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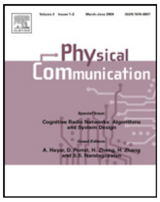

Full length article

On the outage analysis of a D2D network with uniform node distribution in a circular region

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ABSTRACT

This paper studies the coverage probability of a device-to-device (D2D) link between a pair of nodes uniformly distributed in a circular region of arbitrary radius, thereby modeling the D2D network operation for a public safety scenario. The expression for the cumulative distribution function (CDF) of the signal-to-interference ratio (SIR) at the destination is derived, where the transmissions are affected by multipath fading, path loss and interference. The analysis involves finding the ratio distribution of two random variables, representing the desired signal and the intra-region interference, respectively. The expression for CDF helps in ascertaining the outage probability at the destination. Hence, given a desired outage probability, a limit on the number of simultaneously active D2D links in a circular region can be determined, thereby allowing interference avoidance. Numerical simulations are conducted to validate the theoretical model. The Kolmogorov–Smirnov (K–S) test is applied to further characterize the matching between simulation and analytical model. Moreover, the results also help in identifying the minimum transmit power that ensures the desired quality-of-service (QoS), providing efficient transmission in energy constrained environments.

1. Introduction

Fourth generation (4G) communication technology was introduced to accommodate the rising demands of mobile user equipments (UE)s in terms of data rates and network reliability. However, a sharp rise in the number of mobile users has resulted in recent years and is expected to grow exponentially in the years to come, which prompted the need for a futuristic standard which could support a rather complex infrastructure of communication. Moreover, there is a consensus in the research community that in order to realize the concept of future smart cities, new communication technologies are required which provide a resource efficient and reliable connectivity [1]. The concept of 5G technology was introduced as a futuristic solution for applications involving peer-to-peer (P2P) high data rate links. The proposed technologies, which would undergo standardization under the aegis of 5G network development, include heterogeneous networks (HetNets), Machine-to-Machine (M2M) communications, D2D networks and internet of things (IoT) among others.

In D2D communication, the users are able to establish direct links for communication instead of routing transmissions through a base station (BS) [2], allowing energy and resource efficiency due to cellular offloading. D2D networks could be specifically helpful in establishing networks for disaster recovery and public safety networks, utilizing minimum energy and providing reliable connectivity [3]. Several network topologies have been identified in literature for establishing D2D networks, depending upon the role of BS in the transmissions. In the *In-coverage* D2D networks, the BS is responsible for network setup and other signaling activities. On the other hand, in *Out-of-coverage* D2D networks, the D2D users form a cluster and sustain transmissions by forming a cooperative network [4]. Cluster-based D2D networks have gained significant importance as 3rd Generation Partnership Project (3GPP) has envisioned cluster-based D2D networks for public safety [5]. It is envisioned that the communication systems that are developed for future smart cities would be resource efficient, providing a green solution for resource constrained environments. The network based on hand-held devices are energy constrained, highlighting the need for energy efficient communication protocols [6]. Public safety is one of the key aspects that is being researched with regards to smart cities [7]. In situations such as a disaster, the initial few hours are very critical for planning a rescue and rehabilitation

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operation. Due to the destruction of traditional communication infrastructure, the devices need an alternative mechanism to establish communication. The main objective of public safety networks is to transmit initial information such as an SOS message, which helps the government agencies in planning a timely response. The presence of several devices in a close proximity in a smart city makes D2D networks a favorable solution for providing connectivity in case of any unforeseen events such as disasters or security threats.

Out-of-coverage D2D cluster networks could aid the users affected by a safety threat to establish cooperative communication and ensure transmission of information to the government agencies. In this mode, multiple D2D pairs lie in a circular region (CR), and the entire cell is covered using many such regions, as shown in Fig. 1. Each CR may contain a cluster-head that controls various signaling activities [8]. Cluster-based D2D networks allow devices in proximity to enjoy services such as content sharing and social networking [9]. It is important to characterize the impact of interference in D2D public safety networks, as high levels of interference could compromise the end-to-end transmission success [10]. Moreover, the real-time information generated in such scenarios is of sensitive nature, signifying the need for employing an interference avoidance mechanism. The analysis of cluster-based D2D networks strictly depends on the node distribution within an area [11]. Some prior works in literature propose spatial models with fixed distance between D2D pairs [12], however, a more practical approach lies in removing the restriction of fixed D2D distance and assuming a random uniform distribution of nodes [13]. In [14], the authors model the coverage probability and area spectral efficiency (ASE) of a clustered D2D network with random node distribution. The analytical expression of SIR observed between a transmitter–receiver pair in a cluster is derived. The analysis conducted in this work is aimed at identifying the number of simultaneously active D2D transmitters that can ensure content delivery without causing significant interference at the receivers.

