SNR ESTIMATION FOR NON-COHERENT M-FSK RECEIVERS IN RICE FADING ENVIRONMENT

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2010-NUST-MS-EE(S)-60

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A thesis submitted in partial fulfillment of the requirements for the degree of MSEE

In

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Islamabad, Pakistan.

(March 2014)

Approval

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Abstract

We estimate signal-to-noise (SNR) ratio for a communication system employing M-ARY frequency shift keying for non-coherent receivers over fading channels particularly rice fading. Maximum likelihood and Moment based estimator (DA) are proposed. The probability density function of the received data is derived which is used to obtain the estimates of the parameters of interest. In Rice fading environment, there is Line-of-sight (LoS) component whose strength is determined by K-factor. We assume different K-factor, for which the channel has different fading effects. Estimation for various scenarios are developed including data-aided (DA), non-data aided (NDA) and joint estimation using both the data and the pilot sequences. We have also derived the Cramer-Rao bound (CRB) for the estimators.

Certificate of Originality

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I also declare that the intellectual content of this thesis is the product of my own work, except for the assistance from others in the project's design and conception or in style, presentation and linguistics which has been acknowledged.

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Acknowledgment

I would like to thank Almighty Allah, who is the most beneficent and the greatest forgiver, who guided me in through every difficulty and then to my late mother, who supported me throughout my education. At the beginning I was totally unaware what to do but thanks to my thesis supervisor Dr. Syed Ali Hassan and co-supervisors who lead me to the correct direction and extended every support to me any time I needed it. Behind the scene he was always a reason for me to do what I did in SEECS. He made sure that every facility is extended towards the students even in late hours and through internet as well. I would also like to thank all the teachers who taught me in my two years of education and made me capable of doing all this. It was all possible because of the support of my friends morally and technically. And more importantly I express my extreme gratitude towards my family as their patience, support, and, tolerance was the driving force for me. Without them I would have not been able to do all this work.

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Chapter 1

Introduction

In this project, the problem of estimating the average signal-to-noise (SNR) in a non-coherent M-ary frequency shift keying (NCMFSK) receivers is considered. The signal is transmitted over a noisy Ricean Fading Channel. Maximum likelihood estimators involving parameters of interest for data-aided as well as non-data aided are derived which involve approximations of exponentials and modified Bessel functions. The performance of estimator is evaluated by comparing with Cramer-Rao Bound (CRB).

1.1 Motivation

SNR is used in communication systems in order to measure the quality of communication. The energy of the signal fades along with the distance which is known as the path loss. The function of a receiver is to amplify a signal even at very small power level and detect information from it and thermal noise over-shadows it at such low power levels. Receivers performance is majorly based on SNR. Therefore, in wireless communication, many receiver functions rely on SNR estimates. For instance, for any set of channel conditions, a higher transmitted signal power would produce a higher SNR at the receiver end which would reduce the bit error rate of the communication link which is really useful for power control algorithms including sensor networks. Due to channel degrading effects, timely response is required to minimize the bit error rate. In this matter, SNR and feedback channel plays a vital role in adaptive modulation and coding. It uses SNR estimates at the receiver to select a modulation scheme so that spectral efficiency can be improved and desired performance can be achieved.

SNR estimation can be used in wireless sensor networks for efficient routine mechanism. SNR thresholds are used for broadcasting in alternating opportunistic large arrays. The energy of the nodes in the network drains efficiently, increasing network life using SNR thresholds [1].The energy efficiency of the network is quantified against SNR margins and end-to-end success probability constraints, achieving the network reliability by limiting the node participation resulting in energy efficiency[2].Contention avoidance in wireless sensor networks with multiple flows has been proposed. The performance of opportunistic large array concentric routing algorithm over deterministic channels (OLACRA) and on diversity with limited orthogonality has been analyzed. Energy efficiency is enhanced by limiting the broadcasting in the initial upstream level along with limiting the down-link step-sizes[3]. In wireless communication, when a signal is transmitted via wireless medium, routing mechanism in ad-hoc and/or mesh networks can be established via SNR information. In cooperative wireless sensor networks, the selection of relay nodes and adopting amplify-and-forward (AF) or decode-and-forward (DF) mechanisms are depending on SNR estimates [4].

It is necessary in wireless sensor network nodes that are energy constrained to have a simple architecture. In order to ensure such a simple architecture, there is requirement of a robust modulation scheme that lowers the computational load of a wireless sensor node. For that purpose, Non-Coherent frequency shift keying (NC-FSk) provides an efficient transmitter that generates a constant envelope using low cost amplifiers and also a receiver with a simple architecture along with energy detection on every receiver branch[5]. Non-coherent FSK receiver does not have to be precisely phase locked with the sender carrier wave, providing energy efficiency, which is of prime importance in these networks [6].

The channel between the sensor nodes in a wireless sensor network is usually a line-of-sight (LOS) path along with multiple non-LOS (NLOS) paths, a Rice fading environment is appropriate as the wireless channel. In Rice fading environment, the signal is received from several paths exhibiting multipath interference where at least one of the paths is changing. One of the paths typically a LOS is much stronger than the others [7].

1.2 Problem Statement

For energy constrained wireless sensor network, a precise SNR estimation is required to reduce the complexity of the receiver to minimal. The problem of estimating the average signal-to-noise ratio in a non-coherent M-ary frequency shift keying (NMFSK) receiver is considered, where the signal is transmitted through a noisy Ricean fading channel.

1.3 Proposed Solution

We have estimated the average SNR in an NC-FSK receiver where data has been received over the Rice fading environments with additive Gaussian noise (AWGN) as the additional impairment. Closed-form maximum likelihood estimator is derived for parameters of interests, for both data-aided and non data-aided scenarios using approximations of exponentials and modified Bessel functions. The performance of the estimator is evaluated by comparing the numerical results with a Cramer-Rao bound (CRB).

1.4 Thesis Organization

The rest of the report is organized as follows: Chapter 2 describes the literature review. Chapter 3 presents the methodology. Chapter 4 evaluates the results. Chapter 5 concludes the report.

Chapter 2

Literature Review

Research in the area of SNR estimation is wide spread. A lot of work has been done for different fading environments including Rayleigh fading, slow flat fading and fast fading. Some of the earlier work done incorporates the performance of several SNR estimation techniques in order to identify the best estimator. The mean square error is considered as the performance indicator.

2.1 Multi-hop Networks

A lot of work has been in multi-hop network modeling area. Nodes are randomly placed against Bernoulli process for a stochastically modelled single-dimensional cooperative network. To characterize the multi-hop transmission, a discrete time quasi-stationary Markov chain model is considered and its transition probability matrix is derived. Useful information about the coverage of network and SNR margin is derived by eigen-decomposition of the matrix[8]. A special geometry where nodes are aligned on one-dimensional horizontal grid with equal spacing in a way that cooperating clusters are made adjacent. The transition probability matrix of Markov chain based on hypo-exponential distribution of received power at given time instant is also derived[9].Optimal deployment of nodes in one-dimensional multi-hop cooperative network in order to have maximum network coverage under a quality of service has been undertaken. Transmissions that originate from one cluster to another are modelled as discrete time Markov chain. Deploying quasi-stationary theory of Markov chains, it is shown that there is a loss of one-hop success probability when random deployment of relay nodes is done according to Bernoulli random process as compared to a regular deployment. SNR Ratio margins have been quantified for obtaining a desired QoS[10].

In a two-dimensional strip-shaped network, transmission from one group of nodes to the next is modelled as discrete-time Markov chain. It has been shown that a specific SNR Ratio margin is required for obtaining a desired hop distance for various network topologies with a constraint of packet delivery ratio[11]. Different topologies for deployment of nodes have been modelled using quasi-stationary Markov chains. One in which nodes are equally spaced on a line and second deploys group of co-located nodes, such that the groups are equally spaced on the line. It has been shown that co-located groups deployment gives better performance, specifically for higher path loss exponent[12]. Decode and forward wireless multi-hop cooperative transmission system forming Opportunistic Large Arrays has been modelled using quasi-stationary Markov Chains. Transition probability matrix of Markov chain has been derived based on the hypo-exponential distribution of the received power at a given time instant assuming that all the nodes have equal transmit power and the channel has Rayleigh fading and path loss with an arbitrary exponent. SNR Ratio margins enables a given hop distance [13].

