

Opportunistic Large Array with Limited Participation: An Energy-Efficient Cooperative Multi-Hop Network

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Abstract—This paper studies an energy-efficient scheme for cooperative multi-hop communications in a finite density opportunistic large array (OLA) network. In a cooperative OLA network, a group of nodes transmits the same message signal to another group of nodes, providing range extension and increased reliability by exploiting the spatial diversity in a wireless system. However, in this paper, it is shown that a particular coverage and reliability of the network can be achieved by limiting the node participation in an OLA network, thereby providing energy-efficiency. Two types of networks are studied; a one-dimensional linear network and a two-dimensional strip network where the nodes are aligned on a regular grid. The transmissions originating from one level to another are modeled as a Markov process and the underlying transition probability matrix has been derived. By invoking the Perron-Frobenius theorem, the coverage, reliability, and energy-efficiency of the network has been quantified for a given end-to-end success probability constraint and a given signal-to-noise ratio (SNR) margin.

I. INTRODUCTION

Large-scale wireless sensor networks have an important impact in meeting environmental challenges. Sensor applications in a variety of areas like smart power grids, structural health monitoring, emergency services in vehicular ad-hoc network (VANETS), etc., substantially contribute to cost-effective yet reliable use of resources. If the source and destination are far apart, multi-hop techniques can be employed to uni-cast a message to intended destination or broadcast it over the entire network. Multi-hop wireless communication in conjunction with cooperative transmission has been a potential candidate for reliable data delivery in the presence of multi-path fading. Opportunistic large array (OLA) network is a kind of simple cooperative multi-hop network that propagates a message to a far off destination using multiple hops while exploiting distributed spatial diversity of the wireless channel.

In an OLA transmission, a large number of nodes are placed in an area. The source node broadcasts its message and all the nodes that can decode the message correctly become part of the first hop or level. All these nodes will then relay the same message in the next time slot and this process continues until the destination receives the message. The transmission propagates as a virtual multiple-input single-output (MISO) over multiple hops under the effects of the wireless channel; thereby providing diversity [1].

A number of previous work on OLA has appeared. For instance, the authors in [2] studied large dense network and their assumptions stated that an infinite broadcast is guaranteed for infinite density networks. The authors in [3] studied the infinite density OLA network, however, they restricted the number of decode-and-forward (DF) node participation to obtain energy-efficiency and named the algorithm as OLA with threshold, (OLA-T). However, they provided a bound on the threshold that guaranteed an infinite broadcast for infinite node density network. These infinite propagation assumptions were proved void in [4] and [5], where the authors studied extended networks with finite node density. They showed that for finite density cases, an infinite broadcast is not possible. Finite density analysis for linear networks is also studied in [6]-[9] for various channel models.

In this paper, we study a variant of OLA-T algorithm that is applicable to a finite density extended network and we complement the finding of [5] that there is no condition that guarantees an infinite broadcast in a finite density OLA network. For simplicity, we conduct our analysis for strip-shaped networks, where the nodes are aligned on a one-dimensional or a two-dimensional grid. The channel model includes Rayleigh fading and path loss with an arbitrary path loss exponent. We model the transmissions from one level to another as a Markov process and determine the one-hop success probability with our proposed algorithm. We have shown that limiting the node participation leads to energy-efficient network at the expense of a loss in diversity gain. However, we have quantified the optimal level of cooperation required to obtain a certain quality of service (QoS).

The rest of the paper is organized as follows. In Section II, the layout of the system is discussed. The network is modeled using a discrete-time Markov chain, which is discussed in Section III. The transition probability matrix is derived for the proposed network model. The results and analysis follow Section III with conclusion at the end, featuring the future developments.

