# Maximum Likelihood SNR Estimation for Non-Coherent FSK-based Cooperative Networks in Rayleigh fading channels

By

**Aamra Arshad**

**2012-NUST-MS-EE(S)-60897**

Supervisor

**Dr. Syed Ali Hassan**

**Department of Electrical Engineering**

A thesis submitted in partial fulfillment of the requirements for the degree of Masters in Electrical Engineering (MS EE-TCN)

In

School of Electrical Engineering and Computer Science,

National University of Sciences and Technology (NUST),

Islamabad, Pakistan.

(December 2014)

# Approval

It is certified that the contents and form of the thesis entitled "**Maximum Likelihood SNR Estimation for Non-Coherent FSK-based Cooperative Networks in Rayleigh fading channels**" submitted by **Aamra Arshad** have been found satisfactory for the requirement of the degree.

Advisor: **Dr. Syed Ali Hassan**

Signature: ____________________

Date: ____________________

Committee Member 1: **Dr. Adnan Khalid Kiani**

Signature: ____________________

Date: ____________________

Committee Member 2: **Dr. Khawar Khurshid**

Signature: ____________________

Date: ____________________

Committee Member 3: **Dr. M.Shahzad Younis**

Signature: ____________________

Date: ____________________

# Dedication

I dedicate this thesis to my parents, teachers and colleagues who helped me throughout my research phase.

# Certificate of Originality

I hereby declare that this submission is my own work and to the best of my knowledge it contains no materials previously published or written by another person, nor material which to a substantial extent has been accepted for the award of any degree or diploma at NUST SEECS or at any other educational institute, except where due acknowledgement has been made in the thesis. Any contribution made to the research by others, with whom I have worked at NUST SEECS or elsewhere, is explicitly acknowledged in the thesis.

I also declare that the intellectual content of this thesis is the product of my own work, except for the assistance from others in the project's design and conception or in style, presentation and linguistics which has been acknowledged.

Author Name: **Aamra Arshad**

Signature: ________________

# Acknowledgment

I am very thankful to Almighty Allah who enlightened my mind and gave me the courage and knowledge to be able to accomplish this thesis. It would not have been possible without His blessings.

I am eternally grateful to my family who have always supported me by their encouragement and by moral and emotional support throughout the dissertation. This thesis is a milestone in my academic carrier. Extensive research has increased my knowledge in learning theories and concepts.

My utmost gratitude to Dr. Syed Ali Hassan, who provided me assistance and valuable thoughts in my research work. His recommendations, keen interest and above all sympathetic attitude have enabled me to fulfill the requirements of the dissertation effectively. He has been a great source of inspiration for me and his immense knowledge and valuable guidance helped me to accomplish my task in an effective manner.

My special thanks to my committee members, Dr. Adnan Khalid Kiani, Dr. Khawar Khursheed, Dr. M. Sahzad Younis and to all those friends and teachers who contributed towards the successful completion of my dissertation.

**Aamra Arshad**

# Abstract

Signal-to-noise ratio (SNR) is one of the most important and critical performance metrics for analysing the performance of wireless communication systems. SNR estimates are used at the receiver side for the purpose of symbol decoding, power control algorithms, turbo decoding, and assigning adaptive modulation and coding (AMC) schemes. SNR estimation also finds application in many areas of cellular as well as in wireless sensor networks (WSNs). In this thesis, we consider the SNR estimation for single-input multiple-output (SIMO) and multiple-input single output (MISO) communication systems considering independent and correlated Rayleigh fading channels, respectively. Non-coherent Frequency shift keying (FSK) modulation scheme is considered. Data aided and Non data-aided maximum likelihood (ML) estimation techniques are used. Cramer-Rao lower bound expression are derived for analysing the performance of the derived estimators.

# Table of Contents

# List of Figures

# List of Abbreviations

| Abbreveation | Description |
|---|---|
| SNR | Signal-to-noise ratio |
| AMC | Adaptive Modulation and Coding |
| WSN | Wireless Sensor Network |
| SIMO | Single-Input Multi-Output |
| MISO | Multi-Input Single-Output |
| MIMO | Multi-Input Multi-Output |
| NMSE | Normalized Mean Squared Error |
| DA | Data Aided |
| NDA | Non Data Aided |
| CRB | Cramer-Rao Bound |
| EGC | Equal Gain Combning |
| CT | Cooperative Transmission |
| NCFSK | Non-coherent Frequency Shift Keying |
| PLL | Phased Lock loop |

# Chapter 1

# Introduction

Estimation theory deals with the estimation of random parameters based on some measured or empirical data by employing a certain set of techniques on the received data. Estimation in signal processing originated with the origination of analog signals. It has evolved from estimation of analog signals and waveforms and presently a remarkable amount of research has already been conducted on digital signal estimation with different channel conditions and environments.

Signal-to-noise ratio (SNR) is one of the key performance metrics in analysing the performance of a wireless communication system. Prime utilization of SNR estimates dwells in various receiver functions, e.g., all the decisions regarding decoding of data symbols are made on the basis of SNR value of the received data symbol. Moreover, in wireless communication system, the estimated signal SNR value is used in adaptively assigning modulation and coding (AMC) scheme to be used at the transmitter for transmitting the data symbols. It is widely used in power control algorithms and for decision making in turbo decoders. Additionally, SNR estimation finds its application in many areas of cellular as well as in wireless sensor networks (WSNs).

## 1.1 Application of SNR estimation in Wireless sensor networks

In WSNs, the sensor nodes are highly energy constrained. Because of their small size, they can generally be equipped with only a limited amount of power supply. In multihop WSNs, each sensor node has to fulfill dual purpose of data transmitter and data router. Malfunctioning of a single node due to power failure can cause the energy hole for the entire network. Therefore, this limited supply of power needs to be managed efficiently by devising power control algorithms. SNR estimation finds many applications in cellular networks and wireless sensor networks.

### 1.1.1 Receiver Functions

Signal-to-noise ratio is the fundamental metric on the basis of which the demodulation or decoding of the received data symbol is made. A predefined SNR threshold is defined at the receiver, if the estimated SNR value is greater than this threshold value then it is assumed to be decodable otherwise it is not. For example, Decode and forward in different levels of opportunistic large array (OLA) networks is based on the estimated SNR value at every individual node [30].

### 1.1.2 Power Efficient Routing

It serves a major role in finding candidate co-operators in the cooperative communication environment by letting power efficient routing. An SNR threshold is defined for relay recruitment by limiting the number of nodes participating in the data transfer, hence, making the routing process more energy-efficient [1-2]. Similarly, they can be used to find the one-hop neighbors of a wireless sensor node [3].

## 1.2 Motivation

This thesis targets two types of scenarios in which SNR estimation needs to be done:

1. In first scenario, we consider a WSN in which sensor nodes forward their data to a central entity or a fusion center equipped with multiple collocated antennas [4]. The SNR of the received data is estimated at this fusion center and is used in assigning adaptive modulation and coding scheme (AMC) to be used at the transmitter or other receiver functions. Therefore, every individual transmitting sensor node and multiple collocated antennas at the fusion center forms a single-input multiple-output (SIMO) communication system.

2. In the second case, we again consider WSN in which sensor nodes are highly energy constrained. Because of the impairments of the wireless channel and receiver noise, single link between two sensor nodes is prone to errors and imperfect decoding. Therefore, such scenarios are desirable in which power gain along with efficient data transfer

is obtained. Cooperative transmission (CT) is a way to improve the decoding capability by providing array gain. In a multi-hop WSN, a group of nodes participate to transfer their data to the next group of nodes cooperatively [20]. Hence, for one receiving node, a virtual multiple-input single-output (MISO) is created as many sensors send the same message to this node [29]. Due to simultaneous transmission of multiple nodes, power gain is achieved at the central receiving node. The data from each sensor node follows a separate channel but different channels might have correlation with each other due to insufficient spacing between the sensor nodes [21, 22].

### 1.2.1 Modulation Scheme

Keeping in view the above mentioned scenarios of WSNs, such a modulation scheme is desirable, which is relatively power efficient and offers less receiver complexity. Non-coherent frequency shift keying (NCFSK) fulfills both the criteria; power efficiency at the transmitter side by having constant signal envelope and less receiver complexity because of squared envelope detection. This scheme does not require sophisticated signal processing algorithms based on phased-locked loops (PLLs) for carrier phase synchronization. Because of the non-coherent nature of modulation scheme, equal gain combining (EGC) is employed at the receiver side to combine the data arriving in all the receiver branches [5].

## 1.3 Problem Statement

To estimate the average SNR of

1. A single-input multiple-output (SIMO) system employing NCFSK modulation scheme and having single transmit antenna and multiple collocated receive antennas considering independent Rayleigh fading channels.

2. A multiple-input single-output (MISO) system employing NCFSK modulation scheme and having multiple transmitting sensor nodes and a single receiving node considering correlated Rayleigh fading channels.

## 1.4 Thesis Organization

The rest of the thesis is organized as follows.

### 1.4.1 Chapter 2

In chapter 2, the literature review of the research done on SNR estimation has been discussed in the light of different scenarios which we come across in wireless communication systems.

### 1.4.2 Chapter 3

In chapter 3, the system architecture of the considered SIMO communication system has been discussed and derivation of the SNR estimator using maximum likelihood estimation technique is carried on considering data aided and non data-aided estimation cases. Cramer-Rao lower bound is derived for the performance evaluation of the derived estimators. Estimator's performance has been analyzed by changing several system parameters.

### 1.4.3 Chapter 4

Chapter 4 discusses the system architecture of the considered MISO communication system considering correlated Rayleigh fading channels and derivation of the SNR estimator using maximum likelihood estimation (MLE) technique is derived on considering data aided and non data-aided estimation cases. Cramer-Rao lower bound is derived for the performance evaluation of the derived estimators. Estimator's performance has been analyzed by changing several system parameters in the presence of correlation among all channels.

### 1.4.4 Chapter 5

Conclusion and future work has been discussed in chapter 5.

# Chapter 2

# Literature Review

Several authors have worked on designing SNR estimators for different environments and receiver architectures. Similarly, an extensive introduction has been given on WSNs and factors effecting wireless communication in [31]. Effect of spatial and temporal correlation in WSN is studied in [21, 22]. Most of the research work related to SNR estimation has been done on the derivations of M-ary phase shift keying (MPSK) and frequency shift keying (MFSK) estimators. In this chapter, we will be reviewing all the significant research carried out related to SNR estimation.

In [6], the authors have analyzed BPSK and 8-PSK SNR estimators using different estimation techniques and have presented the comparison between the performance of each of them considering real and complex additive white Guassian noise (AWGN) channels, respectively. This study concludes that the best estimator depends upon different system parameters, i.e., block size of data to be estimated, the number of samples per symbol available, modulation scheme to be used and SNR range of interest. After making analysis using different estimation techniques, the author has recommended

Maximum likelihood (ML), method of moments (Mom) and signal to variance ratio (SVR) to perform best in terms of complexity and mean squared error in most of the cases.

