDS method again shows bad behavior at high SNR due to the steepness of curve from Figure 1. To do a fair comparison, we assume that both the pilot and data symbols, in the frame, are available to NDA and EDS approaches for the estimation. It should also be noted that the EDS method also uses the same approximation of high SNR as the NDA and joint estimation approaches use, but the behavior of the curves suggests that the approximation error in tanh(.) function used to derive the ML

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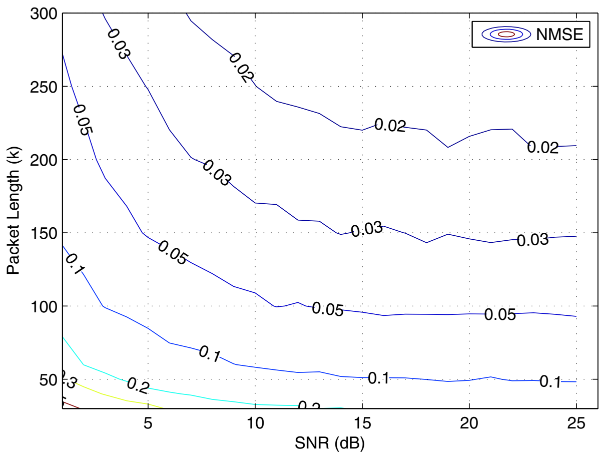

Fig. 4. NMSE contours for various packet lenghts

based estimator is even more sensitive to the value of SNR than the other approximation error. Thus EDS approach performs better even for low SNR estimation. For high SNR estimation, the joint estimation scheme works the best as expected. From the simulation curves, it could be seen that an adaptive mode of SNR estimation can also be derived, which consists of estimation from pilot only during the low SNR regime having no approximation at all while using the entire data packet for estimating high SNR values so that the overall NMSE remains minimum over a wide range of SNR values.

An important observation is that if we compare the NMSE performance of Equation (16) with Equation (17) of [9], for the NDA case, it can be noticed, that the estimators are different not only due to different approximations used, but also due to the known noise variance. In [9], the authors assume unit noise spectral density, thus that approach does not work with any other noise power. But our approach is strictly blind for both the signal as well as noise power. The equivalence of CRB's from Equation (14) of [9] and (30) shows that for data-aided estimation, the NMSE would achieve the same value for both algorithms.

By using the FDA approach (decision feedback), which utilizes the detected data, it can be seen that the performance of the estimator is enhanced significantly and it reaches the CRLB for the longer packet. Using this approach, we gain two advantages: a larger data set and estimation using DA approach which has no approximation errors. The *no errors* assumptions is good in the context of Decode and Forward (DF) scenario, since no errors is a precondition for forwarding the packet. Sometimes, it is desired to choose the packet length such that the NMSE should not exceed some specified value. A contour plot of NMSE for SNR versus the packet length for FDA case is shown in Figure 4. We observe that 5% error can be achieved with 100 symbols.

VI. CONCLUSIONS

We have derived the MLEs and CRB for SNR for a BFSK system assuming different degrees of data knowledge in a packet. It is thus concluded that different scenarios lead to different results based on packet length, availability of pilot sequence, and the region of SNR considered (low/high). If we cannot feedback detected symbols, (i.e., the FDA scheme), then an adaptive scheme is suggested. However, for the Decode and Forward cooperative relay applications, the FDA method gives good results.

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