II. SYSTEM MODEL

In this section, we describe the system level architecture of our network. We consider a regular and deterministic deployment of nodes in a strip-shaped network. First, we analyze a one-dimensional (1D) network and then we discuss

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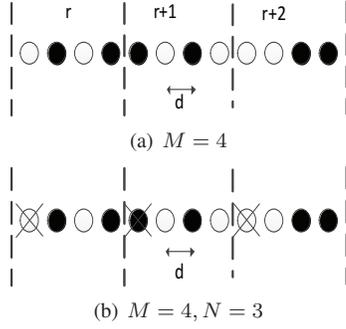


Fig. 1. 1D network layout

a more general 2D network with nodes deployed on a regular 2D grid.

A. 1D Network Layout

Consider a linear network where the adjacent nodes are separated by a distance of d as shown in Fig. 1(a). A window size M depicts the level of cooperation between the nodes, i.e., each ‘level’ or ‘hop’ comprises of M nodes, which transmit the same message to the M nodes of the next level. The transmit power of all the nodes is assumed to be equal. A node can successfully decode a message if its received signal-to-noise ratio (SNR), after post detection combining, is above a specified threshold, τ . We consider decode-and-forward (DF) mechanism where the nodes highlighted with black in Fig.1 are the DF nodes at various time instants $r, r + 1$, and so on. Although, we specify a fixed number of nodes in each level, the network, however, is still opportunistic in the sense that it is unknown a priori which nodes will become the DF nodes, and participate in next level’s transmission.

We consider two modes of transmission: basic OLA (B-OLA) and OLA with limited participation (OLA-LP). In B-OLA, the transmission takes place through hops where each hop is characterized by a window size of M . In other words, all the nodes that have decoded in a level forward the packet to the next level. However, in our proposed OLA-LP, we select N nodes to forward the packet such that $N \leq M$. This selection is based on the fact that the nodes at the starting edge of a window do not provide substantial diversity gain because of the large path loss present between them and the next level nodes. However, we may save their energy by not letting them transmit the packet. Although it will cause a slight degradation in the system performance, however, we will later show that if this degradation is tolerable, we can have an energy-efficient transmission using OLA-LP. It is assumed that each node knows to which level it belongs to. This could be achieved by using a geometry-based approach [10]. The selection of the DF nodes could be performed using threshold criterion [11] or an SNR-based approach [12].

Let \mathbb{N}_r represents the indices of the DF nodes in B-OLA at time instant r , then we define a set $\mathbb{R}_r \subseteq \mathbb{N}_r$, which indicates the indices of the DF nodes in OLA-LP. For example, $\mathbb{R}_r = \{2, 4\}$, $\mathbb{R}_{r+1} = \{3\}$, and $\mathbb{N}_{r+1} = \{1, 3\}$ as shown in Fig. 1.

In OLA-LP, the power received at the j th node of the next

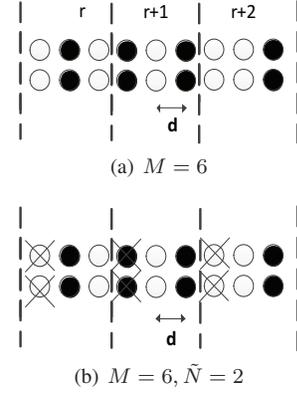


Fig. 2. 2D network layout

level, e.g., $r + 1$ is given by

$$Pr_j(r+1) = \frac{P_t}{d^\beta} \sum_{m \in \mathbb{R}_r} \frac{\mu_{mj}}{(M - m + j)^\beta}, \quad (1)$$

where the transmit power of the radios is given as P_t and the flat fading Rayleigh channel gain from the m th node in level r to the j th node in level $(r + 1)$ is given by μ_{mj} . The elements of μ_{mj} are drawn from an exponential distribution with unit mean. The path loss exponent is represented by β where its normal range varies between 2-4. Another assumption is that the transmission by all the nodes at a particular level take place in synchronization, which means that all the nodes at the r th level transmit at the same time instant over orthogonal fading channels.

