A Multiple Region Reverse Frequency Allocation Scheme for Downlink Capacity Enhancement in 5G HetNets

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***Abstract*—To cope with the data surge problem and to enhance the coverage of existing cellular systems, heterogeneous networks (HetNets) are deployed in hierarchical manner, comprising of macrocells and overlaid femtocells. A novel interference mitigation technique of Reverse Frequency Allocation (RFA) scheme is introduced, which provides intercell orthogonality by dividing the cell into spatial regions and optimally allocating the frequency resources. RFA enhances the data rates of downlink femto users by eliminating the cross-tier interference from macro base station (MBS). In this paper, we extend the multiple region RFA scheme in multi-cellular network to further mitigate the impact of interference in the adjacent cells. In addition, we also develop a hybrid RFA scheme that merges the benefits of different RFA schemes in terms of large bandwidth and limited interference to achieve higher data rates. Simulation results show that the modified RFA (M-RFA) schemes exhibit superior performance as compared to the conventional RFA schemes in terms of user-fairness and improved sum capacity. For the evaluation of system performance, several metrics such as outage probability, sum rates and outage capacity have been analyzed for satisfying the constraint of minimum capacity requirement of cell edge users.**

***Index Terms*—5G Heterogeneous network, reverse frequency allocation, interference, sum-rate, outage capacity.**

I. Introduction

Meeting the exponential increasing demands of higher data traffic is one of the momentous challenges that could be resolved by either reducing the cell sizes along with interference minimzation algorithms or by adopting the option of additional spectrum in the field of modern wireless communications. According to [1], it is anticipated that the mobile data traffic of the world will be increased by 1000 folds from 2010 to 2020, and the origin of 80% of this traffic will be from indoors. As the licensed frequency spectrum is limited and the existing macrocell infrastructure all alone cannot satisfy the needs of subscribers, the idea of heterogeneous networks (HetNets) has been proposed in 3rd Generation Partnership Project (3GPP) and coined forward as a potential candidate for fifth generation (5G) communications, where femtocells are deployed along with the existing macro cellular network and both utilize the same licensed spectrum.

Femtocells are low-powered, relatively less complex, small-sized base stations that are cost effective. One of the greatest benefits of femtocells is that they can be seamlessly overlaid with the existing cellular networks to provide efficient indoor coverage [2]. However, coexisting infrastructure of macrocells and femtocells gives rise to the challenge of deploying the HetNets in such a way that cause minimum co-tier as well as cross-tier interference [3]. In cross-tier phenomenon, the femto user equipments (FUEs) receive tremendous amount of interference from high-powered macro base station (MBS) or the transmission between macro user equipments (MUE) and MBS can be obstructed by femto base station (FBS). Therefore, it is required to deploy the two tier network such that the effect of interference is reduced in all scenarios. These issues can be resolved by adopting several techniques such as efficient power control, adjusting the transmissions of both tiers in separate time frames or by the orthogonalization of bandwidth resources. Some recent results of deploying HetNets with and without millimeter spectrum are discussed in [4]–[6].

A. Related Work

To mitigate the impact of interference, several frequency allocation schemes have also been proposed in [7]–[9] including static, dynamic, sub-frequency band and shared frequency reuse strategies. In *static frequency reuse* scheme, there are dedicated frequency bands for femtocells and macrocells [7], which leads to the wastage of resources that are already scarce. In *dynamic frequency allocation* scheme, the overall traffic is being monitored by the continuous signaling between the two tiers and the spectrum is allocated dynamically. Thus, dynamic frequency reuse scheme behaves spectrum efficient as compared to static scheme, but at the cost of higher complexity and persistent signaling [8]. In *shared frequency band* scheme, the whole spectrum is utilized by the two tiers, which provides the most spectral efficient solution at the expense of strong interference.

The shared frequency band scheme generally results in user fairness for both macro as well as femto users because the entire spectrum is available to both tiers, with no particular

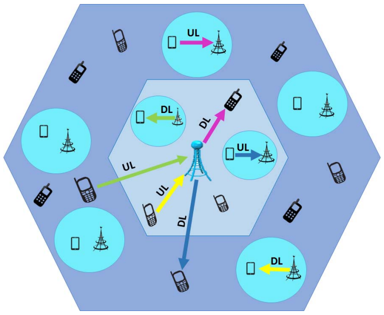

Figure 1: A HetNet with RFA scheme.
(The signaling colors are conversant with the spectrum colors in the Table I

Table I: Frequency Distribution in 2-RFA Scheme.

