Successive Bandwidth Division NOMA Systems: Uplink Power Allocation with Proportional Fairness

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***Abstract*—Non-orthogonal multiple access (NOMA) is considered as a promising candidate for fifth generation (5G) wireless networks. Although, NOMA promises large data rates, however, it also offers significant interference especially when the number of users is large. Therefore, in this paper, we propose a low complexity orthogonal frequency division multiple access (OFDMA)-based NOMA system, which uses the concept of successive bandwidth division (SBD) that not only reduces the complexity of the receiver, but also enhances the overall signal-to-interference plus noise ratio (SINR) of the uplink NOMA by supporting $2N$ users with just N base station (BS) antennas. Power allocation is being performed in SBD-NOMA to maximize the sum rate using a divide-and-allocate approach such that all users are allocated with an optimal transmission power. Simulations results are provided to access and compare the performance of the proposed scheme with other contemporary approaches.**

***Index Terms*—Non-orthogonal multiple access, power allocation, sum rate maximization, fairness, successive interference cancellation (SIC), zero-forcing (ZF) receiver.**

I. INTRODUCTION

ONE of the biggest challenges of future mobile networks is the demand for high data rates, which has been increasing exponentially from the last two decades. To cope with this, researchers are exploring a multitude of techniques to be used in future cellular networks including device-to-device (D2D) communication, ultradensification, millimeter wave (mmWave) communication, massive multiple-input multiple-output (MIMO) and novel multiple access schemes under the umbrella of fifth generation (5G) networks [1].

Traditionally, the multiple access schemes have been classified into orthogonal and non-orthogonal on the basis of resource allocation to the users [3]. Orthogonal multiple access (OMA) schemes such as frequency division multiple access (FDMA), time division multiple access (TDMA), code division multiple access (CDMA) and orthogonal frequency division multiple access (OFDMA) have no cross interference between the users and hence use simple single user detection techniques, however, they achieve low data rates and often suffer from frequency offset, thus failing to achieve the system upper bound i.e., the capacity of the Gaussian broadcast channels. Non-orthogonal multiple access (NOMA), on the other hand, is considered as a promising enabling candidate for 5G wireless networks because of its higher spectral efficiency as compared to its orthogonal counterparts. NOMA offers high data rates and better throughput by simultaneously allowing multiple users cohabitation in the power domain but it requires a rather complex multi-user detection (MUD) at the receiver side. However, by simultaneously allowing multiple users at same frequency/time/code, it achieves the capacity of the Gaussian broadcast channels [4], [6].

So far, resource allocation has been studied extensively in the literature for OMA techniques and have shown promising results. For instance, the authors in [14] considered dynamic resource allocation schemes for sum rate maximization in OFDMA. The energy efficiency maximization of OFDMA system with bit error rate (BER) and data rate constraint is considered in [5]. However, such strategies are in its early stage for a NOMA-based system. System level performance of an OFDM-based NOMA system with fractional transmit power allocation (FTPA) and equal transmit power allocation is considered in [10]. In [9], the author has presented different power allocation schemes for NOMA considering FTPA, full search power allocation (FSPA) and fixed power allