

# Performance Analysis of an Opportunistic Large Array Cooperative Multi-hop Network with Limited Node Participation



By

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# Approval

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# Abstract

This thesis studies an energy-efficient scheme for cooperative multi-hop communications in a finite density opportunistic large array (OLA) network. In a cooperative OLA network, a group of nodes transmits the same message signal to another group of nodes, providing range extension and increased reliability by exploiting the spatial diversity in a wireless system. However, in this thesis, it is shown that a particular coverage and reliability of the network can be achieved by limiting the node participation in an OLA network, thereby providing energy-efficiency. Two types of networks are studied; a one-dimensional linear network and a two-dimensional strip network where the nodes are aligned on a regular grid. The transmissions originating from one level to another are modeled as a Markov process and the underlying transition probability matrix has been derived. By invoking the Perron-Frobenius theorem, the coverage, reliability, and energy-efficiency of the network has been quantified for a given end-to-end success probability constraint and a given signal-to-noise ratio (SNR) margin.

# Certificate of Originality

I hereby declare that this submission is my own work and to the best of my knowledge it contains no materials previously published or written by another person, nor material which to a substantial extent has been accepted for the award of any degree or diploma at NUST SEECS or at any other educational institute, except where due acknowledgement has been made in the thesis. Any contribution made to the research by others, with whom I have worked at NUST SEECS or elsewhere, is explicitly acknowledged in the thesis.

I also declare that the intellectual content of this thesis is the product of my own work, except for the assistance from others in the project's design and conception or in style, presentation and linguistics which has been acknowledged.

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# Chapter 1

## Introduction

### 1.1 Introduction to Cooperative Communication

Cooperative communication, as its name signifies, is a communication in which two or more transmitting nodes participate to help the source message in reaching its destination. The spatial diversity of the network is utilized to create redundancy and reduce errors in transmissions [1]. While increasing the reliability of the communication, cooperative communication helps in extending the range of the network [2]. The range extension may however depend on the network parameters. The individual nodes usually have same transmission capabilities but the load can be shared between individual nodes which impacts the overall power consumed for transmission.

Cooperative transmission can be explained by assuming a simplest scenario of three nodes. If the individual nodes are numbered 1, 2 and 3 respectively as shown in Fig. 1.1 then cooperation by node 2 could ensure successful

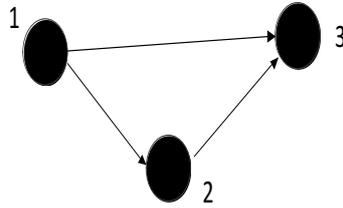


Figure 1.1: Simple cooperative communication

transmission of packet from node 1 to node 3. A direct transmission between node 1 and node 3 could also take place but the fading characteristics could hinder successful transmission or the power consumed for transmission could be higher. The success of transmission may however differ from case to case [3]. Cooperative transmission is also identified as relay transmission as the nodes in the network relay the packet to other nodes. An analysis of the network could help in identifying the range of the network and the network's transmission capabilities.

As the data rates required for communication increase with the advent of third generation (3G) and fourth generation (4G) cellular systems, reliable and spectrum efficient [23] networks are required. Cooperative communication could be used in applications like live video streaming and mobile multimedia as they can provide the required data rates and reliability. Future holds great prospects for further research in cooperative communication as a suitable technique which would be compatible with high data rate applications of 3G and 4G. The mobile multimedia applications could benefit from spatial diversity of cooperative networks for successful end-to-end transmission.

Cooperative communication in cellular networks has been a potential area

of research and techniques like distributed antennas systems (DAS), multi-cell coordination, Group Cell, Coordinated Multiple Point transmission and reception (CoMP) [4] were introduced. Basic cellular systems addressed the issue of capacity and gave a solution to utilizing the spectrum efficiently. As the data rates keep on increasing it is difficult to reach for the desired distances as attenuation characteristics also increase. Cooperative communication could help in overcoming such issues in basic cellular networks.

The data coding techniques used in cooperative transmission and mode of transmission (half or full duplex)[22] are emerging areas of research in cooperative communication. Researchers have shown optimism in utilizing the spatial diversity offered by cooperative communication to design different networks for achieving successful transmission. Cooperative communication helps in replacing a single source to destination link with smaller links which may be regarded as multi-hop network. The decode-and-forward (DF) mechanism is employed in a multi-hop network to achieve successful transmission. The decoding criteria could vary according to the network parameters and the requirement of the user.

A technique called Opportunistic Large Array (OLA) [5] was introduced, which works on the principle of DF while exploiting the spatial diversity of the network. OLA and its variants like opportunistic large array with transmission threshold OLA-T provide redundancy and range extension. In OLA-T the researchers paid special attention to the need of energy-efficiency in cooperative networks. OLA-T worked on the SNR estimation by the individual radios [31] [32] [33]. The radios which are in a particular SNR range and are able to decode, forward the decoded message.

Conservation of energy is important in sensor networks which utilize batteries as a primary power source. The batteries are drained out quickly which is neither feasible nor desirable. In order to overcome the issue energy-efficient techniques for communication were introduced. Research on cooperative networks was carried out with special emphasis on conserving the energy. Researchers have analyzed the utility of these networks in achieving desired transmissions without compromising the quality of service (QoS). The QoS is a variable which differs in each network. In the case of OLA, the end to end success of transmission can be regarded as the desired QoS. This technique is also applicable in networks other than the basic cellular networks.