B. 2D Strip Network

The 2D strip network is shown in Fig. 2 where the length of a level is L in horizontal dimension and its width is W , such that $M = L \times W$. Again, the nodes are spaced at a fixed distance d from each other in both dimensions. The decode-and-forward mechanism remains the same as in the previous case, with the only difference of calculation in the received power. The expression for received power at the j th node of the next level is now given by

$$Pr_j(r+1) = \frac{P_t}{d^\beta} \sum_{m \in \mathbb{R}_r} \frac{\mu_{mj}}{(\sqrt{\delta_{mj}})^\beta}, \quad (2)$$

where all the parameters have the same definition as in (1). The Euclidean distance between a pair of nodes m and j is given by $\sqrt{\delta_{mj}}$; $\delta_{mj} \in \Delta_{[M,M]}$ such that

$$\Delta = (\mathbf{A} \otimes \mathbf{G}_1) + (\mathbf{G}_2 \otimes \mathbf{B}), \quad (3)$$

where \otimes denotes the kronecker product. The matrices \mathbf{A} and \mathbf{B} are given as

$$\mathbf{A} = \begin{bmatrix} L^2 & (L+1)^2 & \cdot & \cdot & (L+(L-1))^2 \\ (L-1)^2 & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ (L-(L-1))^2 & \cdot & \cdot & \cdot & L^2 \end{bmatrix}$$

and

$$\mathbf{B} = \begin{bmatrix} 0 & 1^2 & 2^2 & \cdot & \cdot & (W-1)^2 \\ 1^2 & \cdot & \cdot & \cdot & \cdot & \cdot \\ 2^2 & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ (W-1)^2 & \cdot & \cdot & \cdot & \cdot & 0 \end{bmatrix},$$

where $\mathbf{G}_1 = \mathbf{1}_{[L \times L]}$ and $\mathbf{G}_2 = \mathbf{1}_{[W \times W]}$ are the matrices of all ones of dimensions $(L \times L)$ and $(W \times W)$, respectively.

III. MARKOV CHAIN MODELING

In this section, we model the transmissions that hop from one level to another and consider the 1D case first. A binary indicator random variable (RV), $\mathbb{I}_j(r)$ is used to define the state of a node j at time instant r where $\mathbb{I}_j(r)$ takes value 0 (node j could not decode) or 1 (node j decodes). For B-OLA, at time r , $\mathcal{X}_1(r) = [\mathbb{I}_1(r), \mathbb{I}_2(r), \dots, \mathbb{I}_M(r)]$ represents the state of the network such that the state is represented by an M -bit binary word, for instance from Fig. 1(a), $\mathcal{X}_1(r) = \{0101\}$ and $\mathcal{X}_1(r+1) = \{1010\}$. It can be seen that at any time instant, the state of the system is dependent upon the previous state, making \mathcal{X}_1 a discrete-time Markov process. For a given M , the total number of states is 2^M with *all zeros* being an absorbing state, i.e., a state where transmissions stop propagating. Hence the Markov chain, \mathcal{X}_1 , is defined by the union of two mutually exclusive sets, i.e., $\mathcal{X}_1 \in \{0\} \cup S_1$ where 0 is the absorbing state and S_1 is the transient state space such that $S_1 = \{1, 2, \dots, 2^M - 1\}$, where the set S_1 is the decimal equivalent of the binary words formed from the indicator functions. For instance, if i is a state such that $i \in S_1$, and if $i = \{1010\}$, then $i = 10$ in decimal. The probability of going into an absorbing state is always non-zero and increases asymptotically as $\lim_{r \rightarrow \infty} \mathbb{P}\{\mathcal{X}_1(r) = 0\} \nearrow 1$.