B_1		B_2	
(blue)	(green)	(pink)	(yellow)
Inner UL Femtocell	Inner DL Femtocell	Outer UL Femtocell	Outer DL Femtocell
B_{1x}	B_{1y}	B_{2x}	B_{2y}
Outer DL Macrocell	Outer UL Macrocell	Inner DL Macrocell	Inner UL Macrocell

priority and signaling. However, because of simultaneous transmissions in both tiers, the cross-tier interference becomes a major concern. To cope with it, strict fractional frequency reuse (FFR) scheme is proposed [9], in which the cell is partitioned into two regions. A frequency reuse factor (FRF) of unity is applied to the cell center region while higher values of FRF are assigned to the cell edge region. In a multi-cell FFR scenario, the cells in a cluster share the same frequency band in their center region while the cell edge regions have the remaining dedicated frequency bands but this scheme is spectrally not very efficient. Another relatively efficient scheme is the soft fractional frequency reuse (SFR) scheme, which is similar to the strict frequency reuse scheme but in this case, the users of both regions have the access to full spectrum [10].

In this paper, a novel variant of shared frequency scheme has been analyzed known as reverse frequency allocation (RFA) scheme, which addresses the spectrum sharing challenge and enhances the data rates of downlink FUEs. In RFA, the cell is partitioned into non-overlapping regions and the basic idea is to introduce the complementary spectrum allocation between the macrocells and femtocells, i.e., the direction of radio transmissions of macrocells is reversed and allocated to femtocells that are in the other spatially separated area. This potentially reduces the number of interferers and also the prominent interference from the MBS is entirely avoided. In [12], two region RFA is implemented on multi-cell, however, the neighboring cells caused a great source of interference whereas in [13], multiple region RFA in a single cell has been simulated, however, this work did not consider the problem of inter cell interference (ICI). We accumulate the aspects of previous works and devise a multi-dimensional model of a multi-cell multiple region RFA to enhance the data rates of the network.

The rest of this paper is organized as follows. In Section II, we discuss the frequency allocation of 2-RFA scheme, while in Section III the system model of multi-cell multiple region RFA is presented. In Section IV, we present the modified RFA schemes, whereas Section V shows the details of simulation and results. Finally, Section VI concludes the paper.

II. Frequency allocation in 2-RFA

The main drawback of FFR and SFR is that the femto users have to face severe interference from the MBS. To avert this problem and to use the resources intelligently, RFA scheme has been presented in [11] and this concept is depicted in Fig. 1. We allocate spectrum to MUEs and FUEs such that each user receives minimum interference. For instance, in 2-RFA, where the cell area is divided into two regions, the downlink (DL) frequency spectrum of inner region FUEs is allocated to the uplink (UL) outer region MUEs and the uplink frequency band of the inner FUEs is allocated to the outer region downlink MUEs. On the other hand, the outer UL frequency band of FUEs is allocated to the inner DL MUEs and the outer DL sub-band of FUEs to the inner UL MUEs. On a side role, soft frequency reuse strategy allocates spectrum in such a manner that the DL FUEs have to face strong interference from the MBS whereas in RFA, the FUEs encounter cross-tier interference from low-powered MUEs only, which reduces the level of interference to a considerate amount. When we evaluate the system from the downlink perspective, RFA scheme provides lower cross-tier interference, hence better signal-to-interference plus noise ratio (SINR) is achieved. The frequency allocation for 2-RFA is shown in Table I.