Smart grid communication is one of the candidates which can work on OLA and its variant techniques. Conserving the energy during communication in smart grids is a potential area of research. In order to increase the life time of batteries for smooth functioning of the network, an energy efficient network is required. A network which helps in conserving the transmit energy of the radios might be called the ideal candidate for such communication. Cooperative communication can help in conserving energy as well as providing redundancy to cater the noise and fading affects. Structural health monitoring is also one of the applications which can employ cooperative communication. Sensors to monitor structures are usually placed at considerable heights. Replacing the battery of the sensors every now and then is not feasible. An energy-efficient technique could ensure that the batteries don't drain out quickly to avoid any inconvenience. Patient monitoring sensor networks require a reliable and low power sensor deployment. OLA

and its variants are suitable candidates for such networks. Viewing the sensitivity of these networks, OLA and its variants could provide the reliability and energy-efficient solution.

# Chapter 2

## Literature Review

### 2.1 Opportunistic Large Array

In this chapter we would go through some related literature touching the concept of cooperative communication and OLA networks. We would try to explore different aspects of cooperative communication, the previous research and future prospects. Firstly, we analyze some related work on OLA networks. Opportunistic Large Array networks provide the spatial diversity essential for achieving successful transmission for long distances. Different techniques to ensure long distance transmissions were introduced which included using regenerative repeaters but OLA provided a different dimension to cooperative communication. Previously, single relay mechanisms were introduced, but a new concept of multiple relays in OLA was introduced in [5]. The network design included a sending node also regarded as the leading node which transmits the packet to all the other nodes in its proximity. As the OLA network didn't involve addressing of individual nodes, scalability of the network became easier. The transmission in OLA progressed due to the

DF mechanism. The transmission proceeds like an OLA (also regarded as the Mexican Wave) and the aggregate of the packets received at the destination ensures success of transmission.

A decoding criteria based on the received signal-to-noise-ratio (SNR) helped in catering the effects of noise in propagation. The concept of OLA helped in eliminating the need for a routing mechanism and multiple access techniques. The assumptions in transmission included synchronous transmission at discrete time intervals and equal transmit capabilities of each node [6]. Flooding algorithms introduced previously depended on virtual paths which were preset at the MAC [22] and physical layer [11], for transmission towards the destination. OLA however worked on the physical layer and it didn't require the help of higher layers in determining the success of transmission which in turn increased the connectivity of the network. The results indicated that the OLA could be the best method for transmission in a network with random deployment of nodes. Exploiting the diversity introduced by the random deployment of network, OLA could provide range extension with little complexity.

Wireless sensor networks operating on batteries require an energy-efficient [21] technique so that the batteries don't drain quickly. Opportunistic Large Array with threshold (OLA-T) analyzed for a strip shaped continuum node network, provided a new insight into the working of OLA network on the basis of energy-efficiency. Limiting the participation of nodes, the authors in [7] found out the criteria for conserving the energy during transmission. The analysis depended on a transmission threshold  $\tau$ , explaining the bounds on its value for end-to-end transmission. An SNR threshold [31] [32] value

range identifies the nodes that are used for transmission to the next level.

In OLA-T the trailing section of nodes from the transmission window were selected as they are closer to the next level. The threshold range was chosen so as to not to compromise the success of the transmission. A received SNR greater than a lower transmission threshold  $\tau_l$  signified successful decoding. A upper transmission threshold  $\tau_u$  ensured that only the nodes having received SNR less than  $\tau_u$ , transmit [7]. Fraction of energy saved was calculated to identify the energy consumption by OLA-T as compared to OLA. The energy saved depended on the node density of the network which was highlighted in the results presented by the authors. An idea of an infinite network broadcast was introduced. The authors explained that the range of SNR thresholds helped in determining whether a sustainable transmission would be achieved or not [8]. The results were however analyzed for single path loss exponent and the assumptions included half-duplex transmission.

Along with OLA-T, a upstream routing mechanism called OLA Concentric routing algorithm (OLACRA) was analyzed in [9]. The authors defined a *sink* as the initiator of the downstream broadcast. The *sink* transmits at a particular central frequency to the downlink level 1 which consists of the DF nodes. Now, the nodes in level 1 transmit to next level at some other central frequency. This is how donwstream levels are formed and the transmission proceeds in the form of concentric rings. Upstream transmission takes place in a similar manner. The authors analyzed a performance metric called packet delivery ratio (PDR) for OLACRA. The authors observed that mobility of nodes in a level had little effect on success of transmission [25]. Other observations included low latency in transmission while employing OLACRA

algorithm.

Other diversity routing algorithms included *Proteus* in multi input single output (MISO) system [3]. Authors introduced *Proteus* as a routing mechanism in which the packets contained the information about the cluster size and the routing mechanism alongwith the addresses of the nodes in close proximity. The algorithm worked on acknowledgement by the destination for a route formation. If during the transmission connection failure occurred than re allocation of route was initiated. The authors concluded by presenting results on the success of algorithm in increasing the throughput however, the optimal performance depended on the cluster size of the network. Along with the cluster size [10] the performance metric included the overall grid size.

Alternating OLA (A-OLA) introduced a new concept of optimal energy consumption in OLA networks [12]. The authors identified that the trailing nodes used for transmission in OLA-T for continuum [26] node density strip networks could lose their battery power due to their overuse in transmissions. As the same transmission window was used repeatedly for transmission, the drainage of trailing nodes was inevitable. The authors named this affect as “*network hole*” and asserted that A-OLA could overcome the issue.

The basic concept behind A-OLA was that different transmission window sets were introduced on the basis of threshold. Each time a transmission took place, a different set of transmission window was used so as to equally distribute the burden of transmission among the nodes. The aforementioned concept of using different sets for transmission had been analyzed for non-cooperative networks but the authors provided an interesting insight of this

concept in cooperative networks. A *deterministic model* along with orthogonal transmission was assumed in their analysis. The authors discussed the 2-set A-OLA, moving on to a general m-set A-OLA [12]. The authors concluded by comparing the performance of A-OLA with basic OLA. Their results showed a considerable extension in network life due to energy conservation. However, their results were based on infinite node density networks while they left the practical finite node density networks for future research.