In OLA-LP, we select the trailing nodes used for transmission from the original window of size M , so the Markov chain is defined as $\mathcal{X}_2(r) = [\mathbb{I}_g(r), \mathbb{I}_{g+1}(r), \dots, \mathbb{I}_M(r)]$ where $g = M - N + 1$, i.e., $M - N$ most significant bits are removed. The outcomes of $\mathcal{X}_2(r)$ can be represented in binary form with 2^N states with a transient state space $S_2 = \{1, 2, \dots, 2^N - 1\}$. The transitions to/from 0th state are not included, resulting in a $(2^N - 1) \times (2^N - 1)$ dimensional probability matrix \mathbf{P} . According to the Perron-Frobenius theorem of non-negative matrices, \mathbf{P} has a maximum eigenvalue, ρ , and a corresponding left eigenvector, \mathbf{u} . The transition matrix \mathbf{P} is right sub-stochastic due to elimination of $(M - N)$ nodes because of which the value of Perron-Frobenius eigenvalue ρ is always less than 1. The ρ -invariant distribution is defined by $\mathbf{u} = (u_i, i \in S_2)$ where \mathbf{u} is the left eigenvector of \mathbf{P} . As we are certain that $\forall r, \mathbb{P}\{\mathcal{X}_2(r) = 0\} > 0$, $\mathcal{X}_2(r)$ becomes a Markov chain having a quasi-stationary distribution [5]. The quasi-stationary distribution represents the state of the system just before the killing occurs. Hence, the probability of being in state j is given as

$$\mathbb{P}\{\mathcal{X}_2(r) = j\} = \rho^r u_j, \quad j \in S_2, \quad r \geq 0. \quad (4)$$

The probability that a node m of level $(r+1)$ is able to decode is given by

$$\mathbb{P}\{\mathbb{I}_m(r) = 1\} = \mathbb{P}\{Pr_m(r) > \tau\}, \quad (5)$$

where $\mathbb{P}\{Pr_m(r) \geq \tau\}$ is given by

$$\mathbb{P}\{Pr_m(r) \geq \tau\} = \int_{\tau}^{\infty} f_{Pr_m}(y) dy. \quad (6)$$

In (6) the probability distribution function (PDF) of the received power at the m th node is defined as $f_{Pr_m}(y)$, where the total power received at a node is the sum of the finite powers from nodes in the previous level. The power transmitted by each node is exponentially distributed so the sum of K exponential random variables with distinct parameters λ_k , where $k = 1, 2, \dots, K$, forms hypo-exponential distribution, which is given as

$$f_{Pr_m}(y) = \sum_{k=1}^K C_k \lambda_k^{(m)} \exp(-\lambda_k^{(m)} y), \quad (7)$$

where

$$C_k = \prod_{\varsigma \neq k} \frac{\lambda_{\varsigma}^{(m)}}{\lambda_{\varsigma}^{(m)} - \lambda_k^{(m)}}, \quad (8)$$

and $\lambda_k^{(m)}$ is defined as

$$\lambda_k^{(m)} = \frac{\Delta^\beta d^\beta}{P_t}. \quad (9)$$

In Eq. (9), Δ for 1D is given as $(M - k + m)$ and for 2D it is defined by (3). The success probability at the m th node is thus given as

$$\mathbb{P}\{Pr_m(r) \geq \tau\} = \sum_{k=1}^N C_k \exp(-\lambda_k^{(m)} \tau) \mathbb{I}_k(r-1). \quad (10)$$

Eq. (10) provides the success probability of one node in a level. For all nodes in a level j , the one-hop probability for going from state i to j is given as

$$P_{ij} = \prod_{k \in \mathbb{R}_{r+1}^{(j)}} \left\{ \sum_{m \in \mathbb{R}_r^{(i)}} C_m \exp(-\lambda_m^{(k)} \tau) \right\} \prod_{k \in \mathbb{R}_{r+1}^{(j)}} \left\{ 1 - \sum_{m \in \mathbb{R}_r^{(i)}} C_m \exp(-\lambda_m^{(k)} \tau) \right\}, \quad (11)$$

where the set $\mathbb{R}_r^{(i)}$ represents the indices of DF nodes of i th state at time instant r and $\mathbb{R}_r^{(i)} = \{g, g+1, \dots, M\} \setminus \mathbb{R}_r^{(i)}$, i.e., the indices of the nodes which are 0 at time instant r . Eq. (11) is just a single entry of the transition matrix \mathbf{P} . Similar entries can be calculated for all $i, j \in S_2$.