Optimal deployment of nodes in a one-dimensional multi-hop cooperative network is considered in order to obtain maximum network coverage. Transmission originating from different nodes in forms of clusters is modelled as discrete-time Markov chains. Transition probability matrix of the chain is derived. Small sized cluster of nodes provides maximum coverage to the network if the path loss exponent is large enough[14].Transmission in a cooperative multi-hop line network is modelled as a discrete-time quasi-stationary Markov process. Tranisiton probability matrix is derived considering wireless channel exhibiting composite shadowing modelled as log-normal random variable with multipath fading as Rayleigh fading. SNR margins are quantified to achieve a particular QoS under standard deviation of shadowing[15]. A distributed equi-distant node topology and a co-located group of nodes topology has been considered operating under composite shadowing environment. Transmission between hops is modelled as a Markov process. Optimum level of cooperation required between nodes is dependent on path loss exponent[16].Transmissionn in a one-dimensional network of equally spaced nodes under large scale fading is modelled as Markov process and it's transition probability matrix is derived. Received power is labelled as log normal random variable. Eigen decomposition of the transition matrix gives insightful information about the network coverage. SNR ratio margin values provide a desired coverage distance under a given QoS and standard deviation of shadowing constraints[17].

Simulators performance are compared with a published Cramer-Rao Bound for both real and complex Additive White Gaussian Noise. Estimator structures for both real and complex channels are evaluated [18]. Previous contributions estimated the K-factor and SNR of the Ricean fading environment assuming independent or correlated channel samples [19]-[20]. Performance of turbo coding is evaluated over time varying channels deploying QPSK modulation over Rice flat fading environment. Impact of estimation error is studied in the calculation of channel log-likelihood ratios for a block random in line inter-leaver[21]. SNR estimation is done for time varying fading channels deploying phase-shift keying. Estimation is done for both Data-aided and non data-aided scenarios. Inherent estimation accuracy limitations are examined via CRB. Maximum likelihood estimator is proposed for time varying channel[22].

2.2 Correlated Channel Samples

The authors have derived a maximum likelihood estimator (MLE) for the local average SNR ratio and a slow fading Ricean channel is considered. The mean, variance and the mean square error of the estimator are derived after getting the probability density function (PDF). Estimator performance is evaluated by computing numerical results. Estimation of the K parameter along with local average SNR ratio for a noisy Ricean fading channel but here, independent channel samples are considered. Estimation is done for both data-aided and non data-aided cases. Numerical results are computer and the performance of the estimator is shown in a realistic Ricean fading environment [23]. Some of the previous work proposes SNR estimation for Nakagami-m fading channels with multiple branches equal gain combining (EGC) diversity. Estimation is done using statistical ratio of certain observables over a block of data and the SNR estimates are used in the iterative turbo decoding for Nakagami-m fading channels. The performance of turbo decoder is evaluated using SNR estimates under several fading and diversity cases [24].

2.3 MLE for Coherent Receivers

Some of the early work including SNR estimation has been done for the coherent receivers unlike non-coherent receiver presented in this report. MLE has also been derived using data statistics for non-coherent binary FSK receivers. Data-aided and non data-aided and joint estimation including both data and pilot sequences has been considered. The results of the estimator are evaluated and compared with CRB [25].

Esimation of K parameter of Rice fading environment has been carried out. Unlike other estimators, correlated channel samples are proposed. Both DA and NDA cases are considered. Performance of the estimator is examined and results shows that the estimator performs good in realistic Ricean fading channel. [26]

2.4 MLE for Non-Coherent Receivers

SNR Estimation is also done for a communication system with orthogonal non-coherent MFSK in additive white Gaussian noise (AWGN). Block fading channels as well as symbol by symbol fading channels have been considered. MLE has been derived along with an estimator based on data statistics. Simulation is done for various scenarios including data-aided, non data-aided and joint estimation including both data and pilot symbols. CRB is also computed for Rayleigh fading channels and performance of the estimator is evaluated in comparison with CRB [27]. SNR estimation over different fading channels and over additive white gaussian noise is done deploying Maximum likelihood estimation (MLE) and also using data statistics for both DA and NDA cases and results are compared with CRB[28].

Estimation techniques such as MLE and using data statistics for slow flat fading channels has been done where it is assumed that a block of data is corrupted by constant fade and AWGN[15]. Some of the authors have derived MLEs along with CRB for both data-aided and non data-aided cases. The performance of the estimators is verified by simulation [29].

For non-coherent receivers, carrier frequency offsets has also been considered in the SNR estimation for BFSK but the fading environment assumed is Rayleigh fading and additive white Gaussian noise. CFO is treated as a nuisance parameter and estimation for this parameter is done using data statistics. SNR estimation is done using the estimates for nuisance parameters to propose a maximum likelihood estimator. Both data-aided and non data-aided scenarios have been considered and the performance os the estimator is compared with the CRB [30].

Some of the previous work also includes SNR estimation for flat Rayleigh fading channels for non-coherent BFSK signals. Maximum likelihood estimation is done for both data-aided and non data-aided cases. Simulation results are used to verify the performance of the estimators. CramerRao bound expressions are derived for both data-aided and non data-aided scenarios [31]-[32].

Channel estimation is investigated in the presence of rapidly time-varying frequency selective channel against the issue of data-aided carrier frequency offset (CFO). A joint CFO channel estimators for known and known channel statistics has been derived. For channel statistics to be unknown to the estimator, a parameterization of time varying channel is required and this results in the degradation of performance. Known channel statistics are used to attain the CRB[33]. Recently, SNR estimation is done on non-coherent M-FSK receivers has been done for Rayleigh fading environment with diversity combining [34].

Even though, a lot of work has been done in the domain of SNR estimation for different fading environments but no significant work has been done for Rice fading environment. SNR estimation for rice fading environment is done for Rice fading environment but only for coherent receivers and for data-aided cases only. No work is done for non-coherent receivers in Rice fading environment domain.

Chapter 3

Methodology

Maximum likelihood estimator is proposed for average SNR in noisy Rice fading environment including both data-aided (DA) and non data-aided (NDA) scenarios. The channel samples are assumed to be independent assuming sufficient interleaving. Cramer-Rao bound has been numerically computed to evaluate the performance of the estimator. Moments based estimator for Data-Aided scenario has also been derived [35].

3.1 System Model

Consider a communication system employing non-coherent M-ary FSK modulation in a Rice fading environment. At the ith time instant, the received signal vector $x_i \in R^M$, is given as

$$x_i = |s_i \alpha_i + n_i|^2 \tag{3.1}$$

where x_i denotes the transmitted signal vector such that

$$s_i = [0, .., 0, 1, 0, .., 0]^T \tag{3.2}$$

position of 1 gives the information about the transmitted symbol, where $[.]^T$ shows the transpose operation and $|.|$ shows magnitude operation.

For the transmitted symbol, we have

$$\alpha_i \sim CN(\mu, b_0) \tag{3.3}$$

$$n_i \sim CN(0, N) \tag{3.4}$$

where α_i denotes the fading coefficient drawn from Complex Gaussian distribution with the mean μ and variance b_0 per real dimensions, respectively. Line of sight power is specified by the mean μ , while $2b_0$ specifies the power of the Non line of sight paths. n_i represents noise also drawn from complex Gaussian distribution with 0 mean and variance N.

$$\begin{gathered} \alpha = \alpha_s + j\alpha_i \\ CN(\mu, b_0/2) \end{gathered} \tag{3.5}$$

$$\begin{gathered} n = n_s + jn_i \\ CN(0, N/2) \end{gathered} \tag{3.6}$$

3.1.1 K-Factor

The K-factor in rice fading environment is defined as the ratio between the power in the direct path and the scattered paths, i.e. the ratio between power of line of sight path and the power of non line of sight paths.

