

# Throughput and Energy Efficiency of Two-tier Cellular Networks: Massive MIMO Overlay for Small Cells

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**Abstract**—In this paper, the downlink performance of two-tier heterogeneous network is investigated. We consider a scenario where the macro-tier is empowered by massive antenna-array thus allowing for Massive multiple-input multiple-output (MIMO) transmission scheduling. The small cellular network complements the macro-tier capacity. We propose a novel channel allocation mechanism which optimally splits the spectral resources to maximize network level throughput and energy efficiency. Our proposed channel allocation mechanism is robust to the topological and channel variations. More specifically, the proposed scheme is designed by capturing the random locations of the users in both tiers by a Poisson Point Process (PPP). The channel uncertainty is captured by considering Rayleigh fading complemented by large scale power law path-loss. Our analysis shows that there exists an optimal split which maximizes the network wide throughput and energy efficiency. We also demonstrate that there exists an optimal transmit power which maximizes the energy efficiency for the network. Under different scenarios, massive MIMO plays a vital role in improving sum rate capacity as compared to single antenna femtocells. Finally, using implementation parameters, we obtain the optimal configurations that improve system capacity and energy efficiency.

## I. INTRODUCTION

### A. Motivation

MIMO technology has extensively been studied during the last two decades and applied to many wireless standards because of its ability to considerably improve the capacity and reliability of wireless systems [1] [2]. While the initial research work is concentrated on point-to-point MIMO links, focus has shifted in recent years to more practical massive MIMO systems, where typically a base station (BS) with a very large antenna array simultaneously serves a large number of users so that the multiplexing gain can be shared by all. In this way, expensive equipment is only needed on the BS end of the link, and the user terminals can be relatively inexpensive single-antenna devices [3].

A massive MIMO system is more tolerant of the propagation environment than a point-to-point system. Massive MIMO is of great interest as it provides cost and energy efficient solution for achieving high data rate via very large antenna array. Other

benefits of massive MIMO include simplification of the media access control (MAC) layer, reduced inter-cell interference and robustness against jamming [4] [5].

The ever growing demands for higher data rates and enhanced coverage have created a necessity for efficient and lucrative solutions. Heterogeneous cellular network, consisting of small cells (SCs) overlaid with massive MIMO, are promising solutions to satisfy the rising demand for ubiquitous mobile users. HetNet deployments are promising solution that increases network capacity, expands coverage and improves spectrum and energy efficiency at low deployment cost [6].

Due to the 1000x capacity growth requirement for fifth generation cellular systems, massive MIMO has attracted a lot of attention from both academia and industry. Several research works have investigated various design issues related to massive MIMO cellular networks. Massive MIMO is incorporated in HetNet using time division duplexing and throughput improvement is achieved by exploiting interference nulling [7]. By exploiting the spatial directionality of the channel vectors in massive MIMO, very efficient inter-tier interference management can be obtained with relatively low complexity [8].

Some studies consider deployment of femtocells over an existing macrocell but their assumptions of single femtocell and deterministic locations of femtocells, diminishes the scope of analyses. However, the use of PPP to model the stochastic geometry of wireless networks, has proven to be an effective tool to capture spatial uncertainty in node locations. [9]. SCs are often low- power nodes with single antenna, which provide higher throughput and enhanced coverage when distance between mobile users and base station is reduced.

On the other hand, massive MIMO is equipped with multiple antennas to serve large number of users. Although they seem to be two contradictory paradigms, substantial performance gains are expected if the two networks are designed to coexist [7]. Prior work on massive MIMO and SCs showed energy efficiency by a spatial soft-cell coordination and low-complexity interference coordination [10] [11]. Mas-

sive MIMO is used to determine the energy efficiency of the communication system through interference suppression [12]. A two-tier network with massive MIMO overlaid with SCs is studied in [13]. The performance of randomly distributed nodes in a cellular wireless network through PPP has been investigated in [14] [15].