Signal-to noise ratio (SNR) maximum likelihood estimators (MLEs) have been designed for a system employed with non-coherent BFSK modulation in Rayleigh fading channels and their performances were crossed checked by their respective Cramer-Rao lower bounds (CRBs) in [28]. In this paper, the author has emphasized the use of square law estimation because of its less complexity. Maximum likelihood estimators and their respective CRBs are derived for three data models. In the end, it is also verified through the computer simulations that the performance of the derived estimators matches these bounds very closely.

In [7], the authors have designed an SNR estimator using the method of moments (MoM) for the case of BPSK modulated signals in Nakagami-m fading channels with receiver diversity. Performance comparison curves for turbo decoder have been plotted for both; the estimated SNR value versus knowledge of channel information. It can be inferred from this research that there is no need of transmitting channel information along with data for those SNR values where derived estimator is giving minimum difference in these performance curves. Similarly, many other authors have focused on the problem of SNR estimator design for BPSK modulated signal considering different channels effect, i.e., multipath fading in [3, 25, 26, 27].

In [8] and [9], the authors have designed SNR estimators for non-coherent BFSK and MFSK receivers, respectively, over fading channel in the presence of AWGN using maximum likelihood estimation (MLE) and data statistics approach. Moreover, different scenarios have been taken into account, i.e., data aided (DA), non-data aided (NDA) and method of moments (MoM) and also their comparison is presented in terms of their performance.

Authors in [10] designed SNR estimators for a slow fading environment. Carrier frequency offset effects have been taken into consideration while estimating SNR in [12, 11]. This paper paper points towards the fact that perfect value of the SNR can not be estimated in the presence of CFO factor. Therefore, the author has estimated a nuisance parameter of CFO before SNR estimation. SNR estimator design for NCMFSK receiver for Rice fading environment is presented in [23]. In all of the above research works, it has been proved from analysis of the derived estimators that performance of the estimator is increased on increasing data samples and modulation index. For non-coherent modulation schemes, the selection combining was considered for selecting the data branch with highest SNR value among all [24]. The estimated SNR values in all of the previous given works were applicable for getting the SNR knowledge individually on every diversity branch and on the basis of this information, selection combining (SC) was made. However, in SC the data in all the remaining branches is wasted and we are able to get no diversity gain. Therefore, in this thesis we have employed equal gain combining (EGC) for combining of data coming from all diversity branches [5]. In all of these works the approach is only valid for a single-input single-output (SISO) system which is not sufficient for a WSN where multiple links are formed to transmit the data and constitutes the main motivation behind this study.

# Chapter 3

# SNR Estimation in SIMO communication system

## 3.1 Introduction

In this chapter, the problem of signal-to-noise ratio(SNR) estimator design for a single-input multiple-output (SIMO) communication system employing non-coherent M-ary frequency shift keying (NCMFSK) modulation scheme is considered. The transmitted signal undergoes Rayleigh fading and additive white Gaussian noise (AWGN) and is received at a receiver with $L$ diversity branches. Closed-form expressions of data aided (DA)and non-data aided (NDA) estimators have been derived using the maximum likelihood (ML) estimation approach. Cramer-Rao bound has been evaluated to compare the performance of the designed estimators. The effect of increasing the receiver diversity branches on the performance of estimators has been quantified.

## 3.2 System Model

In this chapter, we are considering a SIMO communication system as shown in Fig. 3.1 having single transmit and $L$ co-located receive antennas. Non-coherent MFSK is employed where each transmitted symbol undergoes independent Rayleigh fading and is corrupted by AWGN.

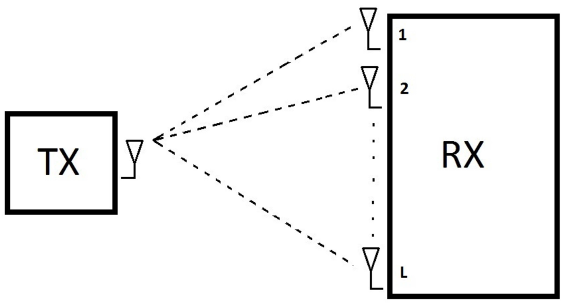


Figure 3.1: SIMO communication system with $L$ co-located receiver antennas

The block diagram of a non-coherent BFSK receiver with $L$ diversity branches and envelope detection is shown in Fig. 3.2. Each diversity branch is further divided into $M$ receiver sub-branches corresponding to the MFSK receiver. For the case of Fig. 3.2, we are considering non-coherent BFSK receiver, so every $\ell^{th}$ diversity branch is further divided into two receiver sub-branches.

We get $L$ copies of the transmitted symbol at the receiver. The copy of the signal acquired on the $\ell^{th}$ diversity branch at any time instant $i$ after correlator is given by

$$\mathbf{v}_{\ell,\mathbf{i}} = \mathbf{s}_{\mathbf{i}}\alpha_{\ell,i} + \mathbf{n}_{\ell,\mathbf{i}}, \tag{3.1}$$

where $\ell$=$\{1, 2, .., L\}$ and $i$=$\{1, 2, .., K\}$ represent the diversity branch and time index, respectively. Each of $\mathbf{v}_{\ell,\mathbf{i}}$, $\mathbf{s}_{\mathbf{i}}$, and $\mathbf{n}_{\ell,\mathbf{i}}$ are independent vectors

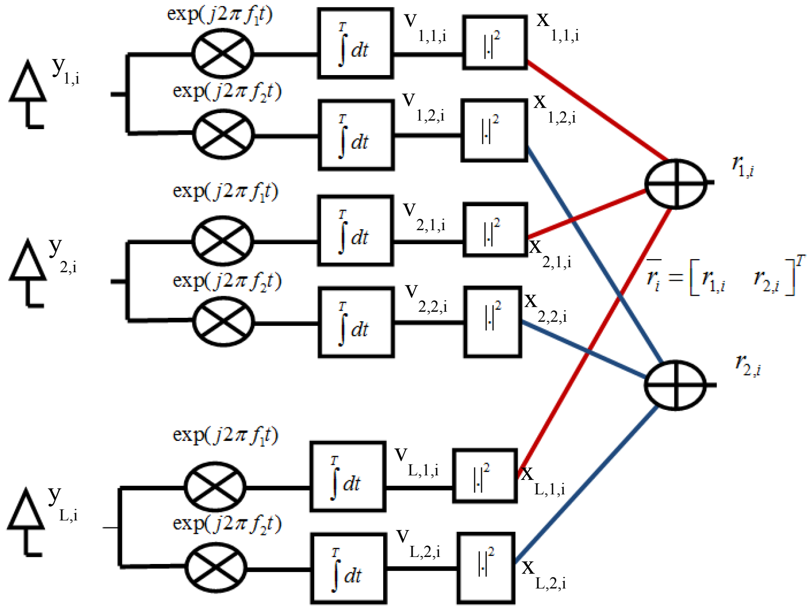


Figure 3.2: Receiver structure of the system employing BFSK modulation and EGC

with a dimension of $M \times 1$, where $\mathbf{v_{\ell,i}} = \{v_{\ell,1,i}, v_{\ell,2,i}, ..., v_{\ell,M,i}\}$. The $v_{\ell,m,i}$ corresponds to the $\ell^{th}$ received copy of the transmitted symbol in the $m^{th}$ receiver sub-branch $(1 \leq m \leq M)$ at the $i^{th}$ time instant. In (4.1), $\mathbf{s_i} = [0, 0, ..., 0, 1, 0, ..., 0]^T$ is the vector transmitted from the MFSK transmitter, where 1 at the $m^{th}$ $(1 \leq m \leq M)$ position corresponds to the transmitted frequency and 0 is set in all the remaining $(M-1)$ positions. Fading in every $\ell^{th}$ diversity branch is represented by $\alpha_{\ell,i}$. Since Rayleigh fading is considered, so the elements of $\alpha_{\ell,i}$ are drawn from a complex Gaussian distribution, i.e., $\alpha_{\ell,i} \in CN(0, S)$, where $S$ is the variance of fading. AWGN in the symbol present in the $m^{th}$ receiver sub-branch of the $\ell^{th}$ diversity branch and at the $i^{th}$ time instant is represented by $n_{\ell,m,i} \in \mathbf{n_{\ell,i}}$. Elements of $\mathbf{n_{\ell,i}}$ belong to complex Gaussian distribution, i.e., $n_{\ell,m,i} \in CN(0, N)$, where $N$ is the noise power.

From Fig. 3.2, we can see that $v_{\ell,m,i}$ will pass from the envelope detector; Thus, $x_{\ell,m,i} = | v_{\ell,m,i} |^2$. Thereafter, equal gain combining (EGC) is employed on the data $x_{\ell,m,i}$ present in all the $M$ sub-branches of $L$ antennas of the receiver to get the vector $\mathbf{r_i} = [r_{1,i}, r_{2,i}, ..., r_{M,i}]^T$. In Fig. 3.2, data from first sub-branch of every antenna has been summed up to get a symbol $r_{1,i} = \sum_{\ell=1}^{L} x_{\ell,1,i}$ and similarly data from the second sub-branch of every antenna has been summed up to get the second symbol $r_{2,i} = \sum_{\ell=1}^{L} x_{\ell,2,i}$. As we are considering non-coherent BFSK in Fig. 3.2, so we will be left with a $(2 \times 1)$ vector i.e., $\mathbf{r_i} = [r_{1,i}, r_{2,i}]^T$. In this chapter, we are interested in estimating average SNR from $[\mathbf{r_1}\ \mathbf{r_2}\ \mathbf{r_3}\ ...\ \mathbf{r_K}]^{\mathbf{T}}$, which is acquired after the post-detection combining of the $K$ received data symbols. This is done for several estimation schemes, which will be discussed in the forthcoming section.

## 3.3 Estimation Techniques

This section contains the derivation of data aided (DA) and non-data aided (NDA) estimator expressions using maximum likelihood (ML) estimation technique.