In this work, we assume a uniform distribution of nodes in a CR and investigate the coverage probability of a typical D2D pair. It is also assumed that the multiple D2D pairs use the same time-frequency resource block for transmissions, leading to intra-region interference at the destination. Specifically, we derive the expression for the CDF of SIR at the destination node, given that the transmitter and the receiver are uniformly placed in a CR and the transmissions are affected by multipath fading, path loss and intra-region interference. The analysis is divided into three steps. We first derive the expression of the CDF of the received power at the destination node, representing the desired signal. The problem of finding the CDF in this case boils down to finding the distribution of the ratio of two random variables (RVs), i.e., an exponential RV representing the channel fading, and a power function of the Euclidean distance between the D2D pair. Second, this ratio distribution is used to determine the statistics of intra-region interference. Finally, the first two steps of the analysis are used to obtain a closed-form expression of the outage probability at the destination.

The distribution of the ratio of different RVs has been widely studied in the literature, e.g., in econometrics [15], statistics [16] and communications theory. In [17], an ad-hoc single-input-single-output (SISO) network is considered, presenting an analytical model for determining the received power at the destination by finding the ratio of exponential random variable and generalized gamma random variable. Similarly, [18] models a multi-hop network by providing a statistical analysis of the ratio of random variables. The results help in characterizing the impact of co-channel interference and signal fading, which helps in readjusting the network parameters to maintain the desired quality-of-service (QoS). References [19,20] model the outage probability of

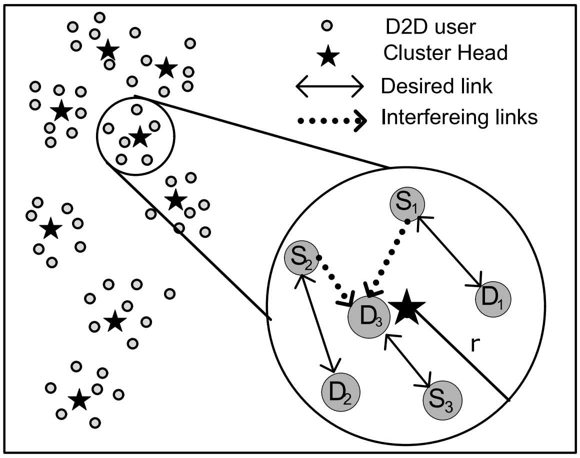

Fig. 1. A realization of D2D network topology with nodes uniformly distributed in a circular region of radius r.

maximum ratio combining (MRC) receiver for multi-hop network environment. Analytical expressions for CDF of sum of ratio of $\alpha-\mu$ RVs are derived that help in obtaining the outage probability of the wireless multi-hop network. In [21], authors derive the ratio of the product of two $k-\mu$ random variables and Nakagami-m random variable for a relay based wireless communication system. However, to the best of our knowledge, the ratio of the specific RVs considered in this paper has not been investigated in the literature before.

The rest of the paper is organized as follows. In Section 2, we discuss our system model. Section 3 presents the mathematical modeling, i.e., the derivation of the distribution of *SIR* at the destination node, which involves finding the ratio distribution of random variables. In Section 4, we validate our analytical model and provide results with regards to network performance. The paper is concluded in Section 5, along with providing some future directions.

2. System model

We consider the case of outband D2D network environment, where the D2D communications take place in the unlicensed spectrum, thereby avoiding interference from cellular users. The proposed model is illustrated in Fig. 1, where several D2D nodes exist in the region [22]. Our analysis is aimed at modeling the transmissions in a CR of radius r, which consists of randomly distributed nodes that form D2D source–destination (S–D) pairs, as shown in Fig. 1. It is assumed that a cluster head (CH) exists at the center of the CR, providing intra-region signaling. The role of the CH is similar to that of the BS, providing synchronization signals and managing D2D user pairing in the absence of any network control from the BS. It is also assumed that the CH possesses the required energy to perform signaling activities in the cluster. Several CH selection techniques have been reported in literature that manage the tradeoff between energy consumption and network throughput to increase network lifetime [23]. Moreover, the rotation of CH helps in distributing the burden of signaling and hence saving energy [24]. The CR can be considered a sub-entity of a larger cluster of D2D nodes distributed in the cell. The objective is to find the outage probability at the destination node given that the transmitter lies in the same CR and transmits with a finite transmit power, P_t. We assume that the system is interference-limited and the simultaneously active D2D transmitters in the CR cause interference at the intended destination. We limit our analysis to finding the expressions of SIR for the case where the

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destination node suffers from the intra-region interference. The modeling of inter-region interference is left as a future direction to this work.

The received power at the D2D destination node is given as

$$P_r = P_t U D^{-\beta}, \tag{1}$$

where U is a unit mean exponential RV representing the squared envelope of a Rayleigh fading channel, and D denotes the Euclidean distance between the intended D2D S–D pair. The β denotes the path loss exponent with a usual range of values between 2 and 4. The received power can be rewritten as $P_r = P_t M$, where $M = \frac{U}{Z}$ and $Z \triangleq D^{\beta}$.

The intra-region interference observed at the destination depends upon p simultaneously active D2D transmitters in the CR and is defined as

$$I_{intra} = \sum_{k=1}^{p} P_t U_k D_k^{-\beta}. \tag{2}$$

Consequently, the SIR at the destination node is given as

$$SIR = \frac{P_t U D^{-\beta}}{I_{intra}} \triangleq \frac{M}{H}. \tag{3}$$

The expression for the distribution of the *SIR* is determined by dividing the analysis into three steps. We first find the ratio distribution M. Using this ratio distribution, we determine the distribution of the sum of p such independent and identically distributed (i.i.d.) ratio RVs, i.e., $H = \sum_{k=1}^{p} M_k$. Finally, we present the closed-form expression for the distribution of *SIR*, which again converges for finding the distribution of the ratio of two RVs namely M and H.

3. Mathematical modeling

The probability density function (PDF) of the Euclidean distance between two nodes uniformly distributed in a circle of radius r is given by [25]

$$f_D(d) = \frac{2d}{r^2}\left(\frac{2}{\pi}\arccos\left(\frac{d}{2r}\right) - \frac{d}{\pi r}\sqrt{1-\frac{d^2}{4r^2}}\right), \tag{4}$$

where $0 \le d \le 2r$. If $Z \triangleq D^{\beta}$, then the PDF of Z is given as

$$\begin{aligned} f_Z(z) &= \frac{1}{\beta} z^{\frac{1}{\beta}-1} f_D(z^{\frac{1}{\beta}}) \\ &= \frac{2z^{\frac{2}{\beta}-1}}{\pi\beta r^2}\left(2\arccos\left(\frac{z^{\frac{1}{\beta}}}{2r}\right) - \frac{z^{\frac{1}{\beta}}}{r}\sqrt{1-\frac{z^{\frac{2}{\beta}}}{4r^2}}\right), \end{aligned} \tag{5}$$

where $0 \le z \le (2r)^{\beta}$. We now proceed to find the CDF of M, which is given in the following theorem.

Theorem 1. *If $M = \frac{U}{Z}$, where U is an exponential RV and the PDF of Z is given by (5), then the CDF of M is given as*

$$F_M(m) = 1 - \frac{2}{\beta r^2}\Gamma\left(\frac{2}{\beta}\right) m^{-2/\beta} + \frac{2}{\pi\beta r^3}\sum_{n=0}^{\infty} \varpi_n m^{-\frac{2n+3}{\beta}}, \tag{6}$$

where $\varpi_n = 2r\gamma_n\Gamma\left((2n+3)/\beta\right) + \sigma_n\kappa_n(m)$, are the series of coefficients that contains the WhittakerM function, $M(\cdot,\cdot,\cdot)$, $a_n = (2n+3)/\beta$ and $\gamma = \left(2r\sqrt{m}\right)^{\beta}$, $\gamma_n \triangleq \frac{\binom{2n}{n}c^{2n+1}}{4^n(2n+1)}$, $\sigma_n = \binom{1/2}{n}\frac{(-1)^n}{4^n r^{2n}}$, such that

$$\kappa_n(m) = \left(\frac{\gamma^{a_n}}{a_n}e^{-\gamma} + \frac{\gamma^{\frac{a_n}{2}}e^{-\frac{\gamma}{2}}}{a_n(a_n+1)} M\left(\frac{a_n}{2}, \frac{a_n+1}{2}, \gamma\right)\right). \tag{7}$$