### B. Contributions & Organization

Massive MIMO systems renders many traditional research problems irrelevant, however, they uncovers an entirely new problem that urgently needs attention to address the throughput requirements of 5G wireless networks. For a two-tier heterogeneous network where the macro-tier operation is provisioned through massive MIMO there are several open design issues:

- 1) What are the potential benefit of using massive MIMO overlay with small cells?.
- 2) How to allocate spectrum across two-tiers to maximize throughput and energy efficiency?
- 3) What degrees of freedom (such as transmit power, density of BSs etc.) are available to maximize the throughput and energy efficiency?

The aim of this paper is to address these design issues. Numerical analysis is carried out when HetNet uses massive MIMO. The performance gains of both tiers is investigated when operating in cooperation under a Rayleigh fading environment. In particular, the performance in terms of sum rate capacity and energy efficiency is analyzed in the downlink of a two-tier network through stochastic geometry and Monte Carlo simulations.

The rest of the paper is organized as follows. Section II introduces the system model. Section III explains the channel and transmit power allocation mechanism. Section IV describes the numerical analysis of the capacity for both tiers. Finally, the conclusions are given in Section V.

## II. SYSTEM MODEL

Consider the downlink of a typical cell in two-tier network consisting of a macrocell base station (MBS) and femtocell access points (FAPs) as shown in Fig. 1. A massive MIMO system is assumed where the base station in macrocell tier uses  $M_t$  transmit antennas and macrocell users are equipped with single receive antenna. The macro tier is overlaid with femtocells where each femtocell is assumed to have one transmit antenna at the femto BS and one receive antenna at each user. We assume that the channel state information (CSI) is perfectly known at both the transmitter and the receiver.

The total number of available channels  $N$ , are distributed among the two tiers such that  $N_m < N$  channels are assigned to the macro tier and  $N_f = N - N_m$  channels are allocated to femto tier. The channel allocation is done in such a manner that the cross-tier interference is completely avoided. Notice that in case of co-channel deployment, small cell user may experience significant interference from the Massive MIMO BS. Thus non co-channel deployment is attractive option for providing better reliability guarantees. The only interference occurs in the femtocell tier where the same spectrum is utilized for all the

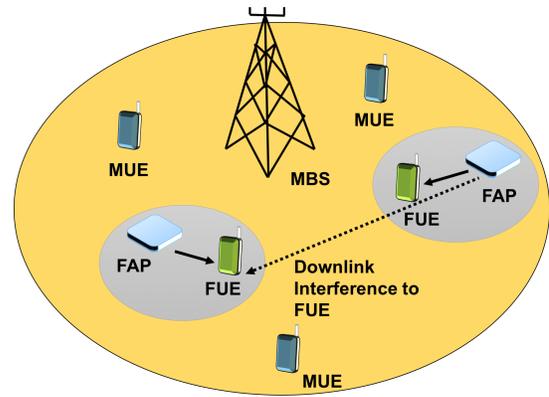


Fig. 1: A massive MIMO-based heterogeneous network

existing femtocells and thus two users in disjoint femtocells may interfere (in the downlink) if both FAPs use the same frequency channel.

The locations of femtocells within the macrocell is characterized by a PPP with density  $\phi_f$  and the users in both the tiers are also distributed according to two independent PPPs  $\phi_u^m$  and  $\phi_u^f$  with densities  $\lambda_m$  and  $\lambda_f$  for macro and femto users, respectively. The propagation model is assumed to be Rayleigh fading with path loss  $(D_l^{j,k})^{-\beta_l}$ , where  $D_l^{j,k}$  is the distance between  $j^{th}$  transmitter and  $k^{th}$  user in tier  $l$ ;  $l \in \{m, f\}$  and  $\beta_l$  is the path loss exponent for that particular tier  $l$ . The transmitted power of a base station in tier  $l$  is denoted as  $P_l$ .