Let F_i and F_o represent the number of inner and outer region femto users respectively, whereas let M_i and M_o depict the respective number of macro users. The downlink SINR of the n^{th} FUE attached to the l^{th} FBS in the inner and outer regions of a single cell is formulated as

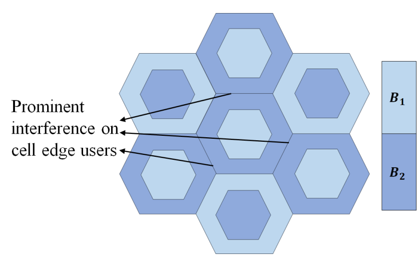

Figure 2: 2-RFA in a multi-cell scenario. The cell area is divided into two regions with frequency bands B_1 and B_2.

$$SINR_{DL\_F_i} = \frac{P_l^n |h_0|^2 d^{-\beta}}{\Sigma_{i=1, i\neq l}^{F_i} P_{fbs} I_i^n + \Sigma_{j=1}^{M_o} P_{mue} I_j^n + \sigma^2}, \quad (1)$$

$$SINR_{DL\_F_o} = \frac{P_l^n |h_0|^2 d^{-\beta}}{\Sigma_{i=1, i\neq l}^{F_o} P_{fbs} I_i^n + \Sigma_{j=1}^{M_i} P_{mue} I_j^n + \sigma^2}, \quad (2)$$

where $SINR_{DL\_F_i}$ is the SINR of a femto user in the inner region and $SINR_{DL\_F_o}$ is the SINR of a femto user in the outer region, $|h_0|^2$ is the fading channel gain between BS and the attached FUE, which is independently and identically distributed (i.i.d) exponential random variable with unit mean.

Here d is the distance between the intended user to the transmitting BS, β stands for path loss exponent and σ^2 represents the noise variance. In (1) and (2) I_i^n, I_j^n represent the channel attenuation of interfering entities from FBS i and MUE j at n^{th} FUE, respectively. It can be observed in (1) that FUEs in the inner region experience interference only from FBS of the same region and MUE interferers of the outer region. This is due to the fact that RFA scheme reverses the direction of transmitting frequencies, thus, any interference from MBS is prohibited, which otherwise is a prominent source of degradation of SINR.

Extending the concept of 2-RFA in a multi-cellular network, it is evident that the interference management becomes a difficult task as the number of interferers increases drastically and the ICI for outer region femtocells becomes inevitable. Fig. 2 depicts the multi-cell 2-RFA scheme, and it shows the interference during the downlink transmission would increase many folds due to the direct obstruction from closely attached 3 macrocells, badly affecting the cell edge users [12]. This motivates the division of a cell into further non-overlapping regions, whereby the intercell interference can be avoided having reduced number of users with same sub-bands, and reducing the cell edge effects. In this paper, we alleviate the problem of interference by dividing the macrocells into multiple regions such as 4, 8 and 16, etc, where the users having same sub-carriers can be placed apart, thus, the path loss between them increases, this results in subsequently mitigating the problem of interference.

III. System Model of K-RFA in a multi-cell scenario

We consider the downlink of a heterogeneous cellular network consisting of macrocells, in which the circular-shaped femtocells are uniformly distributed. The entire region of a macrocell is partitioned into k disjoint regions of equal area, where

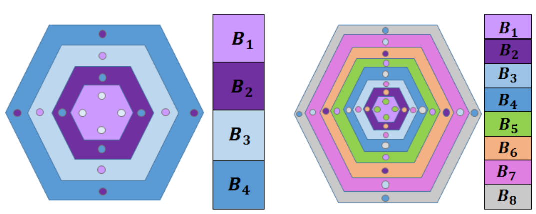

Figure 3: Frequency allocation in a single cell (4-RFA and 8-RFA).

$k = 2^W$, such that $W\epsilon$ and is the set of natural numbers, hence $k = \{2, 4, 8, 16\}$, results in 2-RFA, 4-RFA, 8-RFA and 16-RFA, respectively[1]. The allocated frequency spectrum, B, is partitioned among k orthogonal sub-bands. Sub-carrier allocation is performed by assuming that the users are uniformly distributed within the multiple regions of a macrocell. We consider the in-band transmission of femtocells, i.e., both femto and macro users utilize the same frequency spectrum. Furthermore, frequency division duplexing (FDD) is considered, where each available sub-band further splits into equal uplink and downlink frequency bands of x and y, such that $B_t = [B_{tx} \bigcup B_{ty}]; t = \{1, 2, ..., k\}$.