The concept of infinite network broadcast was limited to the infinite node density cases. The authors in [13] modeled the OLA network for finite node density one-dimensional grid by a discrete time Markov chain. The assumptions included equal transmit power of individual nodes with the nodes placed at equal distances. The authors observed that infinite broadcast was not possible in a finite node density case as the probability of going into an absorbing state was certain. They formulated a transition probability matrix and computed a one hop success probability using Perron Frobenius eigen theorem. The analysis was however limited to a network with fixed OLA windows. Analysis for overlapping windows was carried out in [14]. The authors discussed that the nodes which had participated in transmission in the previous level did not participate in transmission from the next level. The proposed system was modeled by Markov chain and the results presented showed the variation of one-hop success probability with the transmission window size.

All of the previous work on OLA was limited to single-relay network. Authors carried the idea of OLA forward to double-relay networks, which were said to be more robust in terms of the errors and the channel capacity [15]. The authors observed that in single-relay half duplex networks the

spatial diversity was not utilized optimally. They presented the idea of using double-relay networks for optimal use of the channel capacity. Transmit power adjustment was also considered as a performance metric, as it increased or decreased the noise along with the signal [16]. Their results signified that as the SNR increased the dual-relay networks provided highest channel capacity.

# Chapter 3

## System Model

In this chapter we would try to explain the basic system model for the proposed algorithm. System model would be divided into two sections. Section 3.1 would explain the system model of basic OLA and different parameters related to it for one dimensional grid network. We would than compare it with the proposed algorithm. In section 3.2 we would try to develop understanding of the two dimensional grid network deployment and explain how the transmissions proceed with time.

### 3.1 One-Dimensional Grid Network

Our aim is to analyze the deployment of OLA network for one-dimensional grid network with finite density. We wish to determine the relation of the success of transmission with the energy-efficiency of the network. We consider a linear network as shown in Fig. 3.1(a). The adjacent nodes are separated by a fixed distance  $d$ . The network is divided into ‘levels’ where the window size  $M$  denotes the number of nodes in each level. The  $M$  nodes in a particular

level transmit the same message to the next level. The transmit power of the nodes is assumed to be equal. A node is considered to be successful in decoding a message if after post detection combining the received SNR is above a preset threshold  $\tau$ . The transmissions proceed in a DF mechanism. The nodes highlighted with black in Fig. 3.1(a) are the DF nodes at a particular time instant e.g.  $r + 1$ . The window size is  $M = 4$  and the transmission proceeds in a multi-hop manner.

In order to compare and analyze the basic OLA with our proposed algorithm we consider two modes of transmission: basic OLA (B-OLA) and OLA with limited participation (OLA-LP). We have named our algorithm OLA-LP, as we would later show that limiting the node participation could help in achieving the energy-efficient transmission. While operating in B-OLA mode it is pertinent to note that transmission takes place through hops where each hop is of size  $M$ . All the nodes in a hop which have decoded the packet forward it to the next hop and so on. In OLA-LP we select the trailing  $N$  nodes to forward the packet where  $N \leq M$ . In this manner the transmit energy of  $M - N$  nodes is saved at each level. This participation depends on the notion that nodes at the starting edge of the window  $M$  do not provide substantial diversity gain [24] because of the large path loss exponent [31] present between them and the next level nodes. The selection may however cause some degradation in system performance but we would later show that the degradation is tolerable as long as the end-to-end transmission is a success.

Let  $\mathbb{N}_r$  represent the indices of the DF nodes in B-OLA at time instant  $r$  e.g.  $\mathbb{N}_r = \{2, 4\}$  and  $\mathbb{N}_{r+1} = \{1, 3\}$  as in Fig. 3.1(a). Then we define a set  $\mathbb{R}_r \subseteq$

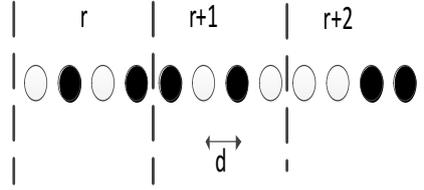
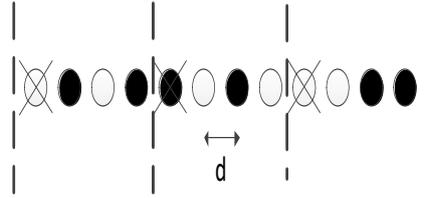
(a)  $M = 4$ (b)  $M = 4, N =$ 

Figure 3.1: 1D network layout

$\mathbb{N}_r$ , which indicates the indices of the DF nodes in OLA-LP. For example,  $\mathbb{R}_r = \{2, 4\}$  and  $\mathbb{R}_{r+1} = \{3\}$  as shown in Fig. 3.1(b). In OLA-LP, the received power at the power received at the  $j$ th node of the next level, e.g.,  $r + 1$  is given by

$$Pr_j(r + 1) = \frac{P_t}{d^\beta} \sum_{m \in \mathbb{R}_r} \frac{\mu_{mj}}{(M - m + j)^\beta}, \quad (3.1)$$

where  $P_t$  represents the transmit power of the nodes. The flat fading Rayleigh channel gain from the  $m$ th node in level  $r$  to the  $j$ th node in level  $(r + 1)$  is given by  $\mu_{mj}$ . The elements of  $\mu_{mj}$  are drawn from an exponential distribution with unit mean.  $\beta$  represents the path loss exponent, where its normal range varies between 2-4. All the nodes in a level are assumed to be transmitting in synchronization [23].