Proof. The CDF of M can be calculated as

$$\begin{aligned} \mathbb{P}(m \le M) &= \int_0^{\infty} F_U(mz) f_Z(z)dz, \\ &= \int_0^{\infty} f_Z(z)dz - \int_0^{\infty} e^{-mz} f_Z(z)dz . \end{aligned} \tag{8}$$

Note that the first term in (8) is $F_Z(\infty) = 1$, with $F_Z(z)$ denoting the CDF of Z. The second term gives

$$\int_0^{\infty} e^{-mz} f_Z(z)dz = \int_0^{\infty} e^{-mz}\left(f_1(z) - f_2(z)\right)dz, \tag{9}$$

where the functions f_1 and f_2 are given by

$$f_1(z) = \frac{4z^{\frac{2}{\beta}-1}}{\pi\beta r^2}\arccos\left(\frac{z^{\frac{1}{\beta}}}{2r}\right), \quad f_2(z) = \frac{2z^{\frac{3}{\beta}-1}}{\pi\beta r^3}\sqrt{1-\frac{z^{\frac{2}{\beta}}}{4r^2}}. \tag{10}$$

Let us denote with I_1 and I_2 the integrals of f_1 and f_2, respectively. To evaluate the integral I_1 in (9), note that the argument of the function *arccos* is less than unity for all values of z, β and r. Hence, using power series expansion,

$$\arccos(c) = \frac{\pi}{2} - \sum_{n=0}^{\infty}\frac{\binom{2n}{n}c^{2n+1}}{4^n(2n+1)}, \tag{11}$$

for $|c| < 1$, yields

$$\begin{aligned} I_1 = {} & \frac{2}{\beta r^2}\int_0^{\infty} e^{-mz} z^{\frac{2}{\beta}-1}dz - \frac{4}{\pi\beta r^2} \\ & \times \sum_{n=0}^{\infty}\gamma_n\left(\int_0^{\infty} e^{-mz} z^{\frac{2n+3}{\beta}-1}\right)dz, \end{aligned} \tag{12}$$

where

$$\gamma_n \triangleq \frac{\binom{2n}{n}c^{2n+1}}{4^n(2n+1)}. \tag{13}$$

Using the definition of Euler gamma function, i.e., $\Gamma(\mu) = \int_0^{\infty} x^{\mu-1}e^{-x}dx$ and introducing the change of variable $mz = x$ in (12), which restricts x such that $|x| \le m(2r)^{\beta}$, we obtain

$$\begin{aligned} I_1 = {} & \frac{2}{\beta r^2} m^{-2/\beta}\Gamma\left(\frac{2}{\beta}\right) - \frac{4}{\pi\beta r^2}\sum_{n=0}^{\infty}\gamma_n m^{-\frac{2n+3}{\beta}} \\ & \times \Gamma\left(\frac{2n+3}{\beta}\right). \end{aligned} \tag{14}$$

Similarly, the second integral I_2 in (9) is given by

$$I_2 = \frac{2}{\pi\beta r^3}\int_0^{\infty} e^{-mz} z^{\frac{3}{\beta}-1}\sqrt{1-\frac{z^{\frac{2}{\beta}}}{4r^2}}\,dz. \tag{15}$$

Since $|z^{2/\beta}| < 4r^2$, we can expand the square-root function into a binomial series expansion, which also brings the above integral in terms of a gamma function such that

$$\begin{aligned} \sqrt{1-\frac{z^{\frac{2}{\beta}}}{4r^2}} &= \sum_{n=0}^{\infty}\binom{1/2}{n}(-1)^n\frac{z^{\frac{2n}{\beta}}}{4^n r^{2n}} \\ &= \sum_{n=0}^{\infty}\sigma_n z^{\frac{2n}{\beta}}, \end{aligned} \tag{16}$$

where σ_n are the coefficients given by,

$$\sigma_n = \binom{1/2}{n}\frac{(-1)^n}{4^n r^{2n}}. \tag{17}$$

Thus

$$I_2 = \frac{2}{\pi\beta r^3}\sum_{n=0}^{\infty}\sigma_n\int_0^{\infty} e^{-mz} z^{\frac{2n+3}{\beta}-1}dz. \tag{18}$$