## III. CHANNEL AND TRANSMIT POWER ALLOCATION FOR MASSIVE MIMO EMPOWERED TWO-TIER HETNET

The received signal-to-noise ratio (SNR) for a macro user equipment (MUE) is given as

$$SNR_m = \frac{P_r^m}{\sigma^2}, \quad (1)$$

where  $P_r^m$  is the received power for a MUE and  $\sigma^2$  is the noise variance, which is assumed to be unity in this study and the received power at a user equipment in macrocell is given as

$$P_r^m = \sum_{q=1}^{M_t} \frac{P_q \alpha_q}{(D_q^{j,k})^{\beta_q}}, \quad (2)$$

where  $P_q$  represents the transmit power of the macrocell antenna and  $\alpha_q$  is the fading power coefficient in macro tier which is drawn from an exponential distribution (corresponding to the squared envelope) with unit mean.

Similarly, the signal-to-interference plus noise ratio (SINR) for a femto user equipment (FUE) can be calculated as

$$SINR_f = \frac{P_r^f}{1 + \sum_{i \in I} P_i \alpha_i (D_i^{j,k})^{-\beta_i}}, \quad (3)$$

where  $P_r^f$  is the received power for FUE,  $P_i$  is the interfering power from  $i$ -th FAP, which is co-tier interference. The set  $I$  denotes the set of all interferers. The Rayleigh fading channel

gain from interfering transmitter is denoted by  $\alpha_i$ , while the distances of the receiver from the interferer is represented as  $D_i^{j,k}$  and the received power at a femto user equipment is given by

$$P_r^f = \frac{P_f \alpha_f}{(D_f^{j,k})^{\beta_f}}, \quad (4)$$

where  $P_f$  is the transmit power of FAP, while  $\alpha_f$  is the fading channel gain in the femtocell.

The system capacity can be modeled for both macro and femtocell on the basis of  $SNR_m$  and  $SINR_f$  given by

$$C_l = B \log_2(1 + SNR_l) \quad (5)$$

where  $C_l$  is the capacity and  $B$  is the system bandwidth.

The number of users are generated randomly through PPP and each user is allocated a unique channel. If the number of users exceed number of channels in any tier, the users with maximum capacity get selected according to (6) and (7)

$$C_m^* = \arg \max_{C_m} B \log_2(1 + SNR_m) \quad (6)$$

$$C_f^* = \arg \max_{C_f} B \log_2(1 + SINR_f) \quad (7)$$

where  $C_m^*$  and  $C_f^*$  represent the macro and femtocells capacity respectively.

The energy efficiency ( $\eta$ ) of a two-tier HetNet is measured in bits/Joule and equals the ratio between the achievable sum rate and the total power consumed by macro and femto cells given as

$$\eta = \frac{C_n}{P_{total}}, \quad (8)$$

where  $C_n$  is the aggregate network throughput and  $P_{total}$  is the total power consumption in dBm. The design issues investigated in this can now be as

**Design Issue #1: Optimal Channel Allocation** From eqs. (5),(6) and (7), the overall network capacity can be written as

$$C_n = C_{mt} + C_{ft} \quad (9)$$

where

$$C_{mt} = \begin{cases} \sum_{m \in \phi_u^m} C_m & \text{if } \|\phi_u^m\| \leq N_m \\ \sum_{m \in \phi_u^m} C_m^* & \text{otherwise} \end{cases}, \quad (10)$$

and similarly

$$C_{ft} = \begin{cases} \sum_{f \in \phi_u^f} C_f & \text{if } \|\phi_u^f\| \leq N_f \\ \sum_{f \in \phi_u^f} C_f^* & \text{otherwise} \end{cases}, \quad (11)$$

Hence our design goal is to determine optimal  $N_f$  and  $N_m$  which maximize the network throughput ( $C_n$ ). Mathematically,

$$\arg \max_{(N_m, N_f)} C_n. \quad (12)$$

**Design Issue #2: Optimal Power Allocation** From eqs. (5), we observe that the energy efficiency of the network is strong function of transmit power. Thus we aim to explore if there