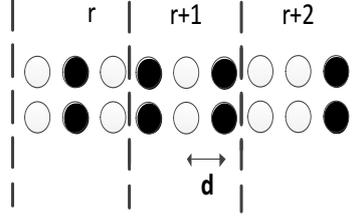
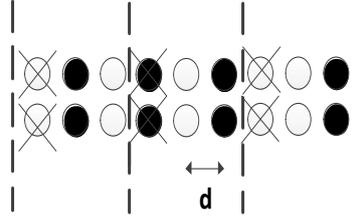
(a)  $M = 6$ (b)  $M = 6, \tilde{N} = 2$ 

Figure 3.2: 2D network layout

### 3.2 Two-Dimensional Grid Network

The 2D strip network is shown in Fig. 3.2(a).  $M = L \times W$  represents the window size where  $L$  is the length in horizontal direction and  $W$  represents the width of a level. As shown in figure, the nodes are placed at a fixed distance  $d$  in both dimensions. The transmissions proceed in the same DF mechanism as in one-dimensional case while the expression for the received power is given by

$$Pr_j(r+1) = \frac{P_t}{d^\beta} \sum_{m \in \mathbb{R}_r} \frac{\mu_{mj}}{(\sqrt{\delta_{mj}})^\beta}, \quad (3.2)$$

where all the parameters are defined as the same way as in (3.1). The Euclidian distance between the nodes in different levels is calculated differently using the geometrical techniques like Pythagoras theorem. We formulate a mathematical expression for computing the whole Euclidian distance matrix.  $\sqrt{\delta_{mj}}$  represents the Euclidian distance between a part of nodes  $m$  and

$j$  where  $\sqrt{\delta_{mj}}$ ;  $\delta_{mj} \in \Delta_{[M,M]}$  such that

$$\Delta = (\mathbf{A} \otimes \mathbf{G}_1) + (\mathbf{G}_2 \otimes \mathbf{B}), \quad (3.3)$$

where  $\otimes$  denotes the kronecker product [17]. The matrices  $\mathbf{A}$  and  $\mathbf{B}$  are given as

$$\mathbf{A} = \begin{bmatrix} L^2 & (L+1)^2 & \cdot & \cdot & (L+(L-1))^2 \\ (L-1)^2 & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ (L-(L-1))^2 & \cdot & \cdot & \cdot & L^2 \end{bmatrix}$$

$$\text{and } \mathbf{B} = \begin{bmatrix} 0 & 1^2 & 2^2 & \cdot & \cdot & (W-1)^2 \\ 1^2 & \cdot & \cdot & \cdot & \cdot & \cdot \\ 2^2 & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ (W-1)^2 & \cdot & \cdot & \cdot & \cdot & 0 \end{bmatrix}$$

where  $\mathbf{G}_1 = \mathbf{1}_{[L \times L]}$  and  $\mathbf{G}_2 = \mathbf{1}_{[W \times W]}$  are the matrices of all ones of dimensions  $(L \times L)$  and  $(W \times W)$ , respectively.

For the ease of analysis, the nodes are numbered from top to bottom and from left to right. Fig. 3.2(a) shows the basic OLA with the DF nodes while Fig. 3.2(b) shows OLA-LP. The selection criteria is a bit different than the one-dimensional case.  $\tilde{N}$  represents the trailing columns e.g. from Fig 3.2(b)  $\tilde{N} = 2$ . The transmissions proceed in a similar multi-hop manner as in one-dimensional network.

# Chapter 4

## Modeling by Markov Chain

### 4.1 Modeling of One-Dimensional Grid Network

The one-dimensional strip network can be modeled by using a Markov Chain. As the transmissions proceed from one level to another the Markov chain helps in understanding the behavior of the network. A binary indicator random variable (RV),  $\mathbb{I}_j(r)$  is used to define the state of a node  $j$  at time instant  $r$  where  $\mathbb{I}_j(r)$  takes value 0 (node  $j$  could not decode) or 1 (node  $j$  decodes). We begin by defining the state of the network for B-OLA.  $\mathcal{X}_1(r) = [\mathbb{I}_1(r), \mathbb{I}_2(r), \dots, \mathbb{I}_M(r)]$  represents the state of the network at time instant  $r$ , by an  $M$ -bit binary word. From Fig. 3.1(a),  $\mathcal{X}_1(r) = \{0101\}$  and  $\mathcal{X}_1(r+1) = \{1010\}$ . As the state of the system at a particular time instant depends upon the previous state, we identify  $\mathcal{X}_1$  as a discrete-time Markov process. The total number of states of a level of window size  $M$  is  $2^M$ .

In all the previous research on the networks based on continuum node

assumption [5] [6] [7], the transmissions also proceeded infinitely. However, the authors in [13] showed that the transmissions would eventually die down due to killing state. We define *all zeros* as the absorbing state, after which the transmissions stop propagating. Hence the Markov chain,  $\mathcal{X}_1$ , is defined by the union of two mutually exclusive sets, i.e.,  $\mathcal{X}_1 \in \{0\} \cup S_1$  where 0 is the absorbing state and  $S_1$  is the transient state space such that  $S_1 = \{1, 2, \dots, 2^M - 1\}$ , where the set  $S_1$  is the decimal equivalent of the binary words formed from the indicator functions. For instance, if  $i$  is a state such that  $i \in S_1$ , and if  $i = \{1010\}$ , then  $i = 10$  in decimal. The probability of going into an absorbing state is always non-zero and increases asymptotically as

$$\lim_{r \rightarrow \infty} \mathbb{P}\{\mathcal{X}_1(r) = 0\} \nearrow 1. \quad (4.1)$$

In OLA-LP the Markov chain is defined in a slightly different manner because of the selection of trailing nodes done for transmission.  $\mathcal{X}_2(r) = [\mathbb{I}_g(r), \mathbb{I}_{g+1}(r), \dots, \mathbb{I}_M(r)]$  now defines the Markov chain where  $g = M - N + 1$ , i.e.,  $M - N$  most significant bits are removed. The outcomes of  $\mathcal{X}_2(r)$  can be represented in binary form with  $2^N$  states with a transient state space  $S_2 = \{1, 2, \dots, 2^N - 1\}$ . The transitions to/from 0th state are not included, resulting in a  $(2^N - 1) \times (2^N - 1)$  dimensional probability matrix  $\mathbf{P}$ .