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Applying the same transformation $mz = x$, I_2 yields

$$I_2 = \frac{2}{\pi \beta r^3} \sum_{n=0}^{\infty} \sigma_n m^{-\frac{2n+3}{\beta}} \int_0^{\left(2r\sqrt{m}\right)^{\beta}} e^{-x} x^{\frac{2n+3}{\beta}-1} dx. \tag{19}$$

Rearranging the terms,

$$\int_0^{\left(2r\sqrt{m}\right)^{\beta}} e^{-x} x^{\frac{2n+3}{\beta}-1} dx \tag{20}$$

$$= \frac{\gamma^{a_n}}{a_n} e^{-\gamma} + \frac{\gamma^{\frac{a_n}{2}} e^{-\frac{\gamma}{2}}}{(a_n)(a_n+1)} M(\gamma) \left(\frac{a_n}{2}, \frac{a_n+1}{2}, \gamma \right), \tag{21}$$

where $a_n = (2n+3)/\beta$, $\gamma = \left(2r\sqrt{m}\right)^{\beta}$ and $M(\cdot, \cdot, \cdot)$ is the WhittakerM function. The final form of the CDF of M is given by $F_M(m) = 1 - I_1 + I_2$, which results in (6) after simple algebraic manipulations. □

Note that $\kappa_n(m)$ in ϖ_n, is a function of m, which is a direct consequence of $|x| \leq m(2r)^{\beta}$ in (19). A relatively simplified form of Theorem 1 can be found by relaxing the restriction on x, i.e., $|x| \leq m(2r)^{\beta}$ in which the coefficients in the series expansion are no longer function of m. For that, we consider the following.

Corollary 1. *For $|x| > m(2r)^{\beta}$, the CDF of M is given by*

$$F_M(m) = 1 - \frac{2}{\beta r^2} \Gamma\left(\frac{2}{\beta}\right) m^{-2/\beta} + \frac{2}{\pi \beta r^3} \sum_{n=0}^{\infty} \tilde{\varpi}_n m^{-\frac{2n+3}{\beta}}, \tag{22}$$

where $\tilde{\varpi}_n = (2r\gamma_n \Gamma((2n+3)/\beta) + \sigma_n)$.

Remark 1. Note that the outage probability of a S–D link operating in the absence of intra-region interference can be found as $P_o = F_M\left(\frac{\tau}{P_t}\right)$, where τ is the modulation-dependent threshold.

The next step is to find the PDF of H in (3), which involves the self-convolution of p i.i.d. RVs M_k, $k = \{1, 2, \ldots, p\}$. For simplicity, we drop the subscript k and the PDF of a single RV M is found by computing the derivative of $F_M(m)$ in (22), with respect to m, which we denote by $f_M(m)$, i.e.,

$$f_M(m) = \mathcal{F}_{r,\beta} m^{-\frac{2}{\beta}-1} - \sum_{n=0}^{\infty} \mathcal{F}_{n,r,\beta} m^{-\frac{2n+3}{\beta}-1}, \tag{23}$$

where $\mathcal{F}_{r,\beta}$ and $\mathcal{F}_{n,r,\beta}$ are redefined coefficients of (22). The self-convolution of the distribution of M is reduced to their product in the frequency domain, i.e.,

$$\mathcal{L}\left[*_{k=1}^{p} f_{M_k}(m_k) \right] = \prod_{k=1}^{p} F_k(s) = [F(s)]^p, \tag{24}$$

where, $*_{k=1}^{p}$ denotes the p-times self-convolution of $f_M(m)$ in (22) and $\mathcal{L}(\cdot)$ denotes the Laplace operator. Since $m > 0$, $f_M(m)$ is a bounded function and there exists an m_0 in its domain, such that $|f_M(m)| < f_M(m_0)$. It is straightforward to verify that the exponent functions in $f_M(m)$ satisfy the exponential order property, i.e., there exist positive numbers $k_1, k_2 > 0$, such that

$$f_M(m) \leq k_1 e^{k_2 m}, \forall\, m > 0. \tag{25}$$

Furthermore, $f_M(m)$ is absolutely integrable, i.e.,

$$\int_0^{\infty} |f_M(m)| dm < \infty. \tag{26}$$

Consequently, the Laplace transform, $F(s)$, can be determined as

$$F(s) = \tilde{\mathcal{F}}_{r,\beta} s^{\frac{2}{\beta}} - \sum_{n=0}^{\infty} \tilde{\mathcal{F}}_{n,r,\beta} s^{\frac{2n+3}{\beta}}, \tag{27}$$

where $\tilde{\mathcal{F}}_{r,\beta}$ and $\tilde{\mathcal{F}}_{n,r,\beta}$ are new coefficients. The above equation can also be expressed as

$$F(s) = \tilde{\mathcal{F}}_{r,\beta} s^{\frac{2}{\beta}} \left(1 - \sum_{n=0}^{\infty} \frac{\tilde{\mathcal{F}}_{n,r,\beta}}{\tilde{\mathcal{F}}_{r,\beta}} s^{\frac{2n+1}{\beta}} \right). \tag{28}$$