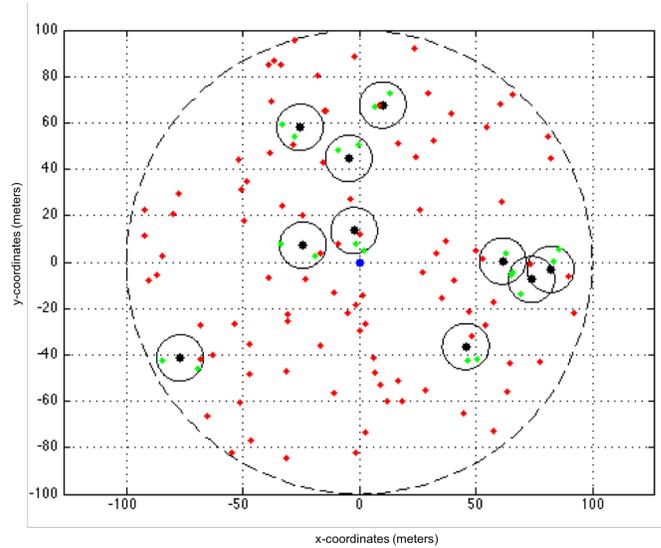


Fig. 2: Heterogeneous network overlaid with massive MIMO

exists an optimal transmit power which will maximize the network wide energy efficiency: Mathematically,

$$\arg \max_{(P_q, P_f)} \eta. \quad (13)$$

Considering these two design issues, the capacity and energy efficiency for both tiers are investigated in next section.

#### IV. NUMERICAL ANALYSIS

In this section, we present the numerical analysis and simulation results for the system under consideration. Under different scenarios, it is shown that the overall system capacity gets improved when HetNet uses massive MIMO. With the help of numerical analysis we investigate the optimal channel allocation which maximizes the throughput and EE for the two-tier network. In our simulation environment, the number of femtocells and the users in both tiers are distributed randomly according to PPP as shown in Fig 2.

Results are evaluated through Monte-Carlo simulations (i.e.  $1 \times 10^5$  runs for each point). The scenarios simulated in this paper consider fixed number of channels for HetNet, i.e.,  $N = 100$  channels with a maximum of  $M_t = 100$  antennas for MBS and single antenna for FAPs. Both macro and femtocell users are equipped with single antenna. The macro and femto cell radius is denoted by  $R_m$  and  $R_f$ , respectively. The parameters used for simulations are given in Table I.

The numbers of channels are distributed among macrocell and femtocells in such a way that the first 15 channels are allocated to femtocells and remaining channels are reserved for macrocell. Channels are allocated to the users on the basis of cardinality values. The cardinality value of matrix  $C_l$  is calculated and if the value is less as compared to the number of channels, then a channel is allocated to each user. However, if the cardinality value is greater than the number of channels, then channels are allocated to only those users which possess the highest data rate. The channel allocation

TABLE I: System Parameters

Parameter	Value
$R_m$	100m
$R_f$	10m
$\beta_q$	2.5
$\beta_f$	2
$\beta_i$	2.2
$B$	1Hz
$P_q$	46 dBm
$P_f$	30 dBm
$P^{total}$	47 dBm

process is repeated throughout the Monte Carlo simulations. The entire process is summarized in Algorithm 1.