$2\mu^2 := p^2$ to the scattered power $2b_0$, given by

$$K = p^2/2b_0 \tag{3.7}$$

3.1.2 SNR Definition

The SNR definition for rice fading environment is defined as the ratio of signal (LOS and NLOS) and noise power. In our case it will take the form shown as:

$$\gamma = \frac{p^2 + 2b_0}{N} \tag{3.8}$$

For the estimation model, we assume that the total packet length is composed of pilot symbols and data symbols, represented by,

$h = g + l$

where g denotes the pilot symbols, l denotes the data symbols and h is the total packet size.

3.2 Estimation Techniques

Maximum likelihood estimation technique is employed for estimation of parameters of interests; in our case Signal power i.e. both LOS and NLOS components power (p^2, b_0) and Noise power N. Estimation is done for both data-aided (DA) and non data-aided (NDA) scenarios. We have also derived moments based estimator for DA case and have compared its performance with MLE.

3.2.1 Data-Aided Estimation

For data-aided case, we assume that the signal is obtained at first branch and rest of the branches contains noise. Without loss in generality, the g pilot symbols are each set to $[1, 0, ..., 0]^T$

We have the received symbols from M branches are represented as $x_{m,i}$, where m denotes the branch index;

$m = 1, 2, ..., M$

while the second index i is the time index, represented by;

$i = 1, 2, ..., g$

Therefore, the symbol received from the first branch is given as;

$$x_{i,1} = |\alpha_i + n_i|^2 \tag{3.9}$$

As both random variables α_i and n_i are Complex Gaussian, the resulting probability density function (PDF) of $x_{i,1}$ will be non-central Chi-squared distribution with the non centrality parameter λ and is represented by

$$p_{x_{1,i}}(x) = \frac{1}{N}\exp(\frac{-x_{1,i} + p^2}{b_0 + N})I_0(\frac{2p\sqrt{x}}{b_0 + N}) \tag{3.10}$$

where $I_0(.)$ is the modified bessel function of zero order first kind.

The rest of the branches contain noise, so with zero mean, the PDF of the rest of the (M-1) branches where $m \in 2, .., M$ is exponentially distributed and is given as

$$p_{x_{m,i}}(x_m) = \frac{1}{N}\exp(\frac{-x_m}{N}), m = 2, ..., M \tag{3.11}$$

ML estimates of the noise power can be obtained from the (M-1) noise only branches.

$$\hat{N} = \frac{\sum_{i=1}^{g}\sum_{m=2}^{M} x_{m,i}}{g(M-1)} \tag{3.12}$$

To estimate the signal power components (LOS and NLOS), we formulate the joint PDF of the received symbol at ith time instant, given by

$$\begin{aligned} p_{x_i}(x) = & \frac{1}{N^{M-1}}\frac{1}{b_0+N} \\ & \exp(\frac{-N(x_i+p^2)-(b_0+N)\sum_{m=2}^{M}x_{m,i}}{N(b_0+N)}) \\ & I_0(\frac{2p\sqrt{x_1}}{b_0+N}) \end{aligned} \tag{3.13}$$

The log likelihood function is obtained by taking log likelihood of the joint PDF of g received symbols from M branches is given as

$$\begin{aligned} \wedge_{x_i}(\mathbf{x};p,b_0,N) = & -g(M-1)\ln(N) - g\ln(b_0+N) \\ & + \sum_{i=1}^{g}\ln(I_0(\frac{2p\sqrt{x_1}}{b_0+N})) - \frac{1}{N(b_0+N)} \\ & [N\sum_{i=1}^{g}x_i + Ngp^2 + (b_0+N)\sum_{i=1}^{g}\sum_{m=2}^{M}x_{m,i}] \end{aligned} \tag{3.14}$$

To find the MLE of the SNR, $\hat{\gamma}$ we have to find the ML estimates of the SNR parameters (p, b_0, N), by taking partial derivatives of the log likelihood function w.r.t the parameters (p, b_0, N) individually.

Partial derivative w.r.t b_0 gives

$$\begin{aligned} \frac{\partial\wedge_{x_i}(\mathbf{x};p,b_0,N)}{\partial b_0} = & \frac{gp^2+\sum_{i=1}^{g}x_i}{(b_0+N)^2} - \frac{g}{b_0+N} \\ & \sum_{i=1}^{g}[\frac{2p\sqrt{x_i}}{(b_0+N)^2}\frac{I_1(\frac{2p\sqrt{x_i}}{b_0+N})}{I_0(\frac{2p\sqrt{x_i}}{b_0+N})}] \end{aligned} \tag{3.15}$$

Partial derivative w.r.t p gives

$$\frac{\partial \wedge_{x_i}(\mathbf{x}; p, b_0, N)}{\partial p} = \sum_{i=1}^{g}[\frac{2\sqrt{x_i}}{b_0+N}\frac{I_1(\frac{2p\sqrt{x_i}}{b_0+N})}{I_0(\frac{2p\sqrt{x_i}}{b_0+N})}] - \frac{2gp}{b_0+N} \tag{3.16}$$

Partial derivative w.r.t N yields

$$\begin{aligned}\frac{\partial \wedge_{x_i}(\mathbf{x}; p, b_0, N)}{\partial N} &= -\frac{g}{b_0+N} - \frac{g(M-1)}{N} \\ &\sum_{i=1}^{g}[\frac{2p\sqrt{x_i}}{(b_0+N)^2}\frac{I_1(\frac{2p\sqrt{x_i}}{b_0+N})}{I_0(\frac{2p\sqrt{x_i}}{b_0+N})}] \\ &+ \frac{1}{N}\sum_{i=1}^{g}\sum_{m=2}^{M} x_{m,i} + \frac{gp^2 + \sum_{i=1}^{g} x_i}{(b_0+N)^2}\end{aligned} \tag{3.17}$$

If we solve the above partial derivatives w.r.t the unknown parameters individually, an exact solution is prohibitive due to the presence of highly non linear modified Bessel function. However, if we plot the ratio of the modified bessel function i.e. $\frac{I_1(.)}{I_0(.)}$ and this approximation gets better as the SNR as well as K is increased.

So approximating the modified bessel function ratio with 1 in comparison with the unit step function and after putting the approximation value in the partial derivatives equations and solving for the unknown parameter individually, we get the estimates

$$\hat{p} = \frac{1}{g}\sum_{i=1}^{g}\sqrt{x_i} \tag{3.18}$$

$$\hat{b_0} = \frac{g\hat{p}^2 - \sum_{i=1}^{g} 2\hat{p}\sqrt{x_i} + \sum_{m=2}^{M} x_m}{g} - \hat{N} \tag{3.19}$$

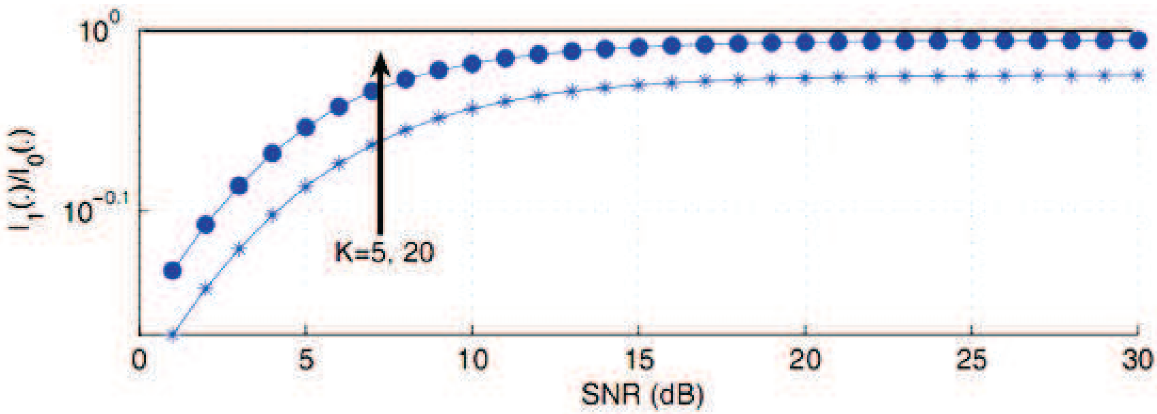

Figure 3.1: *Bessel Function Approximation for DA.*

Putting the values of estimated parameters of signal and noise from Eq (12), (18) and (19) in the SNR definition, data-aided estimation of SNR and K factor can be obtained.