### 3.3.1 Data Aided MLE

The objective here is to estimate the average SNR of $K$ consecutively received symbols. Hence without losing generality, we set $\mathbf{s_i} = [1\ 0\ 0...\ 0]^T$ for each of the $K$ symbols. On the basis of this assumption, the data is received in the first branch of receiver and all the remaining $(M-1)$ branches contain noise. As we are considering postdetection combining, the data to be used is $r_{m,i} \in \mathbf{r_i} = [r_{1,i}, r_{2,i}, ..., r_{M,i}]^T$. Because of the fact that $\alpha_{\ell,i}$ and $n_{\ell,m,i}$ are Complex Gaussian random variables, so the first symbol from every diversity branch, i.e., $x_{\ell,1,i} = \mid \alpha_{\ell,i} + n_{\ell,m,i} \mid^2$, follows an exponential distribution having mean of $\mathbb{E}\mid \alpha_{\ell,i} \mid^2 + \mathbb{E}\mid n_{\ell,m,i} \mid^2$. Moreover, the data received in the first branch of receiver is he sum of $L$-exponentially distributed terms, i.e., $x_{1,1,i} + x_{2,1,i} ... + x_{L,1,i}$ having same mean and will result in $r_{1,i}$ to be gamma distributed. The probability density function (PDFs) of the received pilot symbols $\mathbf{r_i} = [r_{1,i}, r_{2,i}, ..., r_{M,i}]^T$, are given as

$$p_{r_{1,i}}(r_{1,i}) = \frac{(S+N)^{-L}(r_{1,i})^{L-1}}{(L-1)!} exp\left(\frac{-r_{1,i}}{S+N}\right) \tag{3.2}$$

and PDF of the symbols in $\{m = 2, ..., M\}$ receiver sub-branches is

$$p_{r_{m,i}}(r_{m,i}) = \frac{(N)^{-L}(r_{m,i})^{L-1}}{(L-1)!} exp\left(\frac{-r_{m,i}}{N}\right).$$

The joint PDF of the received vector, $\mathbf{r_i}$, becomes

$$p_{\mathbf{r_i}}(r_{m,i}) = \frac{(S+N)^{-L}(N)^{-L(M-1)}\prod_{m=1}^{M}(r_{m,i})^{L-1}}{((L-1)!)^{M-1}} \times \exp\left(\frac{-r_{1,i}}{S+N} - \sum_{m=2}^{M}\frac{r_{m,i}}{N}\right). \tag{3.3}$$

Hence the log-likelihood function of $K$ received symbols can be found as

$$\mathbf{\Lambda}_{\mathbf{r_i}}(r_{m,i}; S, N) = -KL\ln(S+N) - KL(M-1)\ln(N) + (L-1)\sum_{i=1}^{K}\sum_{m=1}^{M}(r_{m,i})M\ln((L-1)!) - \frac{1}{S+N}\sum_{i=1}^{K}r_{1,i} - \frac{1}{N}\sum_{i=1}^{K}\sum_{m=2}^{M}(r_{m,i}). \tag{3.4}$$

Using the fact that ML estimate of the ratio of two parameters is equal to their individual estimates, we can write the estimated SNR expression as

$$\hat{\gamma}_{DA} = \frac{\hat{S}_{ML}}{\hat{N}_{ML}}. \tag{3.5}$$

For this purpose, we want to extract the parameters of our interest, i.e., signal power $\hat{S}_{ML}$ and noise power $\hat{N}_{ML}$ from log-likelihood expression. Differentiating (3.4) with respect to $S$ and $N$ individually and setting these derivatives equal to zero results in $\hat{S}_{ML}$ and $\hat{N}_{ML}$. Putting these values in (6) and solving, we will get the data aided estimates of SNR as

$$\hat{\gamma}_{DA} = \frac{(M-1)\sum_{i=1}^{K} r_{1,i} - \sum_{i=1}^{K}\sum_{m=2}^{M} r_{m,i}}{\sum_{i=1}^{K}\sum_{m=2}^{M} r_{m,i}}. \tag{3.6}$$

### 3.3.2 Non-Data Aided MLE

In NDA, we have no knowledge about the transmitted data symbol. So we assume that all transmitted symbols have equal priori probabilities. The conditional PDF of the received symbol given a 1 at the $n^{th}$ position was transmitted is

$$p_{r_{n,i}}\left(r_{n,i}|s_n = 1\right) = \frac{(S+N)^{-L}\left(r_{n,i}\right)^{L-1}}{(L-1)!} exp\left(\frac{-r_{n,i}}{S+N}\right), \tag{3.7}$$

and the conditional PDF of the received symbol given a 0 at the $n^{th}$ position was transmitted becomes

$$p_{r_{n,i}}\left(r_{n,i}|s_n = 0\right) = \frac{(N)^{-L}\left(r_{n,i}\right)^{L-1}}{(L-1)!} exp\left(\frac{-r_{n,i}}{N}\right). \tag{3.8}$$

Now there are $M$ different possibilities of the received symbol. We can express the joint unconditional PDF of the $M$ received symbols using the law of total probability as

$$p_{\mathbf{r_i}}\left(\mathbf{r_i}\right) = (\frac{1}{M}(S+N)^{-L}(N)^{-L}\prod_{m=1}^{M} r_{m,i}{}^{(L-1)})\left[\left(\exp(\frac{-r_{1,i}}{S+N}) - \sum_{m=2,m\neq 1}^{M}\frac{r_{m,i}}{N}\right)+ \right.$$

$$\left. .... + \left(\exp(\frac{-r_{2,i}}{S+N}) - \sum_{m=1,m\neq M}^{M-1}\frac{r_{m,i}}{N}\right)\right], \quad (3.9)$$

The above equation is very complex to solve, so simplifying the above expression by factoring the term $\exp(\frac{-\sum_{m=1}^{M} r_{m,i}}{N})$, we get

$$p_{\mathbf{r_i}}\left(\mathbf{r_i}\right) = \frac{1}{M}(S+N)^{-L}(N)^{-L}\prod_{m=1}^{M}(r_{m,i})^{L-1}$$

$$\exp(\frac{-\sum_{m=1}^{M} r_{m,i}}{N})\left[\sum_{m=1}^{M}\exp(-r_{m,i}\psi)\right], \quad (3.10)$$

where $\psi = \frac{1}{S+N} - \frac{1}{N}$. We get the log-likelihood function as

$$\mathbf{\Lambda}_{\mathbf{r_i}}(r_{m,i}; S, N) = -K\ln(M) - KL\ln(S+N) - KL(M-1)\ln(N)+$$

$$(L-1)\sum_{i=1}^{K}\sum_{m=1}^{M}\ln(r_{m,i}) + \sum_{i=1}^{K}\sum_{m=1}^{M}\frac{r_{m,i}}{N} + \sum_{i=1}^{K}\ln\left[\sum_{m=1}^{M}\exp(-r_{m,i}\psi)\right]. \quad (3.11)$$

We can have $\hat{S}$ and $\hat{N}$ by differentiating log-likelihood function in (3.11) with respect to $S$ and $N$, exactly in the same way as done for the data aided case in the previous section. We have the following expression

$$\hat{S} + M\hat{N} = \frac{1}{KL}\sum_{i=1}^{K}\left[\frac{\sum_{m=1}^{M} r_{m,i}\exp(-r_{m,i}\psi)}{\sum_{m=1}^{M}\exp(-r_{m,i}\psi)}\right]. \quad (3.12)$$

Finding a closed-form solution of the above non-linear expression is prohibitive, so we approximate it to a feasible form. Let us consider this expression for the case of $M = 2$, i.e., let

$$A = \sum_{i=1}^{K}\left[\frac{\sum_{m=1}^{2} r_{m,i}\exp(-r_{m,i}\psi)}{\sum_{m=1}^{2}\exp(-r_{m,i}\psi)}\right],$$

$$A = \sum_{i=1}^{K} \frac{r_{1,i}\exp(-r_{1,i}\psi) + r_{2,i}\exp(-r_{2,i}\psi)}{exp(-r_{1,i}\psi) + \exp(-r_{2,i}\psi)}. \tag{3.13}$$

It can be observed for the case of very high SNR, i.e., $S >> N$, $\psi = \left[\frac{1}{S+N} - \frac{1}{N}\right]$ reduces to $\psi \cong \frac{-1}{N}$, Thus the above approximation becomes

$$A = \sum_{i=1}^{K} \frac{r_{1,i}\exp(r_{1,i}/N) + r_{2,i}\exp(r_{2,i}/N)}{exp(r_{1,i}/N) + \exp(r_{2,i}/N)}, \tag{3.14}$$

Rearranging the above equation, we get

$$A = \sum_{i=1}^{K} \frac{r_{1,i}}{1 + \frac{\exp(r_{2,i}/N)}{\exp(r_{1,i}/N)}} + \frac{r_{2,i}}{1 + \frac{\exp(r_{1,i}/N)}{\exp(r_{2,i}/N)}}. \tag{3.15}$$

Among $[r_{1,i}, r_{2,i}]$, only one branch will contain signal and the other will contain noise. Let us consider that the $r_{1,i}$ contains signal. For the case of high SNR, $S >> N$, $(1 + \frac{\exp(r_{2,i}/N)}{\exp(r_{1,i}/N)}) \rightarrow 1$ and $(1 + \frac{\exp(r_{1,i}/N)}{\exp(r_{2,i}/N)}) \rightarrow \infty$. Thus the above expression reduces to

$$A \approx \left( \sum_{i=1}^{K} \max_{m=1,2,3,..,M} r_{m,i} \right). \tag{3.16}$$

Using this expression in (3.12) and solving, we get the expression for estimated noise power $\hat{N}$ as

$$\hat{N} = \frac{(M-1)\sum_{m=1}^{M} r_{m,i} - M\sum_{m=1}^{M}\sum_{i=1}^{K} r_{m,i} + M\sum_{i=1}^{K}\max_m r_{m,i}}{\sum_{m=1}^{M}\sum_{i=1}^{K} r_{m,i} + \sum_{i=1}^{K}\max_m r_{m,i}}, \tag{3.17}$$

and the estimate of signal to noise ratio for NDA is given as

$$\hat{\gamma}_{NDA} = \frac{-\sum_{m=1}^{M} r_{m,i} + M\sum_{i=1}^{K}\max_m r_{m,i}}{\sum_{m=1}^{M} r_{m,i} - \sum_{i=1}^{K}\max_m r_{m,i}}, \tag{3.18}$$

where, $r_{m,i} = \sum_{\ell=1}^{L} x_{\ell,m,i}$.