In this paper, we adopt the same strategy of 2-RFA and implemented into higher region RFA. In 4-RFA, the entire spectrum is split into 4 sub-bands B_1, B_2, B_3 and B_4, in the respective 4 regions $\{R_1, R_2, R_3, R_4\}$ of equal area as shown in the left side of Fig. 3. In R_1 region, one of the FBS acts as serving BS for downlink FUE, whereas rest of the FBSs behave as a source of interference, also the MUEs of R_3 act as interferers. Likewise, R_2 DL FUEs encounter cross-tier interference from R_4 MUEs. This is due to increased path loss that provide a negligible interference. Similarly, R_3 FUEs receive interference from R_1, and R_4 FUEs from R_2 MUEs. The benefit of dividing macrocells into higher regions is that the number of FUEs and MUEs are now divided among more regions and the reduction of number of femtocells in a specific region utilizing the same frequency spectrum is achieved. Hence, the cell edge interference is minimized because none of the same frequency sub-bands is allocated in adjacent cells, which was the case in 2-RFA multi-cell, thus minimizing the overall system outage probability.

In case of 8-RFA, the whole area splits into 8 equal regions and similarly the frequency spectrum is also divided as shown in the right side of Fig. 3. In this way, the interference regions are located farther apart, i.e., now R_1 will experience cross-tier interference from R_5, R_2 from R_6, R_3 from R_7 and R_4 from R_8, respectively. Hence, the FUEs in every region receive interference from a distant region as compared to their adjacent regions. Moreover, because of reduced area, the number of interfering users also reduces, this results in higher SINR values. On the other hand, as the frequency spectrum is further divided among the regions, we achieve better coverage at the cost of sum capacity reduction. Further reduction in average outage probability can be achieved if we continue dividing the whole area, e.g., in 16-RFA, the interfering regions are now far apart from each other. However, partitioning the cell into higher regions is not a practical solution for interference management as smaller the area of the region, more frequent handover occurs. Similarly, accurate localization systems would be required that become an expensive solution with redundant infrastructure. Therefore, our aim is to find an optimized scheme that divides the cell into higher regions but also generates a notable increase in the data rates. For K region RFA, the generalized expression of SINR of downlink FUEs in multi-cell is expressed as

$$SINR_{DL\_F} = \frac{P_l^n |h_0|^2 d^{-\beta}}{\Sigma_{c\epsilon C_m} \Sigma_{i=1, i\neq l}^{F} P_{fbs} I_i^n + \Sigma_{c\epsilon C_m} \Sigma_{j=1}^{M} P_{mue} I_j^n + \sigma^2}, \quad (3)$$

where the second summation represents the interference caused by FBSs and MUEs from the neighboring cells and $C_m = \{1, 2, ..., C\}$, is a set that represents the number of neighboring cells. As the users at the edge experience enormous ICI, a power factor (λ) is introduced to minimize the impact of this interference.

In this way, if an MUE is located in the center region, the

[1] It can be noticed that, theoretically, a higher region RFA is possible. However, we will later show that the sum rate saturates after a particular division. Hence, we focus on smaller value of W, i.e., W = 4.

Table II: Frequency Distribution in Modified 4-RFA Method.

B_1				**B_2**			
R_1 Femto UL	R_1 Femto DL	R_3 Femto UL	R_3 Femto DL	R_2 Femto UL	R_2 Femto DL	R_4 Femto UL	R_4 Femto DL
B_{1x}	B_{1y}	B_{1x}	B_{1y}	B_{2x}	B_{2y}	B_{2x}	B_{2y}
R_2 Macro DL	R_2 Macro UL	R_4 Macro DL	R_4 Macro UL	R_1 Macro DL	R_1 Macro UL	R_3 Macro DL	R_3 Macro UL

transmit power is given by P_i, whereas if MUE is in outer region near the edges, the transmit power is $P_o = \lambda P_i$, where $\lambda > 1$.