3.2.2 Non Data-Aided Estimation

In non data-aided estimation, it is unknown in prior that which symbol is transmitted from the MFSK transmitter for the desired parameter from the received symbol. The conditional probability will come into place where we will assume that there is equal probability for any branch to carry signal information from the received symbol vector. The conditional joint PDF of the received symbols at time instant i, given the nth frequency is transmitted is given as:

$$
\begin{aligned}
p_{x_i}(x|s_n = 1) = {} & \frac{1}{N^{M-1}} \frac{1}{b_0 + N} I_0(\frac{2p\sqrt{x_m}}{b_0 + N}) \\
\exp[& \frac{-N(x_n + p^2) - (b_0 + N)\sum_{m=2,m\neq n}^{M} x_m}{N(b_0 + N)}]
\end{aligned}
\tag{3.20}
$$

Assuming equal probabilities of the transmitted symbols, the unconditional joint PDF of the received symbol is given by

$$
\begin{aligned}
p_{x_i}(x) = {} & \frac{1}{M} \frac{1}{N^{M-1}} \frac{1}{b_0 + N} \sum_{m=1}^{M} I_0(\frac{2p\sqrt{x_m}}{b_0 + N}) \\
\exp[& \frac{-N(x_n + p^2) - (b_0 + N)\sum_{m=2,m\neq n}^{M} x_m}{N(b_0 + N)}]
\end{aligned}
\tag{3.21}
$$

The log likelihood function is obtained by taking log likelihood of the unconditional joint PDF of l received symbols from M branches is given as

$$
\begin{aligned}
\wedge_{x_i}(\mathbf{x}; p, b_0, N) = {} & -l(M-1)\ln(N) - l\ln(b_0 + N) \\
& + \sum_{i=1}^{l} \ln \sum_{m=1}^{M} (I_0(\frac{2p\sqrt{x_(m,i)}}{b_0 + N})) \\
- \frac{1}{N(b_0 + N)} [N \sum_{i=1}^{l} x_i & + Ngp^2 + (b_0 + N) \sum_{i=1}^{g} \sum_{m=2}^{M} x_{m,i}]
\end{aligned}
\tag{3.22}
$$

Unlike in data-aided, where noise was calculated from the rest of branches other than the one containing signal information when it was known in prior about the information of the transmitted symbol, the noise for non data-aided will be calculated from the partial derivatives like signal power components w.r.t each component individually.

Partial derivatives w.r.t (p, b_0, N) are given as

$$\frac{\partial \wedge_{x_i}(\mathbf{x}; p, b_0, N)}{\partial p} = \frac{-2lp}{b_0 + N} + \sum_{i=1}^{l} [\frac{\sum_{m=1}^{M} \exp(\frac{x_{m,i} b_0}{N(b_0+N)}) I_1(\frac{2p\sqrt{x_{m,i}}}{b_0+N})(\frac{2\sqrt{x_{m,i}}}{b_0+N})}{\sum_{m=1}^{M} \exp(\frac{x_{m,i} b_0}{N(b_0+N)}) I_0(\frac{2p\sqrt{x_{m,i})}}{b_0+N})}] \tag{3.23}$$

$$\frac{\partial \wedge_{x_i}(\mathbf{x}; p, b_0, N)}{\partial b_0} = \frac{-l}{b_0 + N} + \frac{lp^2}{(b_0+N)^2} + \sum_{i=1}^{l} [\frac{\sum_{m=1}^{M} \exp(\frac{x_{m,i} b_0}{N(b_0+N)}) I_1(\frac{2p\sqrt{x_{m,i}}}{b_0+N})(\frac{-2p\sqrt{x_{m,i}}}{(b_0+N)^2})}{\sum_{m=1}^{M} \exp(\frac{x_{m,i} b_0}{N(b_0+N)}) I_0(\frac{2p\sqrt{x_{m,i})}}{b_0+N})} + \sum_{i=1}^{l} \frac{\sum_{m=1}^{M} \exp(\frac{x_{m,i} b_0}{N(b_0+N)}) I_1(\frac{2p\sqrt{x_{m,i}}}{b_0+N})(\frac{x_{m,i}}{(b_0+N)^2})}{\sum_{m=1}^{M} \exp(\frac{x_{m,i} b_0}{N(b_0+N)}) I_0(\frac{2p\sqrt{x_{m,i})}}{b_0+N})}] \tag{3.24}$$

$$\frac{\partial \wedge_{x_i}(\mathbf{x}; p, b_0, N)}{\partial b_0} = \frac{-l(M-1)}{N} - \frac{l}{b_0 + N} + \frac{lp^2}{(b_0+N)^2} - \sum_{i=1}^{l} \sum_{m=1}^{M} \frac{x_{m,i}}{N^2} + \sum_{i=1}^{l} [\frac{\sum_{m=1}^{M} \exp(\frac{x_{m,i} b_0}{N(b_0+N)}) I_1(\frac{2p\sqrt{x_{m,i}}}{b_0+N})(\frac{-2p\sqrt{x_{m,i}}}{(b_0+N)^2})}{\sum_{m=1}^{M} \exp(\frac{x_{m,i} b_0}{N(b_0+N)}) I_0(\frac{2p\sqrt{x_{m,i})}}{b_0+N})} + \sum_{i=1}^{l} \frac{\sum_{m=1}^{M} \exp(\frac{x_{m,i} b_0}{N(b_0+N)}) I_1(\frac{2p\sqrt{x_{m,i}}}{b_0+N})(\frac{-b_0(b_0+2N)}{N^2(b_0+N)^2})}{\sum_{m=1}^{M} \exp(\frac{x_{m,i} b_0}{N(b_0+N)}) I_0(\frac{2p\sqrt{x_{m,i})}}{b_0+N})}] \tag{3.25}$$

However, just like in data-aided case, the modified Bessel function ratio prohibits the closed form solution. Here we also encounter the exponential function ratio, which is also prohibiting an exact solution. For Bessel function ratio, when we plot the ratio from the expression, we see a similar but inverse trend of the curve like in data-aided the curve approaching 1 for higher SNR values.

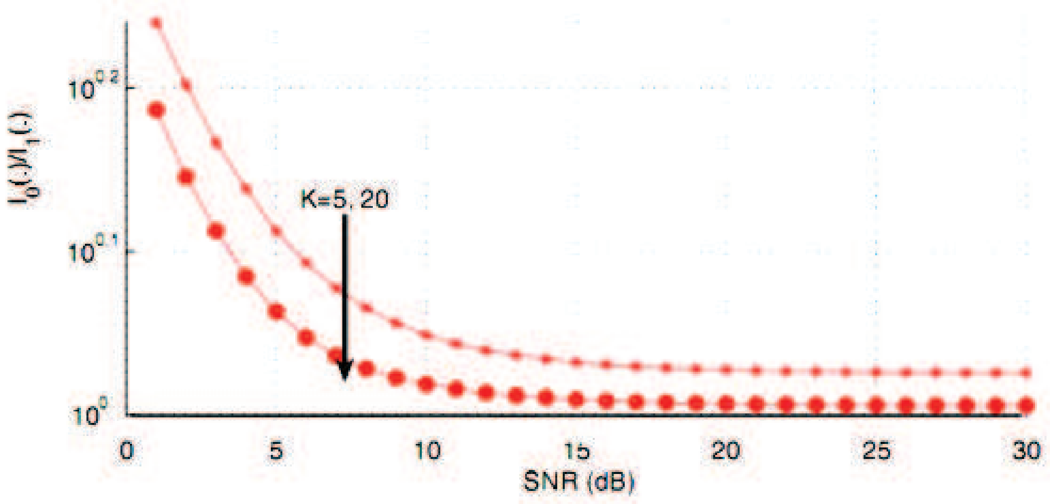

Figure 3.2: *Bessel Function Approximation for NDA.*

So here, we have to come up with double approximation including the exponential function ratio and the modified Bessel function ratio. Using some approximations of the modified Bessel function along with the exponential function as shown in Appendix, the estimates of p, b_0 and N from the received symbols is given as:

$$\hat{p} = \frac{1}{l}\sum_{i=1}^{l}\max_{m=1}^{M}\sqrt{x_{m,i}} \tag{3.26}$$

$$\Delta = 2\hat{b}_0 + \hat{N} = \hat{p}^2 - \frac{2\hat{p}\sum_{i=1}^{l}\max_m \sqrt{x_{m,i}} + \sum_{i=i}^{l}\max_{m=1}^{M} x_{m,i}}{l} \tag{3.27}$$