## 3.4 Cramer-Rao Lower Bound

In order to evaluate the performance of the derived estimators, we find the Cramer-Rao bound (CRB), which is the lower bound on the variance of any estimator. In other words, it states that the variance of the derived estimator must be greater than or equal to this bound. We have derived the CRB for the data aided (DA) case and compared it with the normalized mean squared error (NMSE) to judge the performance of estimator. Although we can have CRB for non-data aided case as well, however as the benchmark performance is given by DA method, we use its CRB to evaluate the performance of the derived estimators. We have two unknown parameters, i.e., signal power, $S$ and the noise power, $N$. We consider the unknown vector parameter $\boldsymbol{\theta}$=$[S\ N]^T$. We have

$$CRB = \frac{\partial g(\theta)}{\partial \theta}\mathbf{I}^{-1}(\theta)\frac{\partial g(\theta)}{\partial \theta}^T, \tag{3.19}$$

where $g(\theta)$ is a function of parameter $\theta$ and $\mathbf{I}$ is the Fisher information matrix. Taking partial derivative of $g(\theta) = \frac{S}{N}$ with respect to $\theta = [S\ N]^T$, we get

$$\frac{\partial g(\theta)}{\partial \theta} = \begin{bmatrix} \frac{1}{N} & \frac{-S}{N^2} \end{bmatrix}, \tag{3.20}$$

The Fisher information matrix (FIM), $\mathbf{I}(\theta)$ is given by

$$\mathbf{I}(\theta) = \begin{bmatrix} -\mathbb{E}\left(\frac{\partial^2 \Lambda_{DA}}{\partial S^2}\right) & -\mathbb{E}\left(\frac{\partial^2 \Lambda_{DA}}{\partial SN}\right) \\ -\mathbb{E}\left(\frac{\partial^2 \Lambda_{DA}}{\partial NS}\right) & -\mathbb{E}\left(\frac{\partial^2 \Lambda_{DA}}{\partial N^2}\right) \end{bmatrix}, \tag{3.21}$$

where $\mathbb{E}$ is the expectation operator. Solving the elements of the above matrix, we get

$$\mathbf{I}(\theta) = \begin{bmatrix} \frac{KL}{(S+N)^2} & \frac{KL}{(S+N)^2} \\ \frac{KL}{(S+N)^2} & \left(\frac{KL}{(S+N)^2} + \frac{KL(M-1)}{N^2}\right) \end{bmatrix}. \tag{3.22}$$

Putting the values of Equations (3.22) and (3.20) in (3.19), we get the expression for CRB as

$$CRB_{DA} = \frac{M}{KL(M-1)}(\gamma+1)^2. \tag{3.23}$$

## 3.5 Simulation Results

In this section, the performance of the estimators designed in the previous sections is presented in terms of normalized mean squared error (NMSE), which is given as

$$NMSE(\hat{\gamma}) = \mathbb{E}\frac{\{(\gamma-\hat{\gamma})^2\}}{\gamma^2}, \tag{3.24}$$

where $\gamma$ and $\hat{\gamma}$ being the true and estimated SNR, respectively. A perfect estimator is the one which always results in the least difference between estimated value and true value of the unknown parameter. Different trends of the NMSE versus SNR have been analyzed for several parameters, i.e., diversity branches, $L$, receiver sub-branches, $M$ and the number of symbols $K$. All the results presented in this section are averaged over 25,000 trials of simulations.

Fig. 3.3 shows the NMSE vs. SNR for the DA estimator for the case of $L = 5$, i.e., five diversity branches and for $K = 100$ symbols for various values of $M$. It can be observed from the figure that the NMSE is decreased as the receiver sub-branches, $M$, are increased. This is due to the fact that by adding more and more receiver sub-branches ($M$), the number of data samples go on increasing, forming a large data set. Thus, the sample mean of the large number of data samples converges towards the actual mean, resulting in a better performance of the estimator. Same trend has been

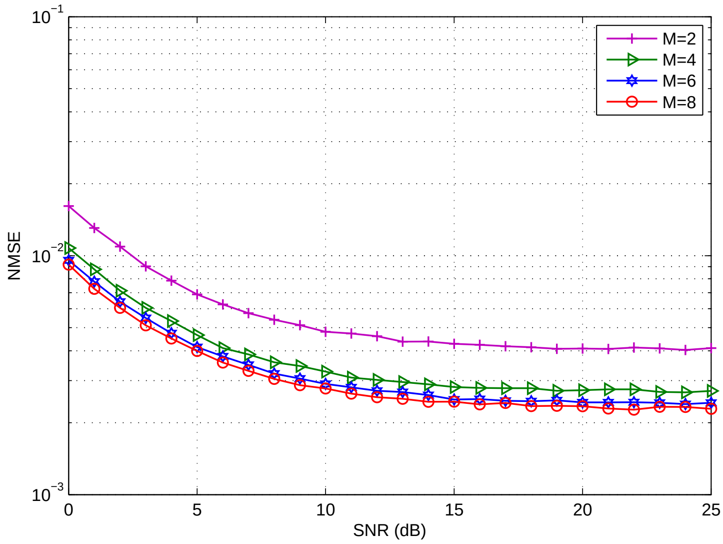


Figure 3.3: Effect of increasing $M$ on NMSE for 100 symbol-long packet for the DA estimator for $L = 5$

depicted by the NDA estimator, but is not shown here to avoid repetition.

Fig. 3.4 presents the effect of increasing diversity branches, $L$ on NMSE values of the DA estimator for $K = 100$ symbols. It can be seen that by increasing the value of $L$, the NMSE decreases. This decrease in NMSE is the consequence of increased number of branches of the data to be estimated. Therefore, we can summarize that increase in the values of both, the diversity branches, $L$ and receiver sub-branches, $M$, serves the same purpose of increased data samples, thereby lowering the NMSE and improved estimator performance.

It can however, be observed from Fig. 3.4 that the rate of NMSE reduction is large, when branches are increased from $L = 5$ to $L = 10$. This shows the diminishing returns behavior of the diversity gain on NMSE. Performance comparison curves for the DA and the NDA estimators have been

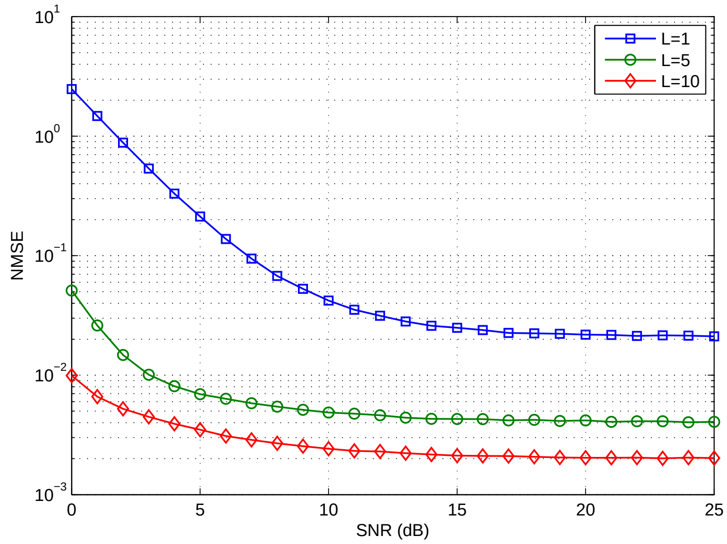


Figure 3.4: Effect of increasing diversity branches on data aided estimator for $K = 100$ symbols and $M = 2$

shown in Fig. 3.5 for $L = 5$ and $K$=100 symbols. Also the CRB derived for DA estimator has been plotted. For low SNR region, the NDA estimator gives large NMSE as compared to the DA estimator. Larger NMSE for the case of NDA in low SNR region can be attributed to the use of approximations derived for high SNR region (3.13)-(3.16). The difference between the error of DA and NDA estimator in the low SNR region goes on decreasing as the number of diversity branches are increased. Although not shown here but this difference is high for the cases of $L = \{1 \to 4\}$ in comparison with $L = 5$ that is shown in Fig. 3.5. Moreover, it can also be observed that the curve for DA is exactly giving the same NMSE values throughout the SNR region as that of the CRB evaluated for it. This points towards the fact that DA estimator is showing the minimum possible variance and the performance margin is high. This motivates the use of DA estimator in

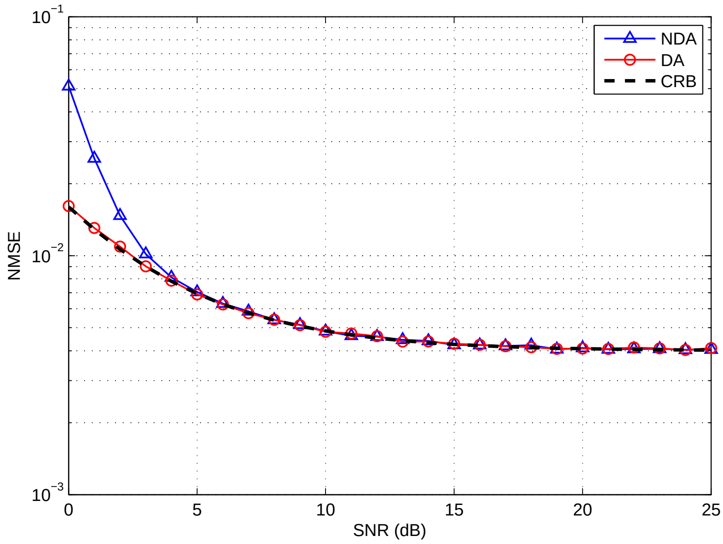


Figure 3.5: NMSE for $K = 100$ symbols, $M = 2$ and $L = 5$

decode-and-forward (DF)-based wireless sensor networks [14, 15, 16, 17, 18], where the precondition for forwarding the packet is to successfully decode it. The decoding is generally done by using cyclic redundancy check (CRC), and if the packet is decodable, the entire packet can be treated as pilot symbols to perform SNR estimation, which can then be used in various algorithms such as [1, 14, 15].

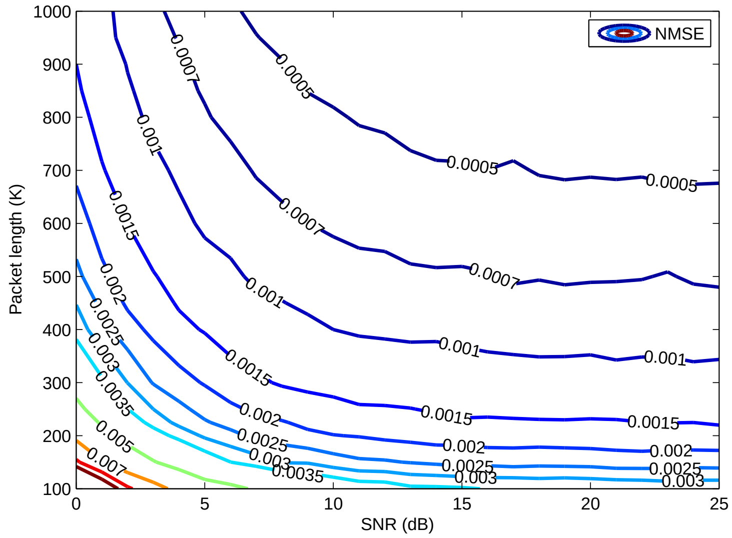


Figure 3.6: NMSE contours for $K = 100$ to 1000 symbols, $M = 4$ and $L = 4$

NMSE contours are presented in Fig. 3.6 for $L = 4$ and $M = 4$ with SNR shown on the x-axis and packet lengths (symbol size) at the y-axis. NMSE decreases with an increase in both the packet length and the SNR. So, for such situation where small NMSE is needed, larger length packets should be chosen and vice versa, e.g., we can see form the figure that less than 2% error can be achieved if $K \geq 200$ at the SNR values$\geq 8dB$.