We now find the product of all such functions, i.e., $l_F(s) = F(s)^p$, which results into

$$l_F(s) = \tilde{\mathcal{F}}_{r,\beta} s^{\frac{2p}{\beta}} \left(1 - \sum_{n=0}^{\infty} \frac{\tilde{\mathcal{F}}_{n,r,\beta}}{\tilde{\mathcal{F}}_{r,\beta}} s^{\frac{2n+1}{\beta}} \right)^p. \tag{29}$$

The required PDF, $f_H(h)$, is derived by finding the inverse Laplace transform. Hence, we arrive at the following result, which can be proved using pth multiplication of power series and through various algebraic manipulations.

Lemma 1. *The required PDF for the intra-region interference is given by*

$$\mathcal{L}^{-1}(l_F(s)) = \tilde{\tilde{\mathcal{F}}}_{r,\beta} h^{-\frac{2p}{\beta}-1} + \sum_{n=0}^{\infty} \left(h^{-\frac{1}{\beta}} \tilde{\tilde{\tilde{\mathcal{F}}}}_{n,r,\beta} - \tilde{\tilde{\mathcal{F}}}_{n,r,\beta} \right) \times h^{-\frac{2(n+p)}{\beta}-1}, \tag{30}$$

where the coefficients are obtained in an algorithmic way using

$$\tilde{\tilde{\mathcal{F}}}_{r,\beta} = \frac{\tilde{\mathcal{F}}^{p}_{r,\beta}}{\Gamma\left(-\frac{2p}{\beta}\right)},$$

$$\tilde{\tilde{\mathcal{F}}}_{n,r,\beta} = \frac{p \tilde{\mathcal{F}}^{p-1}_{r,\beta} \tilde{\mathcal{F}}_{n,r,\beta}}{\Gamma\left(\frac{-2(n+p)+1}{\beta}\right) \Gamma\left(\frac{2p}{\beta}\right)},$$

$$\tilde{\tilde{\tilde{\mathcal{F}}}}_{n,r,\beta} = \frac{\sum_{i=0}^{n} \tilde{\mathcal{F}}_i \tilde{\mathcal{F}}_{n-i}}{\Gamma\left(-\frac{2n+5}{\beta}\right)},$$

$$\tilde{\mathcal{F}}_{r,\beta} = \frac{-2\pi\beta}{\beta^2 r^2 \sin(2\pi/\beta)},$$

$$\tilde{\mathcal{F}}_{n,r,\beta} = \frac{-2(2r\gamma_n + \sigma_n)}{\beta r^3 \sin(2\pi(2n+3)/\beta)},$$

in which we have employed the identity $\Gamma(x)\Gamma(-x) = \frac{-\pi}{x \sin(\pi x)}$.

After deriving the PDF of H, we derive the expression for the CDF of *SIR*. The problem again converges to finding a ratio of RVs; the received power at the destination node with distribution $F_M(m)$ given in (22), and the intra-region interference I_{intra} represented by random variable H with distribution given in (30). The CDF of *SIR*, G, can be calculated from an improper integral

$$\mathbb{P}(g \leq G) = \lim_{t \to c} \int_t^{\infty} F_M(gh) f_H(h) dh, \tag{31}$$

where c is an arbitrarily number chosen close to 0. After carefully taking the product in the integrand, we can evaluate the above integral and obtain the following result.