After putting the values of estimates of p, we get the estimates

$$\Delta = 2\hat{b}_0 + \hat{N} = \hat{p}^2 - \frac{\sum_{i=i}^{l}\max_{m=1}^{M} x_{m,i} - \hat{p}^2}{l} \tag{3.28}$$

$$\hat{N} = \frac{1}{l(M-1)}[\sum_{i=1}^{l}\sum_{m=1}^{M} x_{m,i} - \sum_{i=1}^{l}\max_{m} x_{m,i}] \tag{3.29}$$

3.2.3 Moments Based Estimator(DA)

Moments based estimator is a method of estimating the population parameters by deriving the equations that relate the population moments to the parameters of interest, Signal power and the noise power in our case.

From the received data symbols, we define an MxM matrix Z given by:

$$Z = E[X_m]E[X_m]^T(E[X_m X_m^T])^{-1} \tag{3.30}$$

We have the received symbol,

$$x_i = |s_i \alpha_i + n_i|^2 \tag{3.31}$$

where,

$$\begin{gathered} \alpha = \alpha_s + j\alpha_i \\ CN(\mu, b_0/2) \end{gathered} \tag{3.32}$$

$$\begin{gathered} n = n_s + jn_i \\ CN(0, N/2) \end{gathered} \tag{3.33}$$

So, for second moment, we have

Mean $\mu_1^2 = m^2$ and variance $\sigma_1^2 = b_0$ - for signal power

Mean $\mu_2^2 = 0$ and variance $\sigma_2^2 = N/2$ - for noise power

After computing the expected value of signal and noise, we get

$$E[\alpha_i^2] = E[\alpha_r^2] = b_0 + m^2 \tag{3.34}$$

$$E[n_i^2] = E[n_r^2] = N/2 \tag{3.35}$$

Solving for the moments, we get the expected values

$$E[X_1] = E[X_M] = \frac{2m^2 + 2b_0 + 2N}{M} \tag{3.36}$$

$$E[X_1^2] = E[X_M^2] = 2(m^2 + 6m^2 b_0 + 3b_0^2) + 8N(m^2 + b_0) + 2(m^2 + b_0)^2 + 2N^2 \tag{3.37}$$

$$E[X_1 X_2] = (2m^2 + 2b_0 + 2N)(N) \tag{3.38}$$

For data-aided approach, noise can be computed by taking expected value / mean of the rest of branches containing noise only, so it will take the form

$$\hat{N} = E(X_{m=2..M}) \tag{3.39}$$

Solving the expected values from above equations for unknown parameters of the signal power, we get

$$\hat{p} = (2E[X_1^2] - E\sqrt{X_1})^{1/4} \tag{3.40}$$

$$\hat{b_0} = \frac{(E[X_1] - 2(\frac{\hat{p}}{\sqrt{2}})^2 - \hat{N})}{2} \tag{3.41}$$

Chapter 4

Results And Discussions

4.1 Cramer Rao Bound

Cramer Rao Bound named in honor of Herald Cramer and Calyampudi Radhakrishna Rao expresses a lower bound on the variance of the estimators of a deterministic parameter. As per this bound, the variance of any unbiased estimator is almost same as the inverse of the fisher information. Any unbiased estimator that meets this lower bound is said to be efficient.

In our case, it is SNR estimator which is dependent on $p, N'b_0$. The CRB is expressed as:

$$CRB = \frac{\partial g(\theta)}{\partial(\theta)} I^{-1}(\theta) \frac{\partial g(\theta)^T}{\partial(\theta)} \tag{4.1}$$

where $g(\theta)$ denotes the parameter to be estimated, in our case the SNR.

$$g(\theta) = \frac{p^2 + 2b_0}{N} \tag{4.2}$$

Differentiating the function $g(\theta)$ w.r.t each of the unknown parameters, we get the Jacobian function $J(\theta)$, given as

$$\frac{\partial g(\theta)}{\partial(\theta)} = J_g(\theta) = [-\frac{(p^2+b_0)}{N^2}\frac{2p}{N}\frac{1}{N}] \tag{4.3}$$

Taking partial derivatives for the Fisher information matrix $I(\theta)$, we get:

$$\begin{aligned}\frac{\partial\gamma^2}{\partial N^2} &= \frac{g(M-1)}{N^2} + \frac{g}{(2b_0+N)^2} - 2(\frac{gp^2+\sum_{i=1}^{g} x_i}{2b_0+N^3} + \frac{\sum_{m=2}^{M}\sum_{i=1}^{g} x_{m,i}}{N^3}) \\ &+ \sum_{i=1}^{g}(\frac{2(p^2x_iI_0(\frac{2p\sqrt{x_i}}{2b_0+N})^2 - 2p^2x_iI_1(\frac{2p\sqrt{x_i}}{2b_0+N})^2 + p\sqrt{x_i}(2(2b_0+N)I_1(\frac{2p\sqrt{x_i}}{2b_0+N})))}{((2b_0+N)^4I_0(\frac{2p\sqrt{x_i}}{2b_0+N})^2)} \\ &+ \frac{2(p\sqrt{x_i}I_2(\frac{2p\sqrt{x_i}}{2b_0+N}))I_0(\frac{2p\sqrt{x_i}}{2b_0+N})}{(2b_0+N)^4I_0(\frac{2p\sqrt{x_i}}{2b_0+N})^2})\end{aligned} \tag{4.4}$$

$$\begin{aligned}\frac{\partial\gamma^2}{\partial N\partial p} &= \frac{2gp}{(2b_0+N)^2} \\ &+ \sum_{i=1}^{g}(\frac{4(px_iI_0(\frac{2p\sqrt{x_i}}{2b_0+N})^2 - 2px_iI_1(\frac{2p\sqrt{x_i}}{2b_0+N})^2 + \sqrt{x_i}((2b_0+N)I_1(\frac{2p\sqrt{x_i}}{2b_0+N})))}{((2b_0+N)^3I_0(\frac{2p\sqrt{x_i}}{2b_0+N})^2)} \\ &+ \frac{4(p\sqrt{x_i}I_2(\frac{2p\sqrt{x_i}}{2b_0+N}))I_0(\frac{2p\sqrt{x_i}}{2b_0+N})}{(2b_0+N)^3I_0(\frac{2p\sqrt{x_i}}{2b_0+N})^2})\end{aligned} \tag{4.5}$$

$$\begin{aligned}\frac{\partial\gamma^2}{\partial N\partial b_0} &= \frac{2g}{(2b_0+N)^2} - 4(\frac{gp^2+\sum_{i=1}^{g} x_i}{2b_0+N^3}) \\ &+ \sum_{i=1}^{g}(\frac{4(p^2x_iI_0(\frac{2p\sqrt{x_i}}{2b_0+N})^2 - 2p^2x_iI_1(\frac{2p\sqrt{x_i}}{2b_0+N})^2 + p\sqrt{x_i}(2(2b_0+N)I_1(\frac{2p\sqrt{x_i}}{2b_0+N})))}{((2b_0+N)^4I_0(\frac{2p\sqrt{x_i}}{2b_0+N})^2)} \\ &+ \frac{4(p\sqrt{x_i}I_2(\frac{2p\sqrt{x_i}}{2b_0+N}))I_0(\frac{2p\sqrt{x_i}}{2b_0+N})}{(2b_0+N)^4I_0(\frac{2p\sqrt{x_i}}{2b_0+N})^2})\end{aligned} \tag{4.6}$$