# Chapter 4

# SNR Estimation in MISO communication system

## 4.1 Introduction

In this chapter, we will discuss signal-to-noise ratio (SNR) estimation for a virtual multi-input single-output(MISO) communication system employing non-coherent M-ary frequency shift keying (NCMFSK) modulation scheme. The transmitted signals from L different nodes undergo correlated Rayleigh fading and additive white Gaussian noise (AWGN) and are combined at a single receiving node via equal gain combining(EGC) scheme. Maximum likelihood (ML) estimation technique is used for deriving the closed-form expressions for data aided (DA) and non-data aided (NDA) estimators. Cramer-Rao bound(CRB) has also been derived to evaluate the performance of the derived estimators. Numerical values have been shown for various parameters such as number of transmitting nodes, modulation order, and varying number of symbols

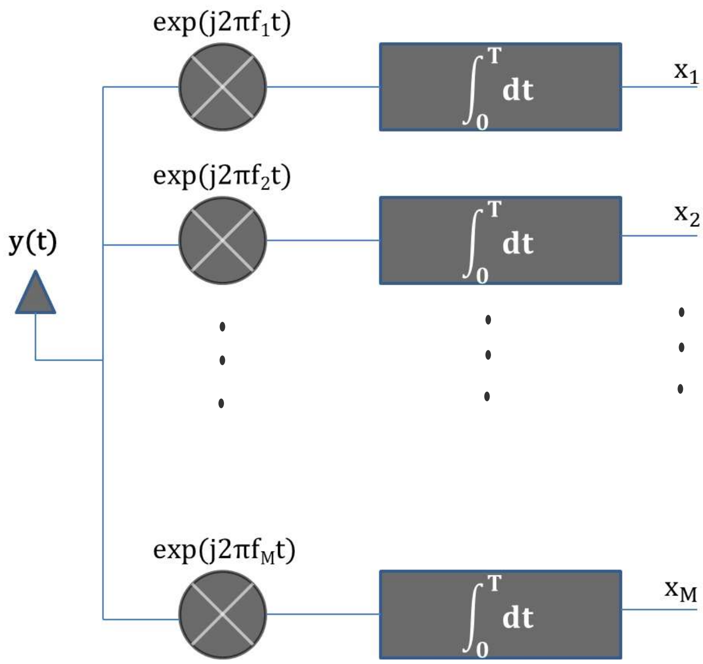


Figure 4.1: Non-coherent MFSK receiver

## 4.2 System Model

Consider a communication system having $L$ transmitting sensor nodes and a single sink or a central receiving node. Non-coherent MFSK modulation scheme is employed where each transmitted symbol undergoes correlated Rayleigh fading and is corrupted by additive white Gaussian noise (AWGN) independently. It can be noticed that transmission system creates a virtual MISO, i.e., all of the $L$ nodes transmit the same signal $s(t)$. The received signals from $L$ nodes are combined at the receiver using non-coherent EGC can be written as

$$y(t) = s(t) \sum_{\ell=1}^{L} \alpha^{(\ell)}(t) + n(t),$$

$$0 \leq t \leq T, m \in \{0, 1, ..., M\} \quad (4.1)$$

where $T$ is the symbol period, $s(t)$ is the message signal modulated at one of the FSK frequency, $\alpha^{(\ell)}(t)$ is the Rayleigh fading envelope and $n(t)$ is the AWGN. The received data symbol $x_m$ from $M$ branches is obtained after quadrature demodulation of the above signal $y(t)$. The block diagram of the non-coherent MFSK receiver is shown in Fig. 7. Each $x_m$ $\forall m = \{0, 1, ..., M\}$ is a complex number and can be written as

$$x_m = s_m \sum_{\ell=1}^{L} \left(\alpha_c^{(\ell)} + j\alpha_s^{(\ell)}\right) + (n_{c_m} + jn_{s_m}), \quad (4.2)$$

where the subscript $m$=$\{0, 1, ..., M\}$ represents the respective branch of the non-coherent MFSK receiver. Conventionally, a complex number $\eta$ is written as $\eta = \eta_c + j\eta_s$, with $j$=$\sqrt{-1}$. Thus

$$x_m = x_{c_m} + jx_{s_m}, \quad (4.3)$$

where $x_{c_m}$ and $x_{s_m}$ are inphase and quadrature components of the symbol received at $m^{th}$ branch of the receiver. Since Rayleigh fading is considered, the elements of the channel gain, $\alpha^{(\ell)}$, and the additive white Gaussian noise, $n_m$, are drawn from zero mean complex Gaussian distribution with variances of $S/2$ and $N/2$ per real dimensions, respectively. Additionally, we assume that all the channels are not independent from each other, therefore, correlation between any two different channels, i.e., $\alpha^{(i)}$ and $\alpha^{(j)}$ is represented by $\rho_{ij}$, where $i, j \in \{1, 2, ..., L\}$. However, $x_m$, $s_m$ and $n_m$ are the elements of the vector $\mathbf{x_{M\times 1}}$, $\mathbf{s_{M\times 1}}$ and $\mathbf{n_{M\times 1}}$ respectively. Thus at one time instant the received data is represented as $\mathbf{x}$=$[x_1, x_2, ..., x_M]^T$, where $T$ is the transpose operator. In this chapter, we are interested in estimating the average SNR of the received complex data vector, that is

$$\mathbf{x} = \mathbf{x_c} + \mathbf{j}\mathbf{x_s}, \tag{4.4}$$

Hence, $\mathbf{x}_{\mathbf{c}(M\times 1)}$ and $\mathbf{x}_{\mathbf{s}(M\times 1)}$ can be viewed as two zero mean real Gaussian random vectors

$$\begin{bmatrix} x_1 \\ \vdots \\ x_M \end{bmatrix} = \begin{bmatrix} x_{c_1} \\ \vdots \\ x_{c_M} \end{bmatrix} + j \begin{bmatrix} x_{s_1} \\ \vdots \\ x_{s_M} \end{bmatrix}, \tag{4.5}$$

with covariance matrices $\mathbf{K}_{\mathbf{cc}(M\times M)}$ and $\mathbf{K}_{\mathbf{ss}(M\times M)}$ and cross-covariance matrices $\mathbf{K}_{\mathbf{cs}(M\times M)}$ and $\mathbf{K}_{\mathbf{sc}(M\times M)}$. Thus

$$\mathbb{E}\big[\mathbf{x_c}\mathbf{x_c}^T\big] = \mathbf{K_{cc}}, \mathbb{E}\big[\mathbf{x_s}\mathbf{x_s}^T\big] = \mathbf{K_{ss}}, \mathbb{E}\big[\mathbf{x_c}\mathbf{x_s}^T\big] = \mathbf{K_{cs}}, \tag{4.6}$$

Furthermore, $\mathbf{x}$ is assumed to be proper random vector with vanishing pseudocovariance matrix [19]. That is

$$\mathbb{E}\left[(\mathbf{x_c} + j\mathbf{x_s})(\mathbf{x_c} + j\mathbf{x_s})^T\right] = 0, \tag{4.7}$$

which implies

$$\mathbf{K}_{cc} = \mathbf{K}_{ss}, \mathbf{K}_{cs}^T = -\mathbf{K}_{cs}. \tag{4.8}$$

We also assume that there is no correlation between the inphase and quadrature parts of any two different channel gains, i.e., $\mathbb{E}\left\{\alpha_c^{(i)}\alpha_s^{(j)}\right\}$=0. Thus $\mathbf{K_{cs}}$=$\mathbf{K_{sc}}$=0. However, correlation among inphase-inphase and quadrature-quadrature is assumed to be present, and the correlation coefficient between two channels is given as

$$\frac{\mathbb{E}\left\{\alpha_c^{(i)}\alpha_c^{(j)}\right\}}{\sigma_{\alpha_c^{(i)}}\sigma_{\alpha_c^{(j)}}} = \frac{\mathbb{E}\left\{\alpha_s^{(i)}\alpha_s^{(j)}\right\}}{\sigma_{\alpha_s^{(i)}}\sigma_{\alpha_s^{(j)}}} = \rho_{ij}, \tag{4.9}$$

where $\sigma_{\alpha_c^{(i)}}$ and $\sigma_{\alpha_s^{(i)}}$ represent the standard deviations of the inphase and the quadrature components respectively, related to channel $i$. Hence, the covariance matrix $\mathbf{K}$ of the complex vector $\mathbf{x}$=$\mathbf{x_c} + j\mathbf{x_s}$ is only a function of $\mathbf{K}_{cc}$=$\mathbf{K}_{ss}$, i.e.,

$$\mathbf{K} = \begin{bmatrix} \mathbf{K_{cc}} & 0 \\ 0 & \mathbf{K_{cc}} \end{bmatrix}, \tag{4.10}$$

In this chapter, we are interested in estimating the average SNR of received data using different estimation schemes. These schemes are discussed in the forthcoming section.

## 4.3 Estimation Techniques

### 4.3.1 Data Aided MLE

In pilot-assisted or data aided estimation, the objective is to estimate the average SNR of the received data $\mathbf{x}$ by transmitting the known data symbols from the transmitter. The $L$ sensor nodes transmit the same data vector, i.e., $\mathbf{s}$=$[s_1 = 1, s_2 = 0, \cdots, s_M = 0]$. Here, $s_1$=1 and $s_m = 0$ for $m$=$\{2, 3, \cdots, M\}$ implies that $f_1$ is transmitted from all nodes and that the data part will be received in the first receiver branch. However, the remaining $(M-1)$ branches will contain noise. The data to be estimated follows complex a Gaussian distribution, therefore, the joint probability density function (PDF) of $\mathbf{x_c}$ and $\mathbf{x_s}$ can be written as

$$p_{\mathbf{x_c},\mathbf{x_s}}(\mathbf{x_c}, \mathbf{x_s}) = \frac{1}{(2\pi)^M \sqrt{\det(\mathbf{K})}} \exp\left\{ -\frac{1}{2} \begin{bmatrix} \mathbf{x_c}^T & \mathbf{x_s}^T \end{bmatrix} \mathbf{K^{-1}} \begin{bmatrix} \mathbf{x_c} \\ \mathbf{x_s} \end{bmatrix} \right\}. \tag{4.11}$$

Assuming the data in the first branch, $\mathbf{K}_{cc}$=$\mathbf{K}_{ss}$ can be easily computed using (4.6) as

$$\mathbf{K}_{cc} = \begin{bmatrix} L\frac{S}{2} + \frac{N}{2} + \frac{S}{2}\Psi & 0 \cdots & 0 \\ 0 & \frac{N}{2} \cdots & 0 \\ \vdots & \vdots \ddots & \vdots \\ 0 & 0 \cdots & \frac{N}{2} \end{bmatrix}, \tag{4.12}$$

where $\Psi = \sum_{a=1}^{L} \sum_{b=1}^{L} \rho_{ab_{(a \neq b)}}$. Hence, using (4.10), we can have

$$\sqrt{\det(\mathbf{K})} = \frac{N}{2}^{M-1} \left[ L\frac{S}{2} + \frac{N}{2} + \frac{S}{2}\Psi \right], \tag{4.13}$$

as both $\mathbf{K}$ and $\mathbf{K}_{cc}^{-1}$ are non-singular matrices, so their inverse can easily be found

$$\mathbf{K^{-1}} = \begin{bmatrix} \mathbf{K_{cc}^{-1}} & 0 \\ 0 & \mathbf{K_{cc}^{-1}} \end{bmatrix}. \tag{4.14}$$