The probability matrix  $\mathbf{P}$  holds the information which can tell us the state of the network at a particular instant and the hops covered before the killing state occurs. The Perron-Frobenius theorem of non-negative matrices [18] could be used to find a maximum eigenvalue  $\rho$  and a corresponding left eigenvector  $\mathbf{u}$ . The transition matrix  $\mathbf{P}$  is right sub-stochastic due to elimination of  $(M - N)$  nodes because of which the value of Perron-Frobenius eigenvalue  $\rho$  is

less than 1. The  $\rho$ -invariant distribution is defined by  $\mathbf{u} = (u_i, i \in S_2)$  where  $\mathbf{u}$  is the left eigenvector of  $\mathbf{P}$ . As we are certain that  $\forall r, \mathbb{P}\{\mathcal{X}_2(r) = 0\} > 0$ ,  $\mathcal{X}_2(r)$  becomes a Markov chain having a quasi-stationary distribution [19]. The quasi-stationary distribution represents the state of the system just before the killing occurs. Hence, the probability of being in state  $j$  is given as

$$\mathbb{P}\{\mathcal{X}_2(r) = j\} = \rho^r u_j, \quad j \in S_2, \quad r \geq 0. \quad (4.2)$$

The probability that a node  $m$  of level  $(r + 1)$  is able to decode is given by

$$\mathbb{P}\{\mathbb{I}_m(r) = 1\} = \mathbb{P}\{Pr_m(r) > \tau\}, \quad (4.3)$$

where  $\mathbb{P}\{Pr_m(r) > \tau\}$  is given by

$$\mathbb{P}\{Pr_m(r) \geq \tau\} = \int_{\tau}^{\infty} f_{Pr_m}(y) dy. \quad (4.4)$$

Now we define the probability distribution function (PDF) of the received power at  $m$ th node as  $f_{Pr_m}(y)$ . The total power received at a node is the sum of the finite powers from the nodes in the previous level. The power transmitted by each node is exponentially distributed so the sum of  $K$  exponential random variable distinct parameters  $\lambda_k$ , where  $k = 1, 2, \dots, K$ , forms hypo exponential distribution [20], which is given as

$$f_{Pr_m}(y) = \sum_{k=1}^K C_k \lambda_k^{(m)} \exp(-\lambda_k^{(m)} y), \quad (4.5)$$

where

$$C_k = \prod_{\varsigma \neq k} \frac{\lambda_{\varsigma}^{(m)}}{\lambda_{\varsigma}^{(m)} - \lambda_k^{(m)}}, \quad (4.6)$$

and  $\lambda_k^{(m)}$  is defined as

$$\lambda_k^{(m)} = \frac{\Delta^{\beta} d^{\beta}}{P_t}. \quad (4.7)$$

In Eq. (4.7),  $\Delta$  is given as  $(M - k + m)$ . Moving on, we define the success probability at  $m$ th node as

$$\mathbb{P}\{Pr_m(r) > \tau\} = \sum_{k=1}^N C_k \exp(-\lambda_k^{(m)} \tau) \mathbb{I}_k(r-1). \quad (4.8)$$

Eq. (4.8) provides the success probability of one node in a level. For all nodes in a level  $j$ , the one-hop probability for going from state  $i$  to  $j$  is given as

$$P_{ij} = \prod_{k \in \mathbb{R}_{r+1}^{(j)}} \left\{ \sum_{m \in \mathbb{R}_r^{(i)}} C_m \exp(-\lambda_m^{(k)} \tau) \right\} \prod_{k \in \bar{\mathbb{R}}_{r+1}^{(j)}} \left\{ 1 - \sum_{m \in \mathbb{R}_r^{(i)}} C_m \exp(-\lambda_m^{(k)} \tau) \right\}, \quad (4.9)$$

where the set  $\mathbb{R}_r^{(i)}$  represents the indices of DF nodes of  $i$ th state at time instant  $r$  and  $\bar{\mathbb{R}}_r^{(i)} = \{g, g+1, \dots, M\} \setminus \mathbb{R}_r^{(i)}$ , i.e., the indices of the nodes which are 0 at time instant  $r$ . Eq. (12) is just a single entry of the transition matrix  $\mathbf{P}$ . Similar entries can be calculated for all  $i, j \in S_2$ .

## 4.2 Modeling of Two-Dimensional Grid Network

The Markov chain modeling for two-dimensional strip network is almost the same as that of one-dimensional case. As already discussed, the nodes are numbered from top to bottom and from left to right. Hence the state of the system at time instant  $r$  is given as

$$\tilde{\mathcal{Y}}_1(r) = \begin{bmatrix} \mathbb{I}_1(r) & \mathbb{I}_{W+1}(r) & \cdot & \cdot & \cdot & \mathbb{I}_{(L-1)W+1}(r) \\ \mathbb{I}_2(r) & \cdot & & & & \cdot \\ \cdot & \cdot & \cdot & & & \cdot \\ \cdot & \cdot & & \cdot & & \cdot \\ \cdot & \cdot & & & \cdot & \cdot \\ \mathbb{I}_W(r) & \mathbb{I}_{2W}(r) & \cdot & \cdot & \cdot & \mathbb{I}_M(r) \end{bmatrix}.$$

For example, for Fig. 4.2(a),  $\tilde{\mathcal{Y}}_1(r) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ .