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**Algorithm 1** Channel Allocation
 

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- 1: Initialize
  - 2: A set of  $K$  users where  $k = \{1, \dots, K\}$
  - 3: A set of  $N$  channels where  $n = \{1, \dots, N\}$
  - 4: Maximum  $N_m$  and  $N_f$  channels allocated to users
  - 5: Channel allocation matrix  $\mathbf{Z} = 0_{\{N \times K\}}$
  - 6: Create  $C_l$  from calculated  $SNR_l$  values
  - 7: Compute the cardinality matrix,  $\mathbf{S}$
  - 8: **while**  $\mathbf{S} \neq 0$  **do**
  - 9:     **if**  $|\mathbf{S}| \leq N$  **then**
  - 10:         Set  $\mathbf{Z}_{\{N \times K\}} = 1$
  - 11:     **else**
  - 12:         Assign  $n$  to  $k$  corresponding to highest value in  $\mathbf{S}$
  - 13:         Set  $\mathbf{Z}_{\{N \times K\}} = 1$
  - 14:     **end if**
  - 15: **end while**
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The overall user intensity  $\lambda_l$  in the HetNet remains fixed as shown in Fig. 3. The abscissa represents the ratio of macro and femto channels. The total number of channels for each observation remains fixed, i.e., 100 but the number of channels assigned to macrocell and femtocells vary. Initially 85 channels are reserved for macrocell and remaining 15 channels are allocated to the femtocells. Similarly, the number of channels allocated to macrocell increase and the channels allocated to femtocells decreases, retaining the overall 100 channels at all times.

For the results in Fig. 3, the total user density (of combined femtocells and macrocell)  $\lambda_l = 120$  is assumed, however, the macro user intensity ( $\lambda_m$ ) and femto user intensity ( $\lambda_f$ ) is varied. In Fig. 3a,  $\lambda_m = 90$  and  $\lambda_f = 30$  is assumed and it is observed that with this user density, the maximum capacity is achieved when 90 channels are allocated to macrocell and 10 channels to femtocell. Similarly, when  $\lambda_m = 100$  and  $\lambda_f = 20$ , the maximum capacity is achieved at 92/8 and in Fig. 3c, the channels ratio of 95/5 provides the highest capacity.

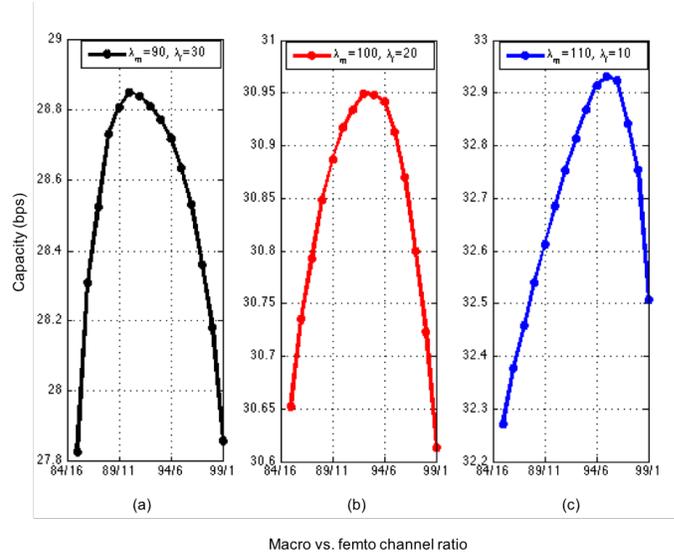


Fig. 3: Variable macro and femto user density

Although, the total intensity of HetNet remains constant, yet by increasing the user intensity of macrocell, the capacity gets improved. Similarly, if we consider a fixed channel ratio in Fig. 3, i.e. 94 channels for macro cells and 6 channels for femto cells, the capacity gets enhanced. For fixed channel ratio, when  $\lambda_m$  is increased from 90 to 110, the sum rate capacity is improved by 15%. Hence, massive MIMO plays a vital role in improving overall system capacity.

In Fig. 4, the macrocell user intensity remains fixed i.e.  $\lambda_m = 100$  whereas, the user intensity of femtocells is varied. The x-axis represents the allocated macro and femto channels ratio. When the femtocell user intensity is increased, a slight improvement in sum rate capacity is observed whereas, the channel ratio almost remains the same. On the other hand, when  $\lambda_f$  is varied from 10 to 20 for a fixed channel ratio (92/8), it is observed that the capacity increases by 1.3% only. Hence, the variable user intensity of femtocells does not show any prominent improvement in overall system capacity.