The average outage probability for a downlink FUE is formulated as

$$\Gamma_{DL} = \quad (SINR_{DL\_F} < \delta), \tag{4}$$

Therefore, the user that is in coverage must have operating SINR greater than the predefined threshold. The achievable sum rate at downlink FUEs can be calculated using the Shannon-Hartley theorem given as

$$\rho_{DL} = \Delta B log_2(1 + SINR_{DL\_F}), \tag{5}$$

where ΔB, is the sub-carrier bandwidth that is allocated to the users in a specific region, $\Delta B = B/N$, where B is the system bandwidth, N is the total number of sub-bands such that $N = 2^M$; $M = \{2, 3, 4, 5\}$ for 2, 4, 8 and 16, RFA respectively.

IV. Modified RFA schemes

The focus of this work is to maximize the sum capacity of downlink FUEs. Although the division of macrocells into higher regions minimizes the number of interfering entities, however, it is accompanied by the reduction of channel bandwidth available to each user (MUE and/or FUE), as a consequence overall throughput of the system is decreased. Thus, in spite of simply dividing the regions and bandwidth, we propose to allocate the frequency in an efficient manner that can provide maximum vulnerability to interference, for gaining desired sum rates along with minimum outage. For instance, we can merge the benefits of both 2-RFA & 4-RFA and devise a scheme known as *modified-4-RFA (M-4-RFA)*. In M-4-RFA scheme, the cell is divided into four regions, just as we do in regular 4-RFA, however, each region now gets co-tier interference from two regions (as opposed to single region in 4-RFA). The femto users of region R_1 receive co-tier interference from the same region and from R_3. Similarly, cross-tier interference from R_2 and R_4. On the other hand, R_2 FUEs experience co-tier interference from R_2 and R_4, whereas cross-tier interference from R_1 and R_3 as explained in Table II. The SINR of DL FUEs for this scheme can be calculated using following expression

$$SINR_{DL\_F} = \frac{P_l^n |h_0|^2 d^{-\beta}}{\Sigma_{c \epsilon C_m} \Sigma_{u \epsilon R_u} \Psi_{fbs} + \Sigma_{c \epsilon C_m} \Sigma_{u \epsilon R_u} \Psi_{mue} + \sigma^2}, \tag{6}$$

where R_u represents the number of regions in each neighboring cell, C_m, causing interference to the intended FUE.

Algorithm 1 Interference calculation algorithm for M-RFA.

Initialization.
A set of F FUEs, where i = {1,....,F}.
A set of FB FBSs, where j = {1,....,FB}.
A set of M MUEs, where m = {1,....,M}
Iteration
`for each FUE f`
FUE association with serving FBS is based on highest received signal power strength.
Calculate the interference from FBSs and MUEs accordingly.
for `i = 1 to F` **do**
  for `j = 1 to FB` **do**
    $s_{ij} \leftarrow$ matrix that contains the interference from all the FBSs of other respective regions
      if i==j **then**
      else
        $e_{ij} \leftarrow$ matrix that contains the interference from the FBSs of the region from which FUE belongs.
        end if
      end for
    end for
    for `m = 1 to M` **do**
      $U_m \leftarrow$ matrix that calculates the interference from the respective region MUEs to the considered FUE.
      end for
      Calculate the SINR according to (6).

$\Psi_{fbs} = \Sigma_{i=1, i \neq l}^{F} P_{fbs} I_i^n$ and $\Psi_{mue} = \Sigma_{j=1}^{M} P_{mue} I_j^n$ symbolize the entities utilizing the same sub-band as the FUE of interest, thus act as interferers. The procedure of calculating the SINR is presented in Algorithm 1. Therefore, by adopting this scheme, the impact of interference is little enhanced as compared to regular 4-RFA (because of additional interfering regions), however, the sub-carriers are divided as in 2-RFA, hence elevating the data rates. The MUEs that share the same sub-band have different powers and also they are moved apart, therefore, the influence caused by the interfering cells is reduced. The ICI when compared to regular 2-RFA is also minimized, as the number of interfering cells in the outer regions of the neighboring cells is reduced. Similar concept can be applied to form M-8-RFA by dividing the cell into 8 regions and utilizing the sub-bands of 4-RFA, alternatively in the regions of each cells. M-16-RFA can also be devised by using sub-carriers of lower RFA schemes (4-RFA or 8-RFA). To evaluate the performance of this hybrid scheme, we define outage capacity, which is formulated as

$$\psi_{\_ot\_DL} = (1 - \Gamma_{DL}) \Delta B log_2(1 + SINR_{DL\_F}), \tag{7}$$

and is used to capture a complete picture of system by taking end-to-end success as well as rates into consideration.