$$\frac{\partial\gamma^2}{\partial p\partial N} = \frac{2gp}{(2b_0+N)^2}$$
$$+\sum_{i=1}^{g}(\frac{2(px_iI_0(\frac{2p\sqrt{x_i}}{2b_0+N})^2 - 2px_iI_1(\frac{2p\sqrt{x_i}}{2b_0+N})^2 + \sqrt{x_i}((2b_0+N)I_1(\frac{2p\sqrt{x_i}}{2b_0+N})))}{((2b_0+N)^3I_0(\frac{2p\sqrt{x_i}}{2b_0+N})^2)}$$
$$+\frac{2(p\sqrt{x_i}I_2(\frac{2p\sqrt{x_i}}{2b_0+N}))I_0(\frac{2p\sqrt{x_i}}{2b_0+N})}{(2b_0+N)^3I_0(\frac{2p\sqrt{x_i}}{2b_0+N})^2}) \tag{4.7}$$

$$\frac{\partial\gamma^2}{\partial p^2} = \frac{-2g}{(2b_0+N)}$$
$$+\sum_{i=1}^{g}(\frac{2x_i(I_0(\frac{2p\sqrt{x_i}}{2b_0+N})^2 - 2I_1(\frac{2p\sqrt{x_i}}{2b_0+N})^2 + I_2(\frac{2p\sqrt{x_i}}{2b_0+N}))I_0(\frac{2p\sqrt{x_i}}{2b_0+N})}{(2b_0+N)^2I_0(\frac{2p\sqrt{x_i}}{2b_0+N})^2}) \tag{4.8}$$

$$\frac{\partial\gamma^2}{\partial p\partial b_0} = \frac{4gp}{(2b_0+N)^2}$$
$$+\sum_{i=1}^{g}(\frac{4(px_iI_0(\frac{2p\sqrt{x_i}}{2b_0+N})^2 - 2px_iI_1(\frac{2p\sqrt{x_i}}{2b_0+N})^2 + \sqrt{x_i}((2b_0+N)I_1(\frac{2p\sqrt{x_i}}{2b_0+N})))}{((2b_0+N)^3I_0(\frac{2p\sqrt{x_i}}{2b_0+N})^2)}$$
$$+\frac{4(p\sqrt{x_i}I_2(\frac{2p\sqrt{x_i}}{2b_0+N}))I_0(\frac{2p\sqrt{x_i}}{2b_0+N})}{(2b_0+N)^3I_0(\frac{2p\sqrt{x_i}}{2b_0+N})^2}) \tag{4.9}$$

$$\frac{\partial\gamma^2}{\partial b_0\partial N} = \frac{2g}{(2b_0+N)^2} - 4(\frac{gp^2+\sum_{i=1}^{g}x_i}{2b_0+N^3})$$
$$+\sum_{i=1}^{g}(\frac{4(p^2x_iI_0(\frac{2p\sqrt{x_i}}{2b_0+N})^2 - 2p^2x_iI_1(\frac{2p\sqrt{x_i}}{2b_0+N})^2 + p\sqrt{x_i}(2(2b_0+N)I_1(\frac{2p\sqrt{x_i}}{2b_0+N})))}{((2b_0+N)^4I_0(\frac{2p\sqrt{x_i}}{2b_0+N})^2)}$$
$$+\frac{4(p\sqrt{x_i}I_2(\frac{2p\sqrt{x_i}}{2b_0+N}))I_0(\frac{2p\sqrt{x_i}}{2b_0+N})}{(2b_0+N)^4I_0(\frac{2p\sqrt{x_i}}{2b_0+N})^2}) \tag{4.10}$$

$$\frac{\partial\gamma^2}{\partial b_0 \partial p} = \frac{4gp}{(2b_0+N)^2} + \sum_{i=1}^{g}\left(\frac{4(px_iI_0(\frac{2p\sqrt{x_i}}{2b_0+N})^2 - 2px_iI_1(\frac{2p\sqrt{x_i}}{2b_0+N})^2 + \sqrt{x_i}((2b_0+N)I_1(\frac{2p\sqrt{x_i}}{2b_0+N})))}{((2b_0+N)^3 I_0(\frac{2p\sqrt{x_i}}{2b_0+N})^2)} + \frac{4(p\sqrt{x_i}I_2(\frac{2p\sqrt{x_i}}{2b_0+N}))I_0(\frac{2p\sqrt{x_i}}{2b_0+N})}{(2b_0+N)^3 I_0(\frac{2p\sqrt{x_i}}{2b_0+N})^2}\right) \tag{4.11}$$

$$\frac{\partial\gamma^2}{\partial {b_0}^2} = \frac{4g}{(2b_0+N)^2} - 8\left(\frac{gp^2 + \sum_{i=1}^{g} x_i}{2b_0 + N^3}\right) + \sum_{i=1}^{g}\left(\frac{8(p^2x_iI_0(\frac{2p\sqrt{x_i}}{2b_0+N})^2 - 2p^2x_iI_1(\frac{2p\sqrt{x_i}}{2b_0+N})^2 + p\sqrt{x_i}(2(2b_0+N)I_1(\frac{2p\sqrt{x_i}}{2b_0+N})))}{((2b_0+N)^4 I_0(\frac{2p\sqrt{x_i}}{2b_0+N})^2)} + \frac{8(p\sqrt{x_i}I_2(\frac{2p\sqrt{x_i}}{2b_0+N}))I_0(\frac{2p\sqrt{x_i}}{2b_0+N})}{(2b_0+N)^4 I_0(\frac{2p\sqrt{x_i}}{2b_0+N})^2}\right) \tag{4.12}$$

$I^{-1}(\theta)$ in the CRB definition is the inverse Fisher information matrix, representing the matrix elements showing partial derivatives of all unknown parameters w.r.t each other respectively, shown as:

$$I^{-1}(\theta) = E\begin{pmatrix} \frac{\partial\gamma^2}{\partial N^2} & \frac{\partial\gamma^2}{\partial N\partial p} & \frac{\partial\gamma^2}{\partial N\partial b_0} \\ \frac{\partial\gamma^2}{\partial p\partial N} & \frac{\partial\gamma^2}{\partial p^2} & \frac{\partial\gamma^2}{\partial p\partial b_0} \\ \frac{\partial\gamma^2}{\partial b_0\partial N} & \frac{\partial\gamma^2}{\partial b_0\partial p} & \frac{\partial\gamma^2}{\partial b_0^2} \end{pmatrix}^{-1}$$

We present the numerical simulation results of the estimator derived earlier and discuss their performance with respect to various parameters like no. of branches, packet length, and the K-factor. The CRB for data-aided estimator is computer numerically, as the closed form solution of Fisher information matrix is highly prohibitied.

4.2 Effect of Branches

Figure 4.1 shows the normalized (w.r.t square of true SNR) mean squared error (NMSE) versus the SNR for two values of M, i.e. M=2 and M=8. The no. of symbols used for this plot are 100 both for data aided and non data-aided cases.

From the curves, it can be seen that, as the number of branches in the received signal increases, SNR estimation gets better and the error decrease because when we increase the number of branches, the data symbols coming from more branches will increase ultimately, which will result in better estimation of the desired parameters. It can be seen in both subplots that both DA and NDA estimators perform better in high SNR region, but as you decrease the SNR their performance deteriorates. This is because of the approximations used for exponential function and modified Bessel functions to get closed form solution for the estimators. We see DA performs better and is much closer to CRB as compared to NDA, due to the fact that only approximation against modified Bessel function is used for DA estimation, while for NDA estimates, approximation for modified Bessel function as well as exponential function is used, which is being reflected in the subplot curves.