We define

$$\mathcal{Y}_1(r) = \{vec[\tilde{\mathcal{Y}}_1(r)]\}^T, \quad (4.10)$$

where  $vec$  is the vector operation and stacks all the columns of  $\tilde{\mathcal{Y}}_1$  into a single column and  $T$  denotes the transpose operation. Using (4.10), the definition of 2D Markov chain is now expressed as 1D case and the state space remains the same as that of 1D, i.e., an absorbing state in addition to  $2^M - 1$  transient states. For OLA-LP, we define  $\tilde{N}$  as the participating columns of  $\tilde{\mathcal{Y}}_1(r)$ . The selection of  $\tilde{N}$  nodes for 2D case refers to the selection of last  $\tilde{N}$  columns of window  $M = L \times W$ . For Fig. 4.2(b), the first column of nodes is restricted to take part in transmission and last  $\tilde{N} = 2$  are now the candidate cooperators.

# Chapter 5

## Results and Analysis

In this chapter we present the results of the simulations and the model. We discuss the system performance with respect to various parameters. Firstly, we try to ascertain the validity of our model by comparing the simulation results and the results of the model. In the numerical simulations setup, the received power at the next level is calculated while assuming a random initial state of nodes in the first level. As discussed before, the indicator functions of nodes constitute a state. The threshold criteria determines the indicator functions. The nodes which have decoded transmit to the next level and so on. We have calculated the one-hop success probability using the proposed model, but for comparison, a value is computed through simulations. The simulations are repeated for 100,000 trials and the results are averaged out to compute an average value of one-hop success probability. The simulation results are then compared with the Perron-Frobenius eigenvalue  $\rho$  found by (4.9).

The algorithm for calculating the one-hop success probability is as follows:

Table 5.1: MSE between theoretical model and simulation

MSE	N=2	N=3	N=4	N=5	N=6	N=7	N=8
$\gamma=12$ dB	1.53e-4	8.10e-5	1.39e-4	6.40e-7	9.82e-6	3.02e-7	1.04e-8
$\gamma=14$ dB	1.50e-4	5.62e-5	1.23e-4	1.60e-7	1.75e-4	2.50e-7	3.16e-6
$\gamma=16$ dB	3.24e-6	9.61e-6	1.60e-5	4.90e-7	8.41e-6	1.60e-7	9.00e-8

1. Initialize the number of hops as  $\text{hop}=1$ .
2. A random initial state at level  $r$  is assumed.
3. The received power is calculated at the next level e.g.  $r + \text{hop}$ .
4. The threshold criteria sets the indicator function which in turn constitutes a state.
5. The state count at level  $r + \text{hop}$  is saved in vector  $V_{\text{hop}}$ .
6. The process is continued for 100,000 trials.
7. The steps 2 to 6 are continued for  $\text{hop}+1$  and the values are saved in  $V_{\text{hop}+1}$
8. As a quasi stationary distribution is assumed so  $V_{\text{hop}+1} = \rho V_{\text{hop}}$ .
9.  $\frac{V_{\text{hop}+1}}{V_{\text{hop}}}$  gives us the required  $\rho$ .

Before proceeding further, we define a normalized parameter

$$\gamma = \frac{P_t}{d^{\beta}\tau}, \quad (5.1)$$

which is the received SNR at a node  $d$  distance away from its transmitter. We call it as SNR margin. The mean squared error (MSE) between the

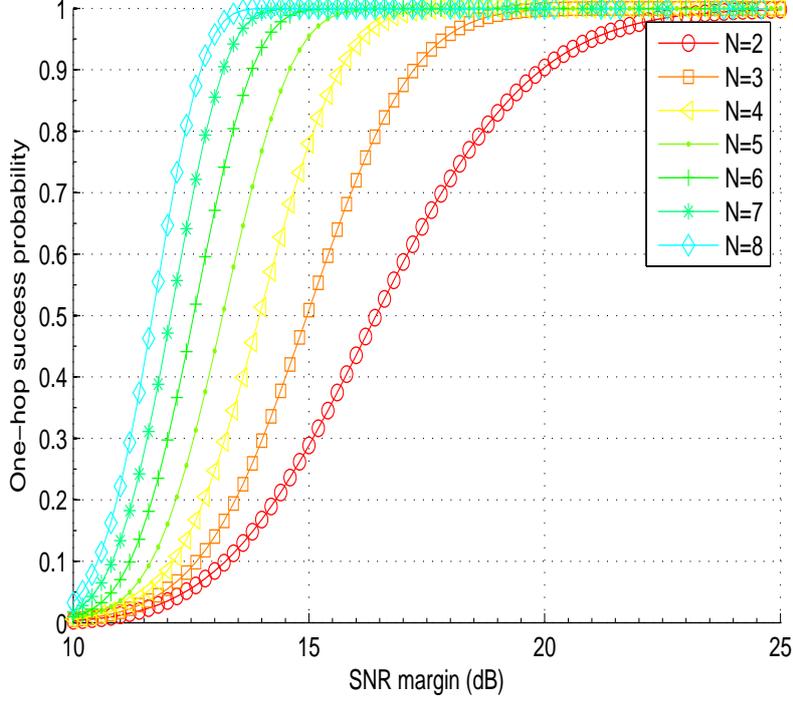


Figure 5.1: SNR margin vs. one-hop success probability for various levels of node participation;  $M = 8$ .

theoretical one-hop success probability and the one obtained from simulations is shown in Table 5.1 for  $M = 8$ , SNR margin =  $\{12, 14, 16\}$  dB, and  $N = \{2, 3, \dots, 8\}$ . In all our simulation results, the value of  $\tau$  is set to 0.1 with  $d = 1$  and  $\beta = 2$ . The table shows that the analytical results are quite close to that of simulations for all values of  $N$  and  $\gamma$  and the model fits the simulation values for an error of order  $10^{-4}$  or less.