V. Simulation Results

In this section, we present the guidelines of proposed multiple region RFA scheme to design the system and evaluate its performance. The simulation environment comprises of two tiers, hexagonal macrocells in which the circular shaped femtocells are randomly distributed. We assumed closed subscriber group (CSG), in which the MUEs are not allowed to access the services of the respective FBSs. For simplicity, it is assumed that one FUE is associated to each FBS. The path loss exponent, β, for MUE and FUE is taken as 2.5 and 2, respectively. The density of MUEs is supposed to be three times greater than that of FUEs is each region. Unless otherwise mentioned, we assume the simulation parameters that are listed in Table III.

Table III: Simulation Parameters.

Parameters	Values
System Bandwidth	10 MHz
Macrocell radius	2 km
Femtocell radius	50 m
Thermal noise	-174 dBm/Hz
FBS power	23 dBm
FUE power	23 dBm
MBS power	43 dbm
Max. MUE power	23 dbm

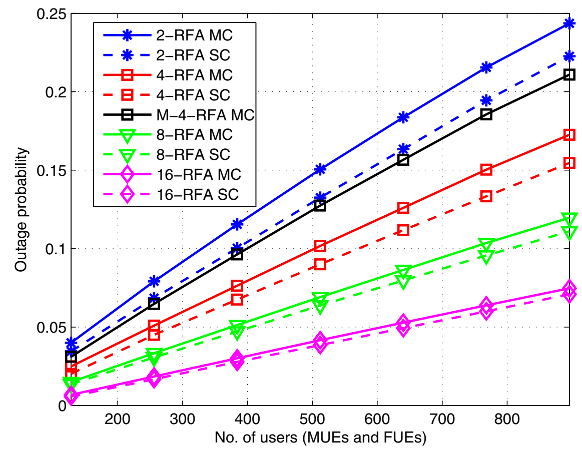

Figure 4: Outage probability of various RFA schemes in single cell and multi-cell scenarios.

Fig. 4 represents the average outage probability of 2-RFA, 4-RFA, M-4-RFA, 8-RFA and 16-RFA, in single cell (SC) and multi-cell (MC) scenarios. The figure shows the effects of cell-edge interference and validates the point that influence of intercell interference in 2-RFA is much larger than the rest of the RFA schemes. Also the hybrid M-4-RFA scheme has a little greater interference than the regular 4-RFA scheme due to enhanced interfering cells. However, it can also be noticed, as we move to higher region RFA, the gap between outage probability of SC and MC reduces because of increased separation among the users, utilizing same sub-band.

The sum capacity of varying number of users in multi-cell is projected in Fig. 5. The gains of proposed scheme in terms of throughput are quite obvious from this figure. It can be observed that the throughput of M-4-RFA scheme is highest among all, as the sources of interference are moved apart, compared to the baseline scheme (2-RFA) that has the highest bandwidth. It is worth mentioning that the proposed modified scheme takes the advantage of 2-RFA and 4-RFA, in terms of bandwidth and partitioning of cells into higher regions, respectively. Moreover, 2-RFA does not provide user fairness, especially to the cell edge users, as the impact of ICI is severe. Whereas, M-4-RFA scheme does not only provide substantial data rate gains over all the other RFA schemes, in addition, it offers considerate higher rates to the cell edge users.

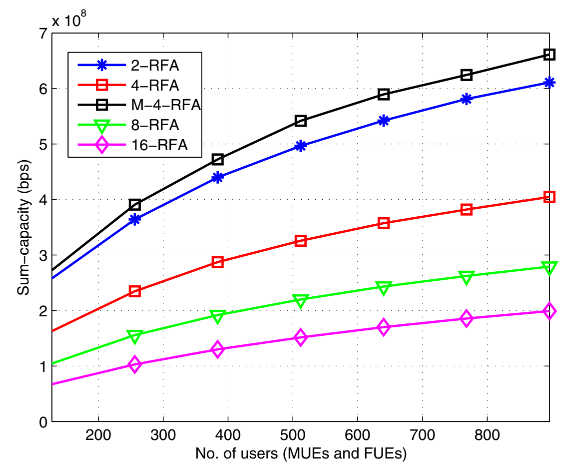

Figure 5: Sum-rate capacity analysis for varying number of downlink FUEs in multi-cell.