In 2D B-OLA case, the nodes are labeled from top to bottom and left to right. Hence the state of the system at time instant r is given as

$$\tilde{\mathcal{Y}}_1(r) = \begin{bmatrix} \mathbb{I}_1(r) & \mathbb{I}_{W+1}(r) & \cdot & \cdot & \cdot & \mathbb{I}_{(L-1)W+1}(r) \\ \mathbb{I}_2(r) & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \mathbb{I}_W(r) & \mathbb{I}_{2W}(r) & \cdot & \cdot & \cdot & \mathbb{I}_M(r) \end{bmatrix}.$$

For example, for Fig. 2(a), $\tilde{\mathcal{Y}}_1(r) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$. We define

$$\mathcal{Y}_1(r) = \{vec[\tilde{\mathcal{Y}}_1(r)]\}^T, \quad (12)$$

where *vec* is the vector operation and stacks all the columns of $\tilde{\mathcal{Y}}_1$ into a single column and T denotes the transpose operation. Using (12), the definition of 2D Markov chain is

TABLE I. MSE BETWEEN THEORETICAL MODEL AND SIMULATION

MSE	N=2	N=3	N=4	N=5	N=6	N=7	N=8
$\gamma=12$ dB	1.53e-4	8.10e-5	1.39e-4	6.40e-7	9.82e-6	3.02e-7	1.04e-8
$\gamma=14$ dB	1.50e-4	5.62e-5	1.23e-4	1.60e-7	1.75e-4	2.50e-7	3.16e-6
$\gamma=16$ dB	3.24e-6	9.61e-6	1.60e-5	4.90e-7	8.41e-6	1.60e-7	9.00e-8

now expressed as 1D case and the state space remains the same as that of 1D, i.e., an absorbing state in addition to $2^M - 1$ transient states. For OLA-LP, we define \tilde{N} as the participating columns of $\tilde{Y}_1(r)$. The selection of \tilde{N} nodes for 2D case refers to the selection of last \tilde{N} columns of window $M = L \times W$. For Fig. 2(b), the first column of nodes is restricted to take part in transmission and last $\tilde{N} = 2$ are now the candidate cooperators.

IV. RESULTS AND ANALYSIS

In this section, we present our numerical results and discuss the system performance with respect to various parameters. We first compare the theoretical results with the results from simulations so as to ascertain the validity of the model. In numerical simulations, the received power at the second level nodes is calculated while assuming a random initial state of the nodes in the first level. The threshold criteria sets the indicator functions of nodes; constituting a state. These nodes then transmit to the next level nodes and this process continues. The one-hop success probability is computed and the simulations are averaged over 100,000 trials to get an average value of one-hop success probability. The simulation results are then compared with the Perron-Frobenius eigenvalue ρ found by (11).

Before proceeding further, we define a normalized parameter

$$\gamma = \frac{P_t}{d^{\beta\tau}}, \tag{13}$$

which is the received SNR at a node d distance away from its transmitter. We call it as SNR margin. The mean squared error (MSE) between the theoretical one-hop success probability and the one obtained from simulations is shown in Table I for $M = 8$, SNR margin={12,14,16}dB, and $N = \{2, 3, \dots, 8\}$. In all our simulation results, the value of τ is set to 0.1 with $d = 1$ and $\beta = 2$. The table shows that the analytical results are quite close to that of simulations for all values of N and γ and the model fits the simulation values for an error of order 10^{-4} or less.