Fig. 5.1 shows the behavior of one-hop success probability for different SNR margins and combinations of  $M$  and  $N$  for a 1D case. It can be observed that regardless of the value of  $N$ , the one-hop success probability increases by increasing the SNR margin, and for a fixed SNR margin, a loss in diver-

sity gain [27] can be seen by decreasing  $N$ . However, the performance of the network can be gauged by defining a quality of service (QoS),  $\eta$ . In this case, the  $\eta$  could be one-hop success probability or end-to-end success probability. For instance, if it is required that the one-hop success probability must be larger than or equal to 90%, then  $\eta \triangleq 0.9$ . Now for Fig. 5.1, an observation at a particular SNR margin e.g., 15dB shows that the required  $\eta = 0.9$  can be achieved at  $N = 5$ , thereby saving the energy of 3 nodes without compromising the QoS.

From a designer's point of view, if a distance of  $D$  is to be achieved than we first calculate the number of hops required to reach the required distance for a particular  $M$ . The number of hops required are  $n_o = \lceil \frac{D}{M} \rceil$ . If  $\rho$  is the one-hop success probability, then the end-to-end success probability for  $n_o$  hops is  $\rho^{n_o}$ . If again, we require that this end-to-end success probability is larger than  $\eta$ , then  $\rho^{n_o} \geq \eta$ , i.e.,

$$\rho \geq \exp \left\{ \frac{\log \eta}{n_o} \right\}. \quad (5.2)$$

The calculated one-hop success probability from (5.2) helps in finding the combinations, which would help in achieving successful transmission and conserving the energy.

In Fig. 5.2, we find the optimal combinations of  $N$  and  $M$  for reaching a distance of  $D = 100d$ . The word 'optimal' means the combination that provides maximum energy-efficiency. We also define a parameter  $N/M$ , which is defined as the number of participating nodes out of the total  $M$  nodes. The Eq (5.2) specifies the one-hop success probability, which is required to cover a distance of  $D = 100d$  and  $\eta = 0.9$ . Now there could be a variety of possible topologies that would guarantee this QoS for a particular value of

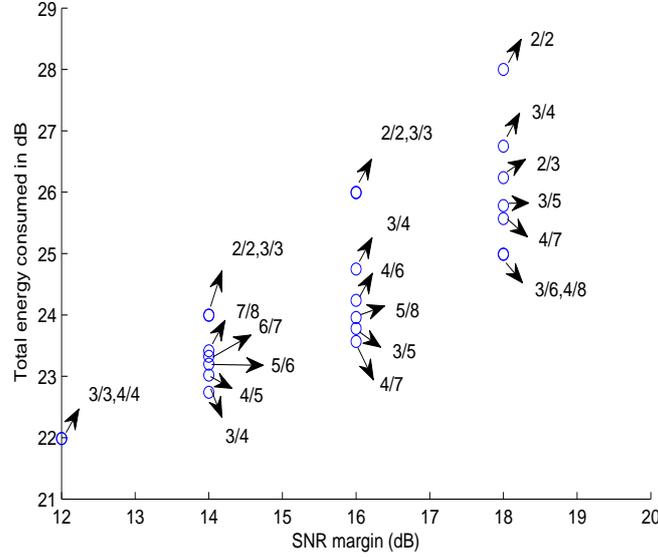


Figure 5.2: Various combinations for reaching distance of 100d vs. energy consumed for each combination.

SNR margin. For instance, one possibility is to deploy nodes as  $M = N = 2$ . In this case, 50 hops are required to reach the destination node. If we operate this topology at an SNR margin of 16dB, then  $N/M = 2/2$  combination in Fig. 4 shows that the required end-to-end QoS can be achieved by having a transmit power of 3.98 per node by using Eq. (5.1). Hence, a total transmit power of 26dB is required for the network to achieve the desired coverage. Other combinations are also plotted and the required energy is shown. It can be seen that at 16dB, the combination 4/7 results in least energy consumption, i.e., deploying a hop of  $M = 7$  nodes, but use 4 trailing nodes for transmission. Hence the fraction of energy saved is  $1 - N/M = 3/7$  for this specific case. Similarly, other cases for other values of SNR margin are also shown and their energies are listed. Hence it can be seen that OLA-LP pro-

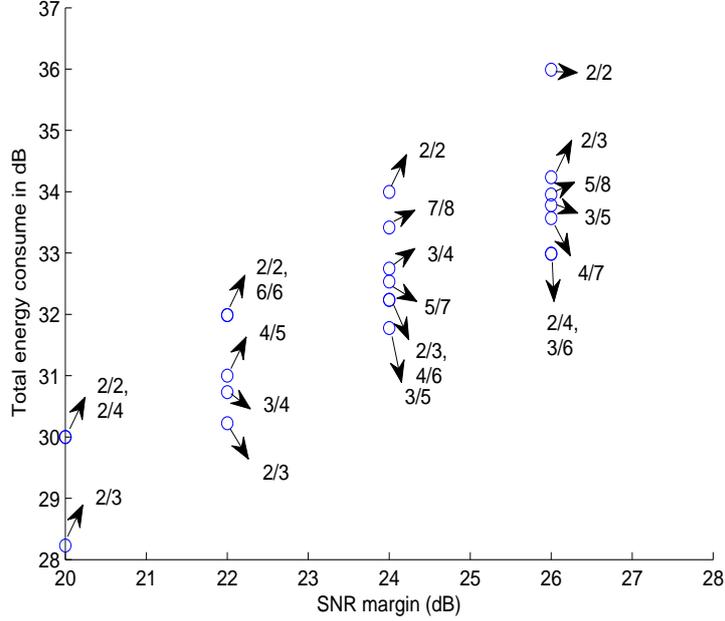


Figure 5.3: Various combinations for reaching distance of  $100d$  vs. energy consumed for each combination.

vides an energy-efficient approach than B-OLA and can be used in various cooperative transmission-based applications.