The effect of variable  $\lambda_m$  on the sum rate capacity is investigated in Fig. 5. Here,  $\lambda_f$  remains fixed and  $\lambda_m$  is varied. The capacity gets enhanced by 17% when  $\lambda_m$  is varied from 95 to 115. Hence, a significant increase in capacity is observed as the macrocell user intensity increases. However, it can be seen that increasing  $\lambda_m$  increases the overall system capacity of the HetNet up to a certain threshold after which the curves start getting saturated. Hence, the sum rate capacity will improve for a certain channel ratio as shown in Table II.

The results observed in Fig. 3-5 are summarized in Table II. When the total user intensity remains fixed, i.e.  $\lambda = 120$ , the capacity gets increased along with an increase in channel ratio. On the other hand, when macrocell user intensity is fixed, the channel ratio also remains constant. Increasing the femtocell user intensity does not play any significant role in improving the sum rate capacity. However, when the user intensity of macrocell is varied, the overall system capacity gets enhanced

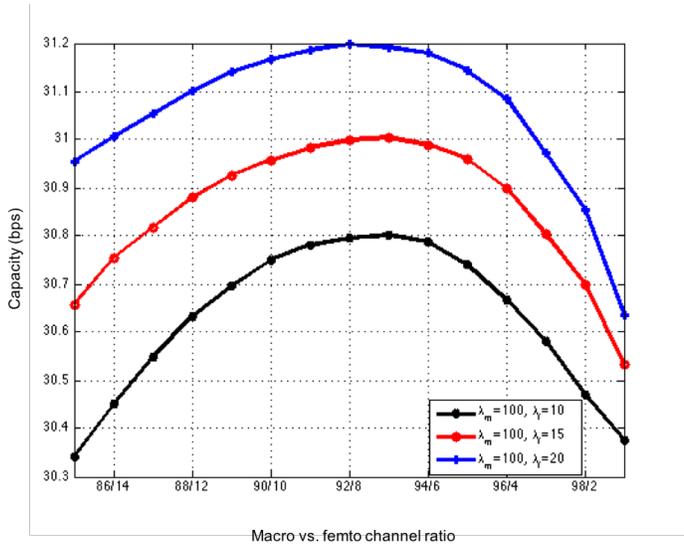


Fig. 4: Fixed macro user density

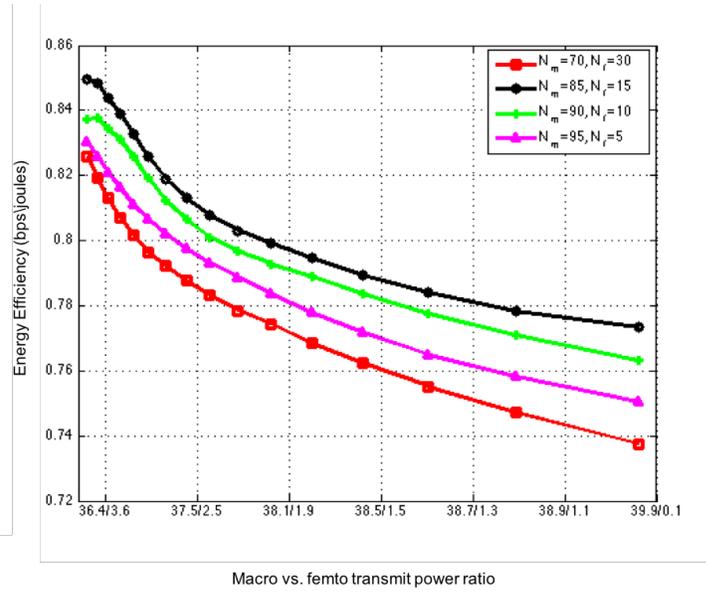


Fig. 6: Energy efficiency in heterogeneous network

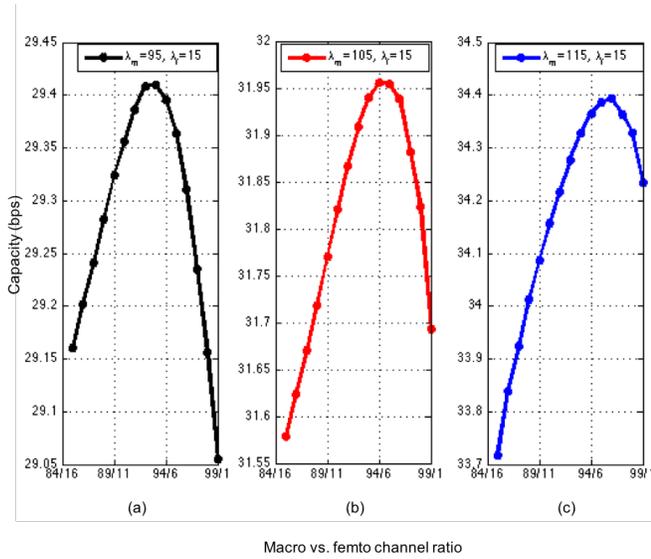


Fig. 5: Fixed femto user density

and channel ratio increases.

On the basis of sum rate capacity, the energy efficiency of HetNet is analyzed in Fig. 6. In this scenario,  $P^{total}$  is distributed among the macro and femto cells such that the total power consumed in the HetNet remains constant i.e. 47dBm. The energy efficiency improves as the number of macro channels increase. As soon as the macro channels are 85 and femto channels are 15,  $\eta$  gets maximized. However, the value of  $\eta$  decreases as the number of macro channels