Fig. 3 shows the behavior of one-hop success probability for different SNR margins and combinations of M and N for a 1D case. It can be observed that regardless of the value of N , the one-hop success probability increases by increasing the SNR margin, and for a fixed SNR margin, a loss in diversity gain can be seen by decreasing N . However, the performance of the network can be gauged by defining a quality of service (QoS), η . In this case, the η could be one-hop success probability or end-to-end success probability. For instance, if it is required that the one-hop success probability must be larger than or equal to 90%, then $\eta \triangleq 0.9$. Now for Fig. 3, an observation at a particular SNR margin e.g., 15dB shows that the required $\eta=0.9$ can be achieved at $N = 5$, thereby saving the energy of 3 nodes without compromising the QoS.

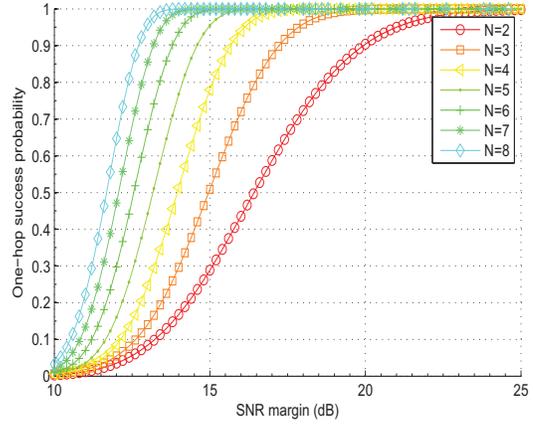


Fig. 3. SNR margin vs. one-hop success probability for various levels of node participation; $M = 8$.

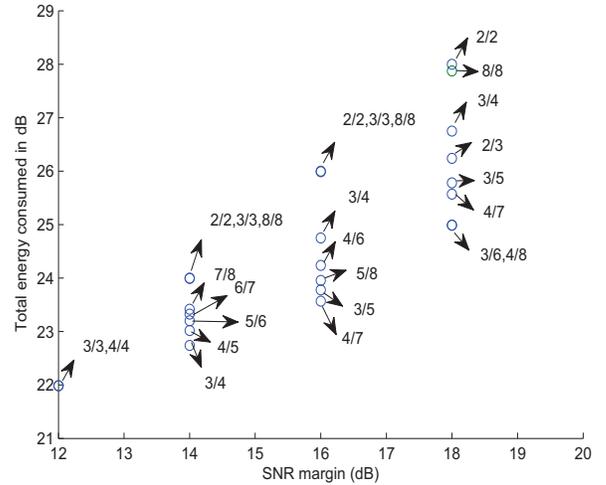


Fig. 4. Various combinations for reaching distance of 100d vs. energy consumed for each combination.

If supposedly, a distance of D is to be achieved by using a particular M , then the number of hops required are $n_o = \lceil \frac{D}{M} \rceil$. If ρ is the one-hop success probability, then the end-to-end success probability for n_o hops is ρ^{n_o} . If again, we require that this end-to-end success probability is larger than η , then $\rho^{n_o} \geq \eta$, i.e.,

$$\rho \geq \exp \left\{ \frac{\log \eta}{n_o} \right\}. \tag{14}$$

The calculated one-hop success probability helps in finding the combinations, which would help in achieving successful transmission and conserving the energy.

In Fig. 4, we find the optimal combinations of N and M for reaching a distance of $D = 100d$. The word ‘optimal’ means the combination that provides maximum energy-efficiency. We also define a parameter N/M , which is defined as the number of participating nodes out of the total M nodes. The Eq.(14) specifies the one-hop success probability, which is required to cover a distance of $D = 100d$ and $\eta = 0.9$. Now there could be a variety of possible topologies that would guarantee this