Therefore, we can re-write (4.11) as follows

$$p_{\mathbf{x_c},\mathbf{x_s}}(\mathbf{x_c}, \mathbf{x_s}) = \frac{\exp\left\{-\frac{1}{2}(\mathbf{x_c}^T \mathbf{K}_{cc}^{-1} \mathbf{x_c} + \mathbf{x_s}^T \mathbf{K}_{cc}^{-1} \mathbf{x_s})\right\}}{(2\pi)^M \sqrt{\det(\mathbf{K})}}. \tag{4.15}$$

Similarly, for a data packet containing $g$ independent data symbols, the likelihood function is the product of their marginal PDFs and is given as

$$L(\mathbf{x_1}, \mathbf{x_2}, \cdots, \mathbf{x_g}; S, N) = (2\pi)^{-Mg} (\sqrt{\det(\mathbf{K})})^{-g}$$
$$\exp\left\{ -\frac{1}{2} \left[ \frac{\sum_{i=1}^{g} \mid x_{1,i} \mid^2}{[L\frac{S}{2} + \frac{N}{2} + \frac{S}{2}\Psi]} + \frac{\sum_{i=1}^{g} \sum_{m=2}^{M} \mid x_{m,i} \mid^2}{\frac{N}{2}} \right] \right\}. \tag{4.16}$$

Note that $\mid x_{m,i} \mid = \sqrt{x_{c_{m,i}}^2 + x_{s_{m,i}}^2}$. Here, the sub-indices $m$ and $i$ denote the receiver branch and time respectively, where $m$=$\{1, 2, \cdots, M\}$ and $i$=$\{1, 2, \cdots, g\}$.

The log-likelihood function is given as

$$\mathbf{\Lambda}_{\mathbf{x_{m,i}}}(x_{m,i}; S, N) = -Mg\log(2\pi) - g\log\sqrt{\det(\mathbf{K})} - \frac{1}{2}\left[\frac{\sum_{i=1}^{g} \mid x_{1,i} \mid^2}{\left[L\frac{S}{2} + \frac{N}{2} + \frac{S}{2}\Psi\right]} + \frac{\sum_{i=1}^{g}\sum_{m=2}^{M} \mid x_{m,i} \mid^2}{\frac{N}{2}}\right]. \quad (4.17)$$

We are interested in finding the individual maximum likelihood estimates of the signal power, $\hat{S}_{ML}$ and the noise power, $\hat{N}_{ML}$, because SNR expression is the ratio of these individual estimates

$$\hat{\gamma}_{DA} = \frac{\hat{S}_{ML}}{\hat{N}_{ML}}. \quad (4.18)$$

By differentiating (4.17) with respect to $S$ and $N$ individually and setting their derivatives equal to zero, we get the estimate of the signal power

$$\hat{S}_{ML} = \frac{(M-1)\sum_{i=1}^{g} \mid x_{1,i} \mid^2 - \sum_{i=1}^{g}\sum_{m=2}^{M} \mid x_{m,i} \mid^2}{g(M-1)(L+\Psi)}, \quad (4.19)$$

and the noise power

$$\hat{N}_{ML} = \frac{\sum_{i=1}^{g}\sum_{m=2}^{M} \mid x_{m,i} \mid^2}{g(M-1)}. \quad (4.20)$$

Now put the value of (4.19) and (4.20) in (4.18) to get the final expression of SNR given as

$$\hat{\gamma}_{DA} = \frac{(M-1)\sum_{i=1}^{g} \mid x_{1,i} \mid^2 - \sum_{i=1}^{g}\sum_{m=2}^{M} \mid x_{m,i} \mid^2}{(L+\Psi)\sum_{i=1}^{g}\sum_{m=2}^{M} \mid x_{m,i} \mid^2}. \quad (4.21)$$

### 4.3.2 Non-Data Aided MLE

In NDA estimation, we do not have any prior knowledge of the transmitted message symbol. Every individual symbol has equal chances to be received at the receiver. The conditional PDF of the received symbol given $f_n$ frequency was transmitted, i.e., 1 at the $n^{th}$ position of vector s is given as

$$p_{\mathbf{x_c},\mathbf{x_s}}(\mathbf{x_c},\mathbf{x_s}|s_n=1) = \frac{1}{M(2\pi)^M(\det\sqrt{\mathbf{K}})}$$
$$\exp\left\{-\frac{1}{2}\left[\frac{\mid x_n\mid^2}{\left[L\frac{S}{2}+\frac{N}{2}+\frac{S}{2}\Psi\right]^2}+\frac{\sum_{m=1,m\neq n}^{M}\mid x_m\mid^2}{\left[\frac{N}{2}\right]}\right]\right\}. \quad (4.22)$$

There are $M$ different possibilities of receiving the data in $M$ receiver branches. Hence by using the law of total probability, we can write the joint unconditional PDF of the received data is given as

$$p_{\mathbf{x_c},\mathbf{x_s}}(\mathbf{x_c},\mathbf{x_s}) = \frac{1}{M(2\pi)^M(\det\sqrt{\mathbf{K}})}\left[\exp\left\{-\left(\frac{\mid x_1\mid^2}{[S(L+\Psi)+N]^2}+\frac{\sum_{m=2}^{M}\mid x_m\mid^2}{N}\right)\right\}+\cdots\right.$$
$$\left.+\exp\left\{-\left(\frac{\mid x_M\mid^2}{[S(L+\Psi)+N]^2}+\frac{\sum_{m=1}^{M-1}\mid x_m\mid^2}{N}\right)\right\}\right]. \quad (4.23)$$

In order to simplify the above expression, we can factor out the common term $\exp\left\{\frac{-\sum_{m=1}^{M}|x_m|^2}{N}\right\}$ such that

$$p_{\mathbf{x_c},\mathbf{x_s}}(\mathbf{x_c},\mathbf{x_s}) = \frac{1}{M(2\pi)^M(\det\sqrt{\mathbf{K}})}$$
$$\exp\left\{\frac{-\sum_{m=1}^{M}\mid x_m\mid^2}{N}\right\}\sum_{m=1}^{M}\exp\left\{-\mid x_m\mid^2\Phi\right\}, \quad (4.24)$$

where $\Phi=\left[\frac{1}{S(L+\Psi)+N}-\frac{1}{N}\right]$. Now generalizing the joint PDF for a packet of $g$ independent symbols, we can write the likelihood function as

$$L\left(\mathbf{x_1},\mathbf{x_2},\cdots,\mathbf{x_g};S,N\right) = (M)^{-g}(2\pi)^{-Mg}\det(\mathbf{K})^{-g/2}$$
$$\prod_{i=1}^{g}\exp\left\{\frac{-\sum_{m=2}^{M}\mid x_{m,i}\mid^2}{N}\right\}\sum_{m=1}^{M}\exp\left\{-\mid x_{m,i}\mid^2\Phi\right\}, \quad (4.25)$$

and log-likelihood function becomes

$$\mathbf{\Lambda_{x_{m,i}}}(x_{m,i}; S, N) = -Mg\log(2\pi) - g\log\det(\sqrt{\mathbf{K}}) - \frac{\sum_{i=1}^{g}\sum_{m=2}^{M} \mid x_{m,i} \mid^2}{N} + \sum_{i=1}^{g} \log\left[\sum_{m=1}^{M} \exp\left\{- \mid x_{m,i} \mid^2 \Phi\right\}\right]. \quad (4.26)$$

Taking partial derivative of (4.26) with respect to $S$ and $N$ individually, setting these derivatives equal to zero and solving them simultaneously result in the following non-linear equations

$$\hat{S}(L + \Psi) + M\hat{N} = \frac{1}{g}\sum_{i=1}^{g}\sum_{m=1}^{M} \mid x_{m,i} \mid^2, \quad (4.27)$$

and

$$\sum_{i=1}^{g}\sum_{m=1}^{M} \mid x_{m,i} \mid^2 - g(M-1)\hat{N} = \frac{\sum_{i=1}^{g}\sum_{m=1}^{M} \mid x_{m,i} \mid^2 \exp(- \mid x_{m,i} \mid^2 \Phi)}{\sum_{m=1}^{M} \exp(- \mid x_{m,i} \mid^2 \Phi)}, \quad (4.28)$$

Finding a closed-form solution for the estimates of $S$ and $N$ is quite complex due to the presence of non-linear term, $\frac{\sum_{i=1}^{g}\sum_{m=1}^{M}|x_{m,i}|^2 \exp(-|x_{m,i}|^2\Phi)}{\sum_{m=1}^{M}\exp(-|x_{m,i}|^2\Phi)}$ in (4.28). In order to make the derivation simpler and to get a closed-form solution, we approximate this term for higher SNR values. Consider the expression for the case of $M = 2$, i.e., let

$$A = \sum_{i=1}^{g} \frac{\mid x_{1,i} \mid^2 \exp\left(- \mid x_{1,i} \mid^2\Phi\right) + \mid x_{2,i} \mid^2 \exp\left(- \mid x_{2,i} \mid^2\Phi\right)}{\exp\left(- \mid x_{1,i} \mid^2\Phi\right) + \exp\left(- \mid x_{2,i} \mid^2\Phi\right)}, \quad (4.29)$$

It can be observed that, if $S \gg N$, then $\Phi$ reduces from $\Phi = \left[\frac{1}{S(L+\Psi)+N} - \frac{1}{N}\right]$ to $\Phi \approx \left[-\frac{1}{N}\right]$. Hence, using this $\Phi$ in (4.29), we get the expression

$$A = \sum_{i=1}^{g}\left[\frac{\mid x_{1,i} \mid^2 \exp(\frac{|x_{1,i}|^2}{N}) + \mid x_{2,i} \mid^2 \exp(\frac{|x_{2,i}|^2}{N})}{\exp(\frac{|x_{1,i}|^2}{N}) + \exp(\frac{|x_{2,i}|^2}{N})}\right], \quad (4.30)$$

Rearranging the above equation, we get

$$A = \sum_{i=1}^{g} \left[ \frac{|\ x_{1,i}\ |^2}{1 + \frac{\exp\left(\frac{|x_{2,i}|^2}{N}\right)}{\exp\left(\frac{|x_{1,i}|^2}{N}\right)}} + \frac{|\ x_{2,i}\ |^2}{1 + \frac{\exp\left(\frac{|x_{1,i}|^2}{N}\right)}{\exp\left(\frac{|x_{2,i}|^2}{N}\right)}} \right]. \tag{4.31}$$