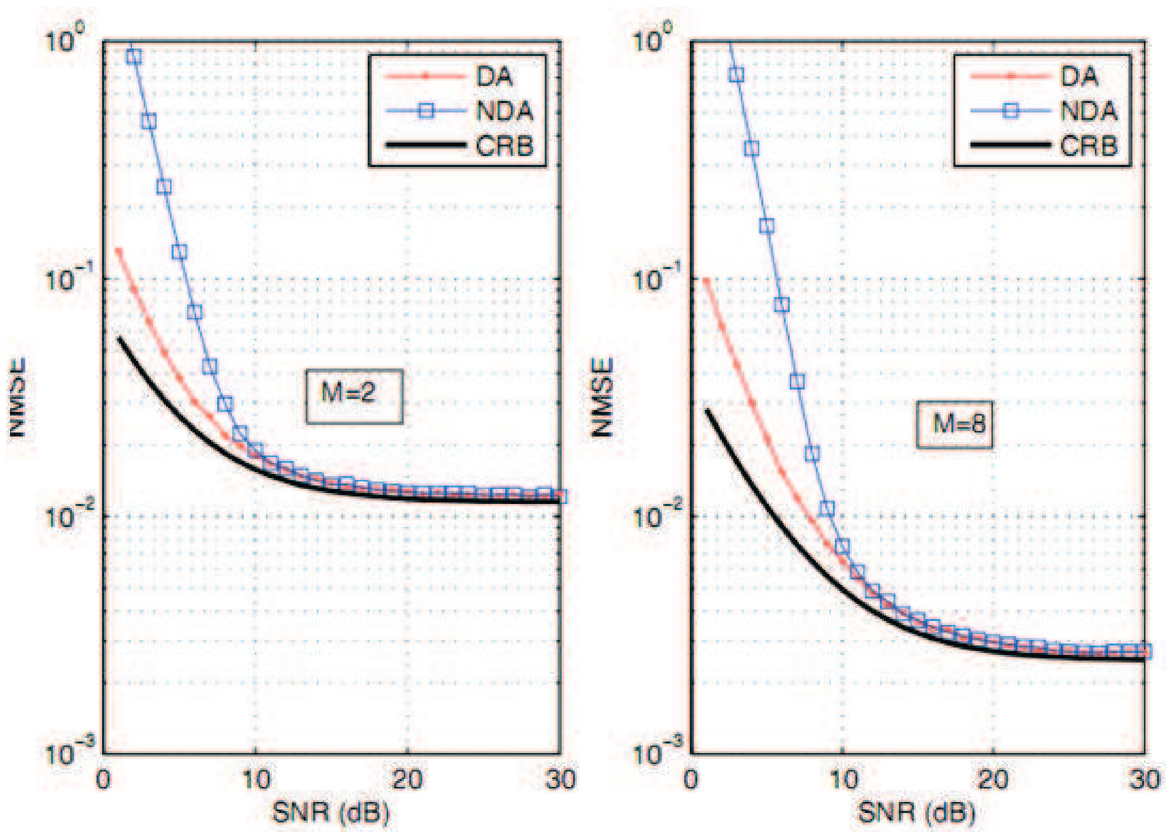

Figure 4.1: *Effect of Branches.*

4.3 Effect of Packet Length

Figure 4.2 shows the estimator performance for short packet transmission where the pilot symbols are 8, i.e. g=8, and the data symbols i.e. l=28. So, total packet h size is 36. We have also depicted Full-data aided (FDA) case in the figure. For FDA case, we assume that the packet is decoded successfully (e.g., passing CRC check) and thus the received packet can now be utilized for SNR estimation. This case is appropriate in cooperative communication scenario where decode and forward mechanism is employed and CRC check pass is must for packet forwarding; the detected data is assumed to be correct regardless of the SNR value.

From the figure 4.2, it can be seen that as we increase the number of data symbols, the estimator performs better, the effect can be seen in FDA as compared to DA as well as NDA case. The plotted CRB in this figure is obtained from FDA case i.e. 36 symbols. FDA estimator performs better for almost complete range of SNR as compared to DA and NDA having errors for low SNR cases because of the reasons discussed earlier.

4.4 Effect of K Factor

For the result discussed earlier, a constant K-factor of 20 has been used. However, for Rice fading environment, the K-factor varies for different values. Figure 4.3 represents a contour plot for NMSE for different values of K-factor and SNR for DA estimator where g=100 and M=8.

From figure 4.3, it can be seen that NMSE decrease as the K-factor or SNR is increased. Estimator gives less than 10 percent error when K-factor