In consideration of random channel conditions and varying traffic load, the performance metrics of this two tier system are formulated accordingly. For example, for dense traffic load conditions, it is required to enhance the spectral efficiency of system, however, for the dense networks, the problem of outage is also prominent. Our performance metric is outage capacity that provides user fairness, as indicated in (7). 2-RFA scheme provides channel fairness, as a consequence, the cell edge users do not achieve favorable data rates, whereas M-4-RFA is a scheme that provides fairness to all the users. As depicted in Fig. 6, for 512 users, where the density of MUEs is three times greater than that of FUEs, it can be observed that the M-4-RFA provides 10.5% higher outage capacity as compared to baseline scheme, which validates that at higher user density, hybrid RFA scheme outperforms the 2-RFA in terms of data rates as well as outage probability.

The comparison between various M-RFA schemes is given in Fig. 7. As it is stated previously, by combining the benefits of different RFA schemes, we can gain better rates. We have compared modified 4 and 8 RFA schemes with the conventional 2-RFA scheme. In order to observe the behavior of prominent interference from the neighboring regions, the radius of macrocells is assumed to be 1 Km. Fig. 8 shows that, at a very dense user population, higher region M-RFA schemes show better performance, for example, at user density of 1024, M-8-RFA provides 42.13% gain as compared to 2-RFA and 8.4% higher outage capacity than M-4-RFA. However, this gain is achieved at the cost of higher complexity and challenge of dealing the frequent handoffs. Therefore, we do not need to divide the cell into higher regions, instead M-4-RFA scheme can provide substantial data rates even at a dense number of users. Thus, it can be concluded that, it is not required to opt for higher region modified RFA schemes, as these schemes provide minimal increase in throughput with much higher resource consumption.

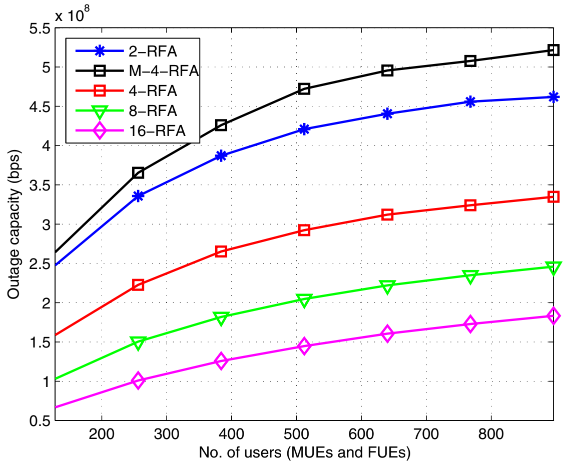

Figure 6: Outage capacity of downlink FUEs in multi-cell.

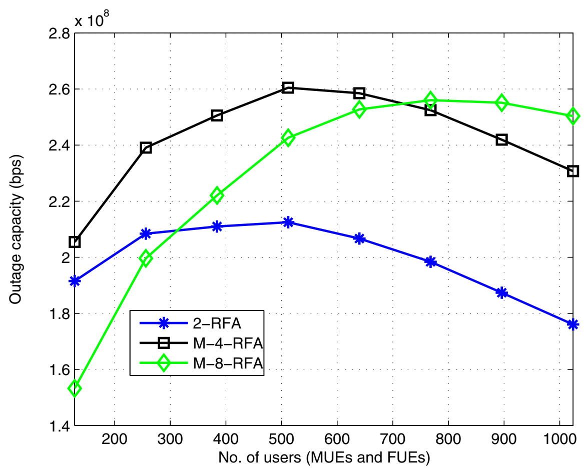

Figure 7: Outage capacity comparison of various modified RFA schemes with 2-RFA scheme in multi-cell.