Now suppose, $f_1$ is transmitted, i.e., the data is transmitted for the first branch of receiver and the second branch contains noise. For higher SNR values, $S \gg N$, the expressions $1 + \frac{\exp\left(\frac{|x_{1,i}|^2}{N}\right)}{\exp\left(\frac{|x_{2,i}|^2}{N}\right)} \to \infty$ and $1 + \frac{\exp\left(\frac{|x_{2,i}|^2}{N}\right)}{\exp\left(\frac{|x_{1,i}|^2}{N}\right)} \to 1$. Hence, the non-linear term gets finally transformed into the following approximation

$$A \approx \sum_{i=1}^{g} \max_{m=1,\cdots,M} |\ x_{m,i}\ |^2 . \tag{4.32}$$

Using (4.32) in (4.28), we get the estimate for noise power, $\hat{N}$, that is

$$\hat{N} = \frac{\sum_{i=1}^{g} \sum_{m=1}^{M} |\ x_{m,i}\ |^2 - \sum_{i=1}^{g} \max_{m=1,\cdots,M} |\ x_{m,i}\ |^2}{g(M-1)} \tag{4.33}$$

Using (4.33) in (4.27) to get the estimate for signal power, $\hat{S}$. The final expression for non-data aided SNR, $\hat{\gamma}_{NDA}=\frac{\hat{S}}{\hat{N}}$ is given as

$$\hat{\gamma}_{NDA} = \frac{-\sum_{i=1}^{g} \sum_{m=1}^{M} |\ x_{m,i}\ |^2 + MA}{M(L+\Psi)\left[\sum_{i=1}^{g} \sum_{m=1}^{M} |\ x_{m,i}\ |^2 - A\right]}. \tag{4.34}$$

## 4.4 Cramer-RAO Lower Bound

In this section, we find the Cramer-Rao bound (CRB) for the performance evaluation of the derived estimators. We will derive the CRB for data aided case and judge its performance by comparing it with normalized mean squared error (NMSE) of the estimator. Here, we have two unknown parameters, i.e., the signal power, $S$ and the noise power, $N$. We can represent them in the form of an unknown parameter vector, i.e., $\theta = [S \quad N]^T$. Thus the CRB for vector parameter is given as [13]

$$CRB = \frac{\partial g(\theta)}{\partial \theta} \mathbf{I}^{-1}(\theta) \frac{\partial g(\theta)}{\partial \theta}^T, \tag{4.35}$$

where $g(\theta) = \frac{S}{N}$ is the function of unknown parameter $\theta$ and $\mathbf{I}$ is the Fisher information matrix. Taking partial derivative of the function $g(\theta)$ with respect to $\theta$, we get

$$\frac{\partial g(\theta)}{\partial \theta} = \begin{bmatrix} \frac{1}{N} & \frac{-S}{N^2} \end{bmatrix}^T, \tag{4.36}$$

The Fisher information matrix (FIM) is given as

$$\mathbf{I}(\theta) = \begin{bmatrix} -\mathbb{E}\left(\frac{\partial^2 \Lambda_{DA}}{\partial S^2}\right) & -\mathbb{E}\left(\frac{\partial^2 \Lambda_{DA}}{\partial SN}\right) \\ -\mathbb{E}\left(\frac{\partial^2 \Lambda_{DA}}{\partial NS}\right) & -\mathbb{E}\left(\frac{\partial^2 \Lambda_{DA}}{\partial N^2}\right) \end{bmatrix}, \tag{4.37}$$

where $\mathbb{E}$ is the expectation operator. Solving the above matrix, we have $\mathbf{I}(\theta)$, given as

$$\mathbf{I}(\theta) = \begin{bmatrix} \frac{g(L+\Psi^2)}{(S(L+\Psi)+N)^2} & \frac{g(L+\Psi)}{(S(L+\Psi)+N)^2} \\ \frac{g(L+\Psi)}{(S(L+\Psi)+N)^2} & \left(\frac{g}{(S(L+\Psi)+N)^2} + \frac{g(M-1)}{N^2}\right) \end{bmatrix}, \tag{4.38}$$

which results in CRB from (4.35) as

$$CRB_{DA} = \frac{M}{g(M-1)} \left[\gamma^2 + \frac{2\gamma}{\Psi} + \frac{1}{\Psi}\right]. \tag{4.39}$$

## 4.5 Simulation Results

In this section, the performance of the derived estimators has been presented in terms of normalized mean squared error (NMSE) values. Different trends for NMSE versus SNR have been analyzed on the basis of several system parameters, i.e., number of transmitting nodes, $L$, number of receiver branches, $M$ and different number of symbols, $g$. All the results shown in this section

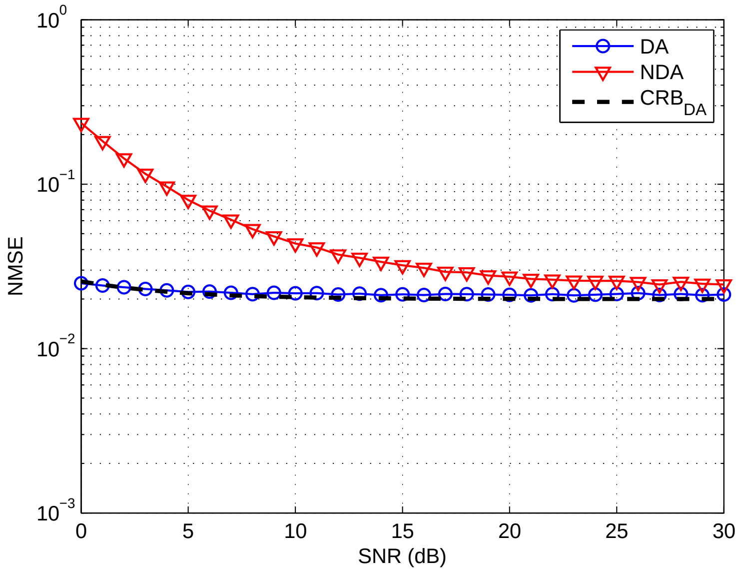


Figure 4.2: NMSE for $g$=100 symbols, $M$=2, $L$=5 and $\rho$=0.3

are averaged over 25000 trials of simulations.

Fig. 4.4 presents the NMSE versus SNR for the derived DA and NDA estimators for the case of $L$=5, $M$=2, $g$=100 symbol size and for constant correlation value between every channel, i.e., $\rho_{ij}$=0.3 $\forall i, j$. It can be observed from the figure that for lower SNR region, data aided (DA) estimator outperforms the non-data aided (NDA) estimator. This difference in NMSE is due to the approximation made in NDA estimator for the case of higher SNR values. Additionaly, DA estimator is showing the minimum possible variance as it gives exactly the same NMSE values throughout the whole SNR region as that of CRB. In Fig. 4.3, the effect of increasing the receiver branches, i.e., $M$ on NMSE value has been analyzed for DA estimator for $L$=2 transmitting nodes and correlation value of $\rho_{ij}$=0.3 $\forall i, j$. While keeping all the other parameters constant, it can be observed that NMSE is decreased

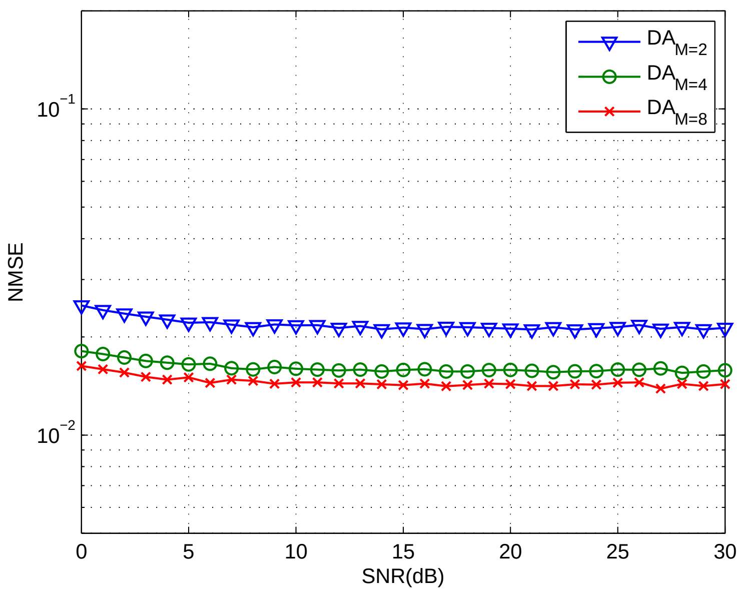


Figure 4.3: Effect of increasing $M$ on NMSE of Data aided estimator for g=100 symbols, $\rho$=0.3, $L$=2

throughout the whole SNR region as $M$ is increased. It is due to the fact that the estimated sample mean approaches the actual mean as we go on increasing the data samples. By increasing the receiver branches, $M$ and transmitting nodes, $L$, we are indirectly increasing the number of data samples, which result in decreased NMSE value. Same trend is followed by the NDA estimator. It can be observed from the Fig. 4.4 that the NMSE for DA estimator, is decreased by increasing the number of transmitting nodes, $L$. This improved performance of the estimator is again due to the reason of a larger data set. NMSE contours for DA estimator can be seen in the Fig. 4.5 for $\rho = 0.2$, M=4 and L=3. The effect of varying symbol sizes and SNR on the NMSE values has been analyzed in it. It can be observed that there is a decreasing trend in NMSE values as both the symbol size or SNR

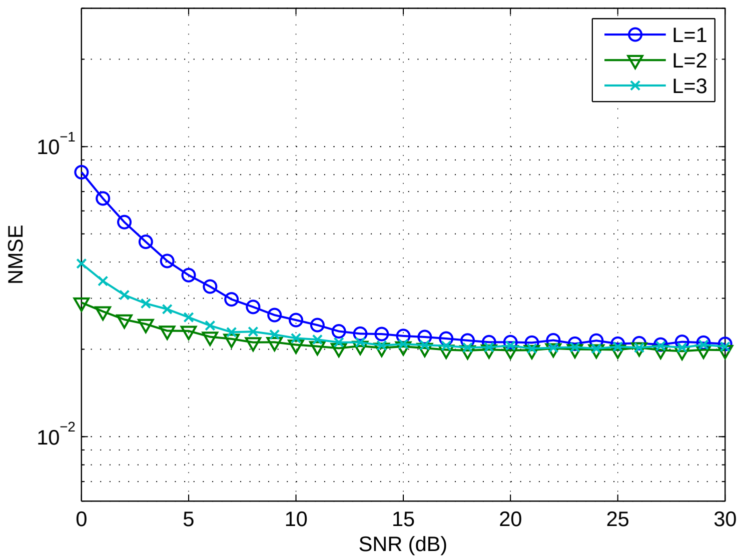


Figure 4.4: Effect of increasing $L$ on NMSE of data aided estimator for $g$=100 symbols, $\rho$=0.3, $M$=2

are increased. So, for such scenarios, where smaller NMSE value is needed, packets with larger number of symbols may be chosen and vice versa. For example, We can see from the figure that NMSE=0.003 can be achieved at the low SNR value of 5dB by choosing symbol size of 540, similarly, high SNR value of 20dB can be estimated with same NMSE by making g=470.