increases afterwards. Thus, the optimal value of  $\eta$  is achieved for  $\lambda_m = 85$  and  $\lambda_f = 15$ . It is also observed that these optimum number of channels achieve an energy efficiency of 3%, 1.5% and 2.3% when  $N_m = 70$ ,  $N_m = 90$  and  $N_m = 95$  are considered respectively. Hence, massive MIMO helps in

TABLE II: Performance Analysis

$\lambda_m, \lambda_f$	$\lambda = 120$		$\lambda_m = 100$		$\lambda_f = 15$			
	C	$\frac{N_m}{N_f}$	$\lambda_f$	C	$\frac{N_m}{N_f}$	C	$\frac{N_m}{N_f}$	
90,30	28.85	90/10	10	30.8	93/7	95	29.43	93/7
100,20	30.95	92/8	15	30.99	93/7	105	31.98	94/6
110,10	32.93	96/4	20	31.20	93/7	115	34.42	96/4
115,5	33.9	96/4	25	31.24	93/7	125	36.75	96/4
119,1	34.59	98/2	30	31.48	93/7	135	38.98	97/3

improving the energy efficiency and the sum rate capacity of a HetNet.

## V. CONCLUSION

A novel channel allocation scheme is proposed in this paper and its performance has been evaluated through stochastic geometry and Monte Carlo simulations. Specifically, in a two-tier system, using PPP and multiple antennas, the overall system capacity and the energy efficiency of the network is improved when the macro and femto channel ratio is considered. With a direct increase in the user intensity of macrocell, the network capacity is increased by 15%-17%. Hence, the capacity improvement is significant in the existing architecture with the deployment of massive MIMO. On the other hand, the energy efficiency also gets optimized under massive MIMO-based single cell HetNet. Therefore, massive MIMO is a promising solution for backhaul networks since the sum rate capacity and energy efficiency gets enhanced in HetNet. In future, multi-cell analysis will be carried out for macro and femtocells simultaneously and their closed-form expressions will be addressed at length.

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