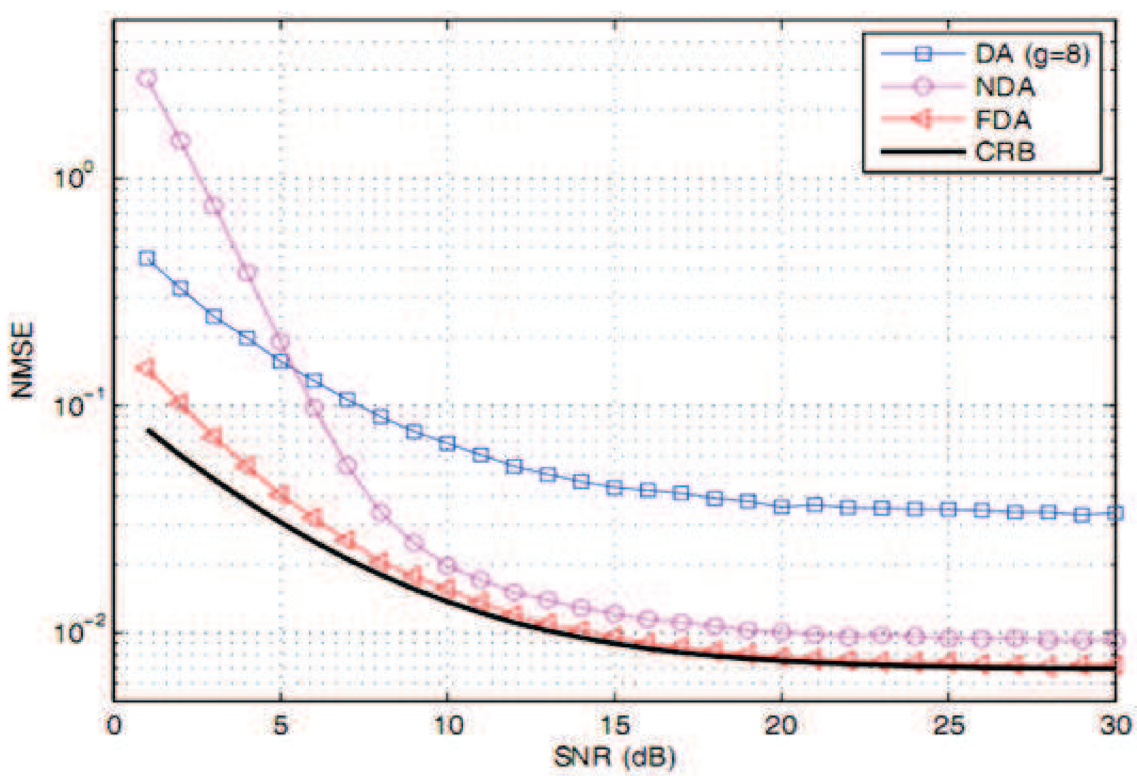

Figure 4.2: *Effect of Packet Length.*

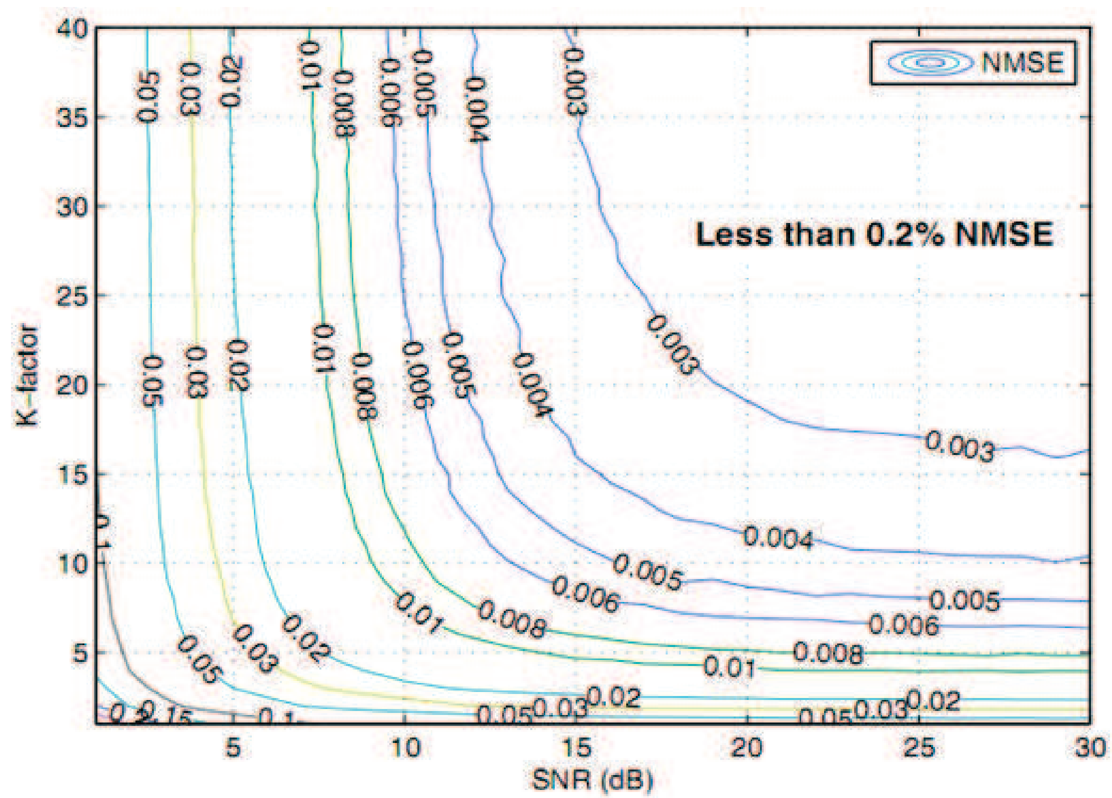

Figure 4.3: *Effect of K Factor.*

> 2 and SNR > 2 dB. Therefore, the proposed estimators work well in Rice fading environments even for moderate packet lengths. It can be seen that NMSE 0.2 percent for large values of both SNR and the K-factor.

4.5 Method of Moments v/s MLE

Figure 4.4 shows the curves obtained for data aided cases from Maximum likelihood estimator and Moments based estimator. Method of moments works well as compared to MLE for large data sets typically g=1000 and above, as in our case for g = 1000. The wide gap between MLE and MoM curve for low SNRs is because of the approximations used in MLE while there isnt any approximations for MoM based estimators.

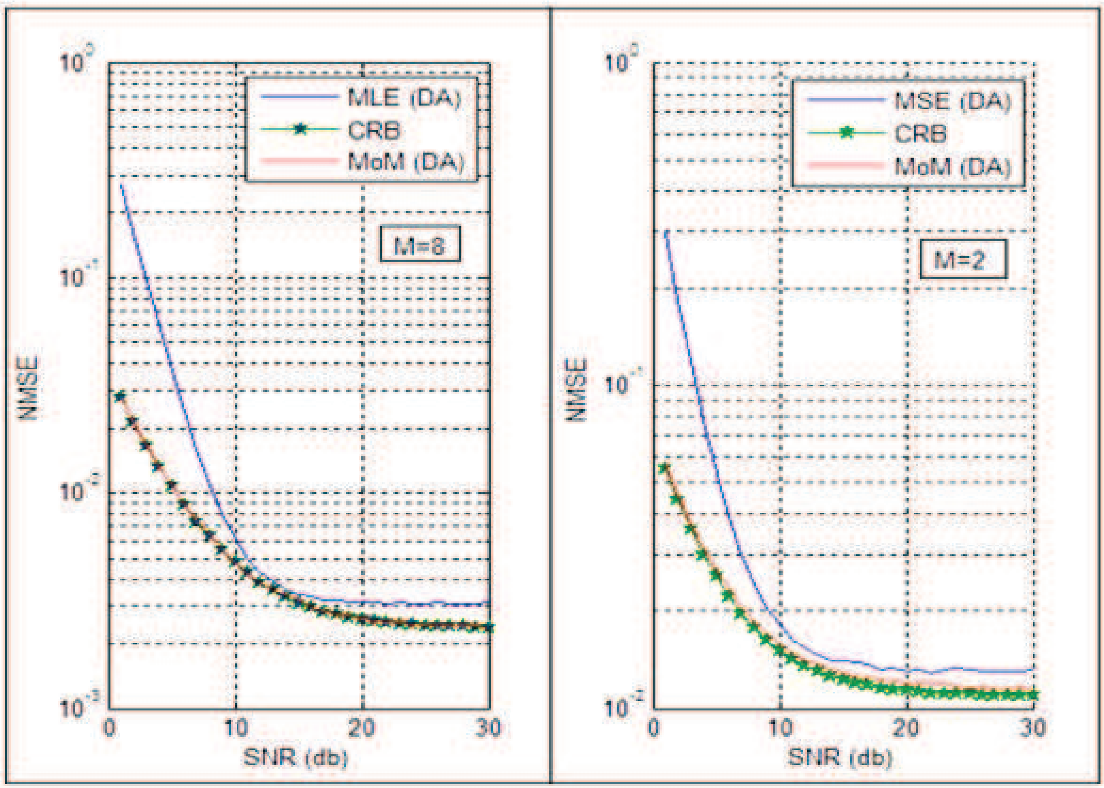

Figure 4.4: *MoM v/s MLE.*

Chapter 5

Conclusion and Future Work

5.1 Conclusion

We have derived closed form maximum likelihood estimator for average SNR in non-coherent MFSK receiver operating in Rice fading environment. Estimation is done for both data-aided and non data-aided cases and performances of the estimators have been compared with a numerically computed CRB. The joint distribution of the received data, in both DA and NDA cases, contains the non-linear modified Bessel function that prohibits the closed form solution. However, it has been shown using some approximations of the modified Bessel functions that estimator performs well specially for high SNR and a high LOS component or high K-factor. We have discussed the effect of packet length on estimator performance and how the estimator would perform with increasing number of branches. In both cases, as the number of branches and the packet length is increased the error floor decreases and the estimator performs better with the estimator curve approaching CRB curve for high SNR values and increasing packet length and number of branches.

Similarly, as the K-factor increases, the error floor is reduced, typically 0.1 percent for lower values of SNR and K-factor but for high K-factor values and high SNR values the estimator performance is up to 0.02 precent which is really optimal for even small packet lengths. We have also compared the performance of MLE with Moments based estimator, which is designed for high packet length typically for 1000 and above for DA case. Also, for low SNR values, the curve of MSE and Moments based estimator has a wide gap. This is due to the exponential function and modified Bessel function approximations used in MLE, where the curve approaches to the unit step approximation for high values of SNR and deviates from the unit step curve for low SNR values, therefore the MLE performs better for high SNR values and for moderate packet length.

5.2 Future Work

The research work can be broadened for SNR Estimation in Rice fading environment, some of the highlighted future work is mentioned under:

5.2.1 Method of Moments for Non-data Aided

We have done estimation based on Method of moments for Data aided case. Future work can be done for non-data aided using Method of moments where it is unknown a priori about which symbol is transmitted from the MFSK transmitter. The conditional probability needs to be calculated where it can be assumed that there is an equal probability for any branch to carry signal information from the received symbol vector.

5.2.2 SNR in Rice Fading with Diversity

A possible future work can be to incorporate diversity along with rice fading environment. Diversity combining is a popular technique in combating with these effects and significantly improve the system bit error performance. Estimation deploying this diversity fading model would incorporate the diversity parameter along with signal and noise.

5.2.3 SNR in Correlated Rice

Work can be done for SNR in correlated Rice fading where fading correlation among the received envelope samples can be studied and its impact on the performance of the estimator can be observed.

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