Lemma 2. *The CDF of SIR at the destination node is given by*

$$F_G(g) = \mathcal{J}_1 - \mathcal{J}_2 + \mathcal{J}_3, \tag{32}$$

where the three terms are given by

$$\mathcal{J}_1 = \frac{\beta\, \tilde{\tilde{\mathcal{F}}}_{r,\beta}\, c^{-\frac{4}{\beta}}}{4} + \beta \sum_{n=0}^{\infty} \left(\tilde{\tilde{\tilde{\mathcal{F}}}}_{n,r,\beta} c^{-1/\beta} - \tilde{\tilde{\mathcal{F}}}_{n,r,\beta} \right) \frac{c^{-\frac{2n+5}{\beta}}}{2n+5},$$

$$\mathcal{J}_2 = \frac{2g^{-\frac{2}{\beta}}}{r^2} \Gamma\left(\frac{2}{\beta}\right) \left(\frac{\tilde{\tilde{\mathcal{F}}}_{r,\beta}\, c^{-6/\beta}}{6} + \sum_{n=0}^{\infty} c^{-\frac{2n+8}{\beta}} \right.$$

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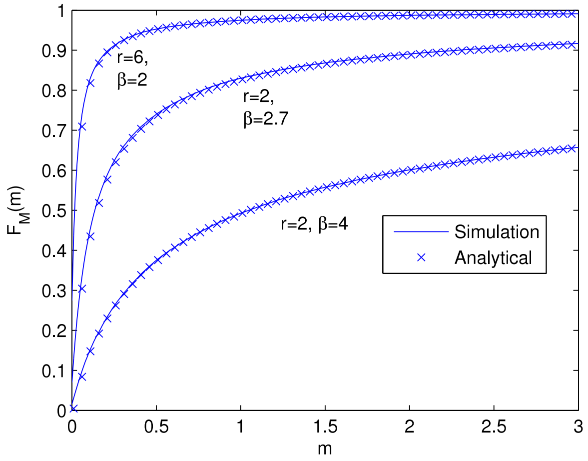

Fig. 2. CDF of received power using the analytical and simulation models.

$$
\times\left(\frac{\tilde{\tilde{\tilde{\mathcal{F}}}}_{n,r,\beta}}{2n+8}-\frac{c\tilde{\tilde{\tilde{F}}}_{n,r,\beta}}{2n+7}\right)\Bigg)\Bigg),
$$

$$
\mathcal{J}_3=\frac{2b}{\pi\beta r^3}\sum_{n=0}^{\infty}(2r\gamma_n+\sigma_n)\Gamma\left(\frac{2n+3}{\beta}\right)g^{-\frac{2n+3}{\beta}}\times
$$

$$
\left(\frac{c^{-\frac{2n+7}{\beta}}\tilde{\tilde{\mathcal{F}}}_{r,\beta}}{2n+7}+\frac{c^{-\frac{4n+9}{\beta}}\tilde{\tilde{\tilde{\mathcal{F}}}}_{n,r,\beta}}{4n+9}-\frac{c^{-\frac{4n+8}{\beta}}\tilde{\tilde{\mathcal{F}}}_{n,r,\beta}}{4n+8}\right). \tag{33}
$$

Thus, the outage probability of the node suffering from intra-region interference is given as $F_G(\tau)$.

4. Simulation results

In this section, we verify the derived analytical results by comparing them with network simulations. First, the analytical expression for the CDF derived in (6) is validated by comparing the analytical results with Monte-Carlo simulations, as shown in Fig. 2. In the simulation setup, a pair of nodes is randomly placed in a circle of radius r and the received power at the destination is calculated. The destination node is able to decode the message if the received power is greater than a predefined threshold τ. The process is repeated n times and the independent samples of the received power are represented by $X_1, \ldots, X_n$. The empirical cumulative distribution function (ECDF) is calculated as

$$
F_1(m)=\frac{1}{n}\sum_{i=1}^{n}\mathbb{I}_{(-\infty,m]}(X_i), \tag{34}
$$

where $\mathbb{I}$ is the indicator function which is 1 if $X_i \le m$, and 0 otherwise. In order to further characterize the results, we apply the Kolmogorov–Smirnov (K–S) test. The hypothesized CDF, found using (6), is denoted by $F_2(m)$. The null hypothesis is defined as,

$$
H_o : F_1(m)=F_2(m), \tag{35}
$$

which determines if both samples come from the population with the same distribution. The goodness-of-fit is determined using K–S statistic,

$$
K_{1,2}=\sup_{m}\left|F_1(m)-F_2(m)\right|, \tag{36}
$$

signifying the maximum distance between the ECDF and hypothesized CDF, while *sup* denotes the supremum operator. The K–S test is performed for a significance level α, which is defined as the probability of rejecting the null hypothesis that is given by,

$$
\alpha \triangleq \mathbb{P}(K_{1,2}\ge v|H), \tag{37}
$$

where v denotes the critical value, which is dependent on α and the sample size. In our work, the K–S test is performed at $\alpha = 0.05$ where the critical value $v = 0.078$. The results are presented in Table 1 with P denoting the asymptotic value. It can be observed that the K–S test does not reject the null hypothesis H_o, as $K_{1,2} \ge v$ for all cases.