All the results for Fig. 5.2 were formulated for path loss exponential  $\beta = 2$ . We also wish to analyze the affect of increasing the  $\beta$ . Fig. 5.3 shows the combinations of  $N/M$  for  $\beta = 3$ . The results show that the path loss exponent has a significant impact on the total energy consumed as the SNR margin required to reach  $100d$  increases. We can also observe that the optimal combinations of  $N/M$  differ from the  $\beta = 2$  case.

Now we show our results for 2D case. In this scenario, we consider the following cases for  $M = 6$ :

- Case A:  $L = 3, W = 2, \tilde{N} = 3$

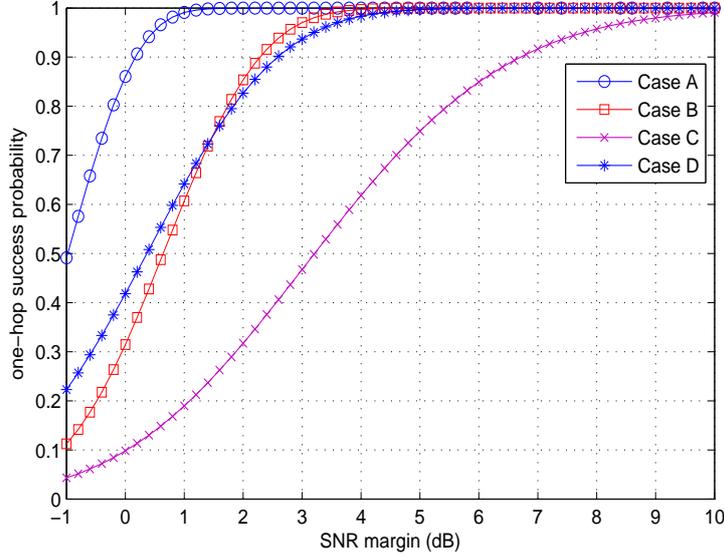


Figure 5.4: Behavior of one-hop success probability in a 2D strip network for  $M = 6$

- Case B:  $L = 3, W = 2, \tilde{N} = 2$
- Case C:  $L = 3, W = 2, \tilde{N} = 1$
- Case D:  $L = 2, W = 3, \tilde{N} = 1$

where  $L$  and  $W$  are the number of nodes in horizontal and vertical dimensions of the network, respectively and  $\tilde{N}$  represents the last participating columns. Hence, Case A is B-OLA, whereas the other cases represent OLA-LP with different levels of participating nodes. Fig. 5.4 shows the trend of one-hop success probability versus the SNR margin for the aforementioned cases. It can be seen that at lower value of SNR margin, B-OLA performs the best. However, as the SNR margin increases, the OLA-LP achieves asymptotically 100% success. If we operate the system at 6dB to reach a distance of  $D = 100d$  for  $\eta = 0.9$ , in horizontal dimension, the one-hop success probability

required to cover the distance could be found by Eq.(5.1) with  $n_o = L$ . Case C doesn't provide the required one-hop success probability and hence can't be used to cover the desired range since only two nodes are participating in the transmissions. Nevertheless, the rest of the cases provide the desired coverage with desired QoS. The total energy consumption of these cases is given as follows:

- Case A=19.01dB
- Case B=17.24dB
- Case D=17.75dB

The results show that Case B consumes the least energy and provides the required QoS. The fraction of energy saved for this particular case is  $1 - ((\tilde{N}W)/M) = 1/3$ . The results show that OLA-LP provides an energy-efficient approach in 2D deployment of network than the B-OLA without compromising the required QoS.

# Chapter 6

## Conclusions and Future Recommendations

In this thesis, we analyzed an energy-efficient cooperative communication technique known as the opportunistic large array (OLA). OLA is a simple technique based on multi-hop communication and the decode-forward (DF) mechanism. OLA network can be utilized in many practical applications e.g. smart metering, structural health monitoring and patient monitoring. A cost effective and low power [28] solution lies in using OLA network for these applications. OLA provides the spatial diversity needed for the sustainability of the transmission. However, we could exploit the spatial diversity further for conserving the energy as we have shown in our proposed algorithm OLA with limited participation OLA-LP.

The transmissions were modeled by a discrete time Markov chain. Mathematical expressions for calculating the received power for both 1-D and 2-D case were formulated. The results for finite node density networks showed that infinite network broadcast was not possible. The transmissions would

eventually die down and reach an absorbing state. The transition probability matrix was analysed by using the Perron-Frobenius eigenvalue theorem to calculate the one-hop success probability. As we were interested in finding the quasi stationary state of the network, the Perron-Frobenius eigenvalue helped in finding the distribution of the states after a particular number of hops.

After computing the Markov chain model we presented some results for 1-D and 2-D cases of OLA-LP. We started by presenting a comparison of the theoretical and simulation results to find the validity of the model. The results were formulated on the basis of a performance metric known as the SNR margin. We calculated the one-hop success probabilities to ascertain the success of transmissions. The results signified the optimal combinations for achieving a particular transmission distance although the number of hops covered to reach the destination varied for each combination. While presenting the results of the 2-D (OLA-LP) network we compared different deployments of 2-D OLA-LP network with the basic OLA (B-OLA) to gather an idea about the success of transmission.

As for the future research on these type of networks, we recommend to extend the analysis of this network to more general random networks. It would provide an interesting insight into the behavior of the network, as the node deployment would be random and the levels might be defined differently. A further analysis could help in determining the amount of energy conserved for random networks and what are the performance metrics which define the sustainability [30] of the transmission.

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