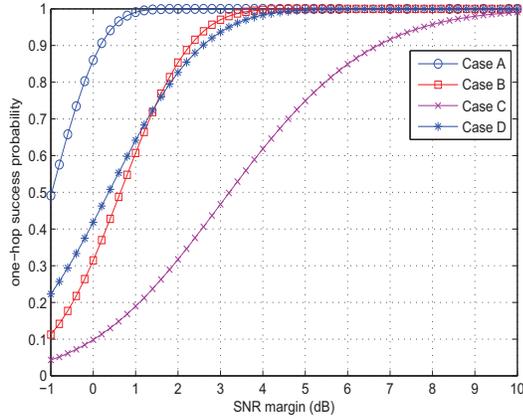


Fig. 5. Behavior of one-hop success probability in a 2D strip network for $M = 6$

QoS for a particular value of SNR margin. For instance, one possibility is to deploy nodes as $M = N = 2$. In this case, 50 hops are required to reach the destination node. If we operate this topology at an SNR margin of 16dB, then $N/M = 2/2$ combination in Fig. 4 shows that the required end-to-end QoS can be achieved by having a transmit power of 3.98 per node by using Eq. (13). Hence, a total transmit power of 26dB is required for the network to achieve the desired coverage. Other combinations are also plotted and the required energy is shown. All $N/M=1$ cases represent the B-OLA results. It can be seen that at 16dB, the combination 4/7 results in least energy consumption, i.e., deploying a hop of $M = 7$ nodes, but use 4 trailing nodes for transmission. Hence the fraction of energy saved is $1 - N/M = 3/7$ for this specific case. Similarly, other cases for other values of SNR margin are also shown and their energies are listed. Hence it can be seen that OLA-LP provides an energy-efficient approach than B-OLA and can be used in various cooperative transmission-based applications.

Now we show our results for a 2D case. In this scenario, we consider the following cases for $M = 6$:

- Case A: $L = 3, W = 2, \tilde{N} = 3$
- Case B: $L = 3, W = 2, \tilde{N} = 2$
- Case C: $L = 3, W = 2, \tilde{N} = 1$
- Case D: $L = 2, W = 3, \tilde{N} = 1$

where L and W are the number of nodes in horizontal and vertical dimensions of the network, respectively and \tilde{N} represents the last participating columns. Hence, Case A is B-OLA, whereas the other cases represent OLA-LP with different levels of participating nodes. Fig. 5 shows the trend of one-hop success probability versus the SNR margin for the aforementioned cases. It can be seen that at lower value of SNR margin, B-OLA performs the best. However, as the SNR margin increases, the OLA-LP achieves asymptotically 100% success. If we operate the system at 6dB to reach a distance of $D = 100d$ for $\eta = 0.9$, in horizontal dimension, the one-hop success probability required to cover the distance could be found by Eq.(13) with $n_o = L$. Case C does not provide the required one-hop success probability and hence cannot be used to cover the desired range since only two nodes

are participating in the transmissions. Nevertheless, the rest of the cases provide the desired coverage with desired QoS. The total energy consumption of these cases is given as: Case A=19.01dB, Case B=17.24dB and Case D=17.75dB.

The results show that Case B consumes the least energy and provides the required QoS. The fraction of energy saved for this particular case is $1 - ((\tilde{N}W)/M) = 1/3$. The results show that OLA-LP provides an energy-efficient approach in 2D deployment of network than the B-OLA without compromising the required QoS.

V. CONCLUSIONS

In this paper, we introduced the concept of limiting the node participation in a decode-and-forward cooperative network for conserving the energy during transmissions in a 1D and 2D finite node density network. The transmissions are modeled by a discrete-time Markov chain and the Perron-Frobenius eigen-decomposition helps in determining the selection of nodes for sustainability of transmissions at a particular SNR margin. It has been shown that to obtain a desired end-to-end reliability of the network, there is no need to exploit full diversity offered by all the transmitting nodes. Limiting the node participation results in an energy-efficient transmission besides fulfilling the required QoS. It is recommended to extend this network analysis to more general random networks where the nodes are randomly placed.

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