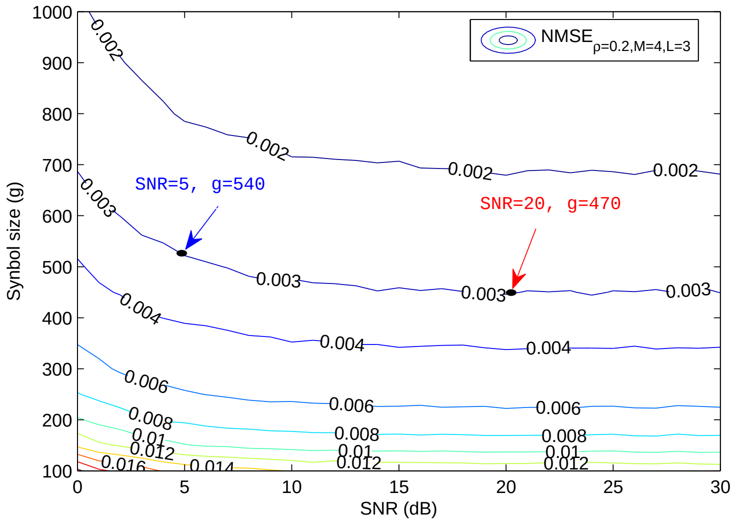


Figure 4.5: NMSE contours of Data aided estimator for $g$=100 to 1000 symbol size, $\rho$=0.2, $M$=4, $L$=3

# Chapter 5

# Conclusion and Future Work

## 5.1 Conclusion

The average SNR is estimated for

1. A single-input multiple-output (SIMO) system employing NCFSK modulation scheme and having single transmit antenna and multiple collocated receive antennas considering independent Rayleigh fading channels.

2. A multiple-input single-output (MISO) system employing NCFSK modulation scheme and having multiple transmitting sensor nodes and a single receiving node considering correlated Rayleigh fading channels.

Data aided (DA) and non-data aided (NDA) signal-to-noise ratio (SNR) estimator expressions have been derived for both the systems using maximum likelihood estimation (MLE) technique in a NC-MFSK receiver. Receiver diversity has been taken into account for the case of Rayleigh fading channel and AWGN. For comparison purposes, Cramer-Rao bound (CRB) for data aided case has been evaluated. On the basis of analysis done in the previous sections, we have found that by adding the diversity in the system, the performance of estimator is increased for both DA and NDA estimators. However, DA estimator performs best in all the cases as compared to NDA because of an approximation used in the NDA scheme to get a closed-form expression. Difference between the performances of both the estimators is large in low SNR regions. However, by increasing the diversity branches, this difference can be minimized. Moreover, the NDA estimator performs equally well for higher order diversity cases by approaching the performance of DA estimator in the higher SNR region.

## 5.2 Future Work

The SNR estimator designed in chapter 3 is for SIMO communication system, i.e., it is assumed that single sensor node is transmitting towards fusion centre having multiple collocated antennas. However, this work may further be extended for the generalized case of multiple-input multiple output (MIMO) communication system, i.e., multiple transmitting sensor nodes considering both independent and correlated Rayleigh fading channels. Secondly, the received data from multiple sensor nodes is correlated due to higher density in the network topology. The data transmitted from each individual node contributes towards increasing the power gain at the receiver node. In order to perfectly estimate the SNR value this correlation factor among the data must be known. We do not have any priori information about the degree of correlation among the received data. However, the estimators designed for MISO communication system assumes the known values of channel correlation however as an extension to this work, it is recommended that the channel correlation may be estimated as a nuisance parameter on the basis of received data as a future work and further this nuisance parameter may be incorporated to eventually estimate the Signal-to-noise ratio.

# Bibliography

[1] A. Kailas and M. A. Ingram, "Alternating opportunistic large arrays in broadcasting for network lifetime extension," *IEEE Trans. Wireless Commun.,* vol. 8, no. 6, pp. 2831-2835, Jun. 2009.

[2] R. I. Ansari and S. A. Hassan, "Opportunistic large array with limited participation: An energy-efficient cooperative multi-hop network," *IEEE International Conference on Computing, Networking and Communications (ICNC)*, Feb. 2014.

[3] C. Ellis, H. Miranda, F. Taiani, "Count on me: light weight Ad-Hoc broadcasting in heterogeneous topologies," *ACM, M-MPAC*, pp. 1-6, Nov. 2009.

[4] Niu, Ruixin, Biao Chen, and Pramod K. Varshney. "Fusion of decisions transmitted over Rayleigh fading channels in wireless sensor networks," *IEEE Trans, Signal Process.,* vol. 54, no. 3, pp. 1018-1027, Mar. 2006.

[5] M.K. Simon and M.S. Alouini, *Digital communications over Fading channels*: *A Unified Appraoch to Performance Analysis, Wiley and Sons.* New York, 2000

[6] D.R. Pauluzzi and N.C. Beaulieu, "A comparison of SNR estimation techniques for the AWGN channel," *IEEE Trans. Commun.,* vol. 48, no.10, pp. 1681-1691, Oct. 2000.

[7] A. Ramesh, A. Chockalingam and L.B. Milstein, "SNR estimation in Nakagami-m fading with diversity combining and its applications to turbo decoding," *IEEE Trans. Commun.,* vol. 50, no. 11, pp. 1719-1724, Nov. 2002.

[8] S. A. Hassan and M.A. Ingram, "SNR estimation of a non-coherent binary frequency shift keying receiver,"' *IEEE GLOBECOM*, Honolulu, Hawaii, Nov-Dec. 2009.

[9] S. A. Hassan and M. A. Ingram, "SNR estimation for M-ARY noncoherent frequency shift keying systems," *IEEE Trans. Commun.,* vol. 59, no. 10, pp. 2786-2795, Oct. 2011.

[10] S. A. Hassan, M. A. Ingram, "SNR estimation for non-coherent M-FSK receiver in a slow flat fading environment," *IEEE Intl. Conf. Commun. (ICC)*, Cape Town, South Africa, May. 2010.

[11] S. A. Hassan and M.A. Ingram, "SNR estimation in a non-coherent BFSK receiver with a carrier frequency offset," *IEEE Trans. Signal Process.,* vol. 59, no. 7, pp. 3481 - 3486, Jul. 2011.

[12] S. A. Hassan and M.A. Ingram, "Pilot assisted SNR estimation in a non-coherent M-FSK receiver with a carrier frequency offset,"' *IEEE Intl. Conf. Commun. (ICC)*, Ottawa, Canada, Jun. 2012.

[13] S.M. Kay,"*Fundamentals of Statistical Signal Processing: Estimation Theory*", *P*rentice Hall. New Jersey, 1993.

[14] S. A. Hassan and M.A. Ingram, "A quasi-stationary Markov Chain model of a cooperative multi-hop linear network," *IEEE Trans. Wireless Commun.*, vol. 10, no. 7, pp. 2306-2315, Jul. 2011

[15] S. A. Hassan and M. A. Ingram, "The benefit of co-locating groups of nodes in cooperative line networks," *IEEE Commun Letters.*, vol. 16, no. 2, pp. 234-237, Feb. 2012.

[16] M. Bacha and S. A. Hassan, "Performance analysis of cooperative linear networks subject to composite shadowing fading" *IEEE Trans. Wireless Commun.,* vol. 12, no. 11, pp. 5850-5858, Nov. 2013.

[17] A. Afzal and S. A. Hassan, "A stochastic geometry approach for outage analysis of ad-hoc SISO networks in Rayleigh fading," *IEEE GLOBECOM*, Atlanta, USA, Dec. 2013.

[18] S. A. Hassan, "Range extension using optimal node deployment in linear multi-hop cooperative networks," *IEEE Radio and Wireless Symposium (RWS)*, Jan. 2013.

[19] R. G. Gallager, "Circularly-symmetric Gaussian random vectors," Jun. 2009.

[20] A. Afzal and S. A. Hassan, "Stochastic Modeling of Cooperative Multi-Hop Strip Networks with Fixed Hop Boundaries," *IEEE Trans. Commun.,* vol. 13, no.8, pp. 4146-4155, Aug. 2014.

[21] M.C. Varun, O.B. Akan and I.F. Akyildiz, "Spatio-temporal correlation: theory and applications for wireless sensor networks," *Elsevier. Comput Networks.,* vol. 45, no.3, pp. 245-259, March. 2004.

[22] M.C. Varun, O.B. Akan and I.F. Akyildiz, "Spatial correlation-based collaborative medium access control in wireless sensor networks," *IEEE Trans. Network.,* vol. 14, no.2, pp. 316-329, Aug. 2006

[23] U. Javed and S. A. Hassan, "SNR Estimation in non-coherent M-FSK Receiver in Rice Fading Environment," *IEEE Communications Letters,* vol. 17, no. 9, pp. 1786-1789, Sept. 2013.

[24] D. R. Pauluzzi and N. C. Beaulieu, "A comparison of SNR estimation techniques for the AWGN channel," *IEEE Trans. Commun.,* vol. 48, no.10, pp. 16811691, Oct. 2000.

[25] Y. Chen and N. C. Beaulieu, "An approximate maximum likelihood estimator for SNR jointly using pilot and data symbols," *IEEE Lett.Commun.,* vol. 9, no. 6, pp. 517519, Jun. 2005.

[26] A. Ramesh, A. Chockalingam, and L. B. Milstein, "SNR estimation in generalized fading channels and its applications to turbo decoding," *Proc. IEEE Intl.Conf. Commun. (ICC),* pp. 10941098, Jun. 2001.

[27] T. Ertas and E. Dilaveroglu, "Low SNR asymptote of CRB on SNR estimates for BPSK in Nakagami-m fading channels with diversity combining," *IEEE Electron. Lett.,* vol. 39, no. 23, pp. 16801682, Nov.2003, DOI: 10.1049/el:20031057.

[28] E. Dilaveroglu and T. Ertas, "CRBs and MLEs for SNR estimation on non coherent BFSK signals in Rayleigh fading," *IEEE Elect. Lett.,* vol.41, No. 2, January 2005

[29] A. Scaglione and Y.W. Hong, "Opportunistic large arrays: Cooperative transmission in wireless multi-hop ad-hoc networks to reach far distances," *IEEE Trans. Signal Process.,* vol. 51, no. 8, Aug. 2003.

[30] M. Bacha and S. A. Hassan, "Coverage Aspects of Cooperative Multi-hop Line Networks in Composite Fading Environment", *IEEE Intl Wireless Commun and Mob Computing Conf. (IWCMC),* Aug, 2014.

[31] I.F. Akyildiz, W. Su, Y. Sankarasubramaniam and E. Cayirci, "Wireless sensor networks: a survey," *Computer networks.,* vol.38,no. 4, pp. 393-422, 2002.