VI. Conclusion

Deployment of heterogeneous networks has provided a paradigm shift for elevating data rates to the subscribers, however, addressing the sources of interference, causing performance degradation, becomes a major concern. RFA is a resilient technique for the elimination of severe cross-tier interference from MBS to FUEs, as a consequence, a significant increase in the data rates is experienced. We proposed the implementation of a K-region RFA scheme in multi-cellular network to minimize the effect of ICI. Moreover, the performance of the network can be further intensified by combining several RFA schemes into single network to form M-RFA scheme that provides fairness along with improved data rates to the users, in particular, to the cell edge users. It can thus be concluded that M-RFA schemes offer significant margin in outage capacity, which is verified for different user densities as well as by varying the threshold values. However, optimizing the handoff mechanism when users move from one region to another and implementation of sectors within the cell to further reduce the density of interferers is left as future extension.

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References

[1] S.Chen, and J. Zhao, “The requirements, challenges, and technologies for 5G of terrestrial mobile telecommunication,” *IEEE Commun. Mag.*, pp. 36-43, 2014.

[2] J. G. Andrews, “Seven ways that HetNets are a cellular paradigm shift,” *IEEE Commun. Mag.*, vol. 51, no. 3, pp. 136-144, 2013.

[3] Yun Li, H. Celebi, M. Daneshmand, Chonggang Wang, and Weiliang Zhao,“Energy-efficient femtocell networks: challenges and opportunities,” *IEEE Wireless Commun.*, vol. 20, no. 6, pp. 99-105, December 2013.

[4] H. Munir, S. A. Hassan, H. Parveiz, Q. Ni, “A Game Theoretical Network-Assisted User-Centric Design for Resource Allocation in 5G Heterogeneous Networks,” *IEEE Vehicular Technology Conference (VTC-Spring)*, May, 2016.

[5] H. Munir, S. A. Hassan, H. B. Parveiz, Leila Musavian, Qiang Ni, “Energy Efficient Resource Allocation in 5G Hybrid Heterogeneous Networks: A Game Theoretic Approach,” *IEEE Vehicular Technology Conference (VTC-Fall)*, Sep, 2016.

[6] S. H. Waqar, and S. A. Hassan, “Fuzzy logic-based downlink subchannel allocation for capacity maximization in OFDMA femtocells,” *IEEE Wireless Commun. and Mobile Computing Conference (IWCMC)*, pp. 1073-1078, 2015.

[7] M. Z. Chowdhury, Y. M. Jang, and Z. J. Haas, “Cost-effective frequency planning for capacity enhancement of femtocellular networks,” *Wireless Personal Commun.*, vol. 60, Issue 1, pp. 83-104, Sept. 2011.

[8] Simsek, M., M. Bennis, and Czylwik, “Dynamic inter-cell interference coordination in HetNets: A reinforcement learning approach,”*IEEE Global Communications Conference (GLOBECOM)*, pp. 5446-5450, 2012

[9] N. Saquib, E. Hossain, and D. Kim., “Fractional frequency reuse for interference management in LTE-advanced hetnets,” *IEEE Wireless Commun.*, pp. 113-122, 2013.

[10] Abderrazak, J. B., and H. Besbes, “A tailored fractional frequency reuse scheme for Heterogeneous Networks,” *IEEE International Wireless Commun. and Mobile Computing Conference (IWCMC)*, pp. 419-424, 2015.

[11] P. Jacob, A. James, and A. S. Madhukumar, “Downlink interference reduction through Reverse Frequency Allocation,” *Int. Conf. on Commun. Systems (ICCS)*, pp. 329-333, Nov. 2012.

[12] P. Jacob, A. James, and A. S. Madhukumar, “Interference mitigation through reverse frequency allocation in multi-tier cellular network: a downlink perspective,” *Wireless Networks*, pp. 1613-1629, July 2015.

[13] R. Zahid, and S. A. Hassan, , “On the performance of multiple region reverse frequency allocation scheme in a single cell downlink heterogeneous networks,” *IEEE Wireless Commun. and Mobile Computing Conference (IWCMC)*, pp. 387-392, August 2014.