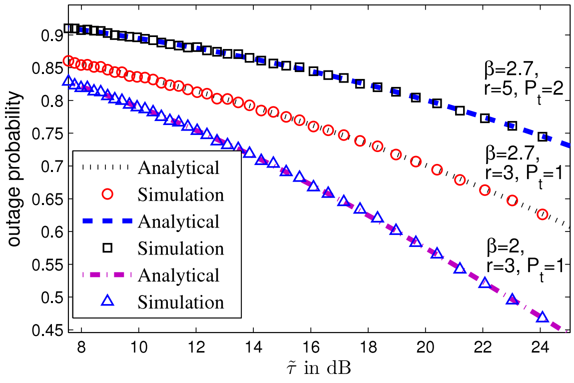

Fig. 3. Threshold $\tilde{\tau}$ versus outage probability of S–D link with no interferers.

Table 1
Kolmogorov–Smirnov test.

Network parameters	P	$K_{1,2}$
$r = 6, \beta = 2$	0.9957	0.0333
$r = 2, \beta = 2.7$	1	0.0164
$r = 2, \beta = 4$	0.8925	0.0467

For the representation of the results, we introduce a normalized threshold $\tilde{\tau} = 10\log(1/\tau)$. First, we consider the case of a D2D link operating in the absence of intra-region interference. The outage probability of a D2D link, for different values of the CR radius r, the transmission power P_t, and the path loss exponent β is plotted in Fig. 3. The outage probability is calculated using both the analytical expression and the Monte Carlo simulations, with the results indicating perfect matching between the two. Moreover, it can be observed that an increase in the path loss exponent or radius increases the outage probability, which is obvious.

Similarly, Fig. 4 shows the outage probability for a destination node in the presence of intra-region interference. The results are generated for path loss exponent of $\beta = 2.7$, modeling the urban network environment. The analytical results are found using (32) and compared with the simulation results for different number of interfering nodes. It can be observed that increasing the number of interferers escalates the outage probability at the destination. Hence, given a desired outage probability, a limit on the number of simultaneously active D2D links in a CR can be determined.

5. Conclusion

In this paper, D2D network environment is considered in which a pair of nodes is uniformly distributed in a circular region of arbitrary radius. Multiple D2D pairs utilize the same time-frequency resource block for transmission, which leads to intra-region interference at the destination. The performance of the network is characterized by deriving an analytical expression for the CDF of the SIR at the destination, where the transmissions are affected by multi-path fading, path loss and intra-region interference. The problem of finding the CDF involves finding the ratio distribution to two

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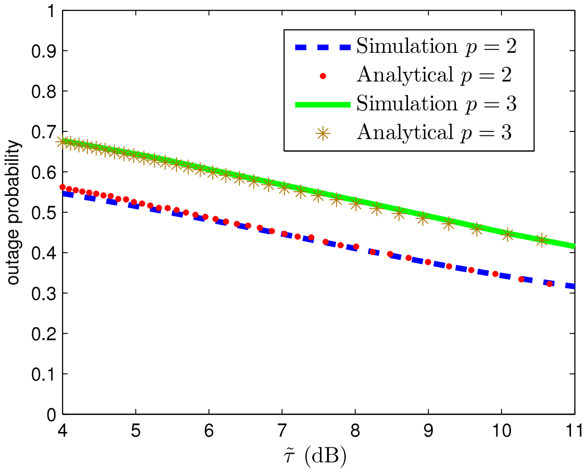

Fig. 4. Impact of interference on outage probability for $\beta = 2.7, r = 5$.

random variables, representing the desired signal and the intra-region interference, respectively. The results are validated using Monte Carlo simulations at different network parameters. The K–S test is applied to further characterize the matching between simulation and analytical model. The analytical results presented in this work could be particularly helpful in interference avoidance for public safety networks in future smart cities. Moreover, the results could also help in determining the minimum required transmit power which provides the desired QoS. The derivation of analytical expressions for modeling inter-region interference could form a future direction of this work. Moreover, an expansion of the analysis to multi-hop D2D network and deriving an analytical model would be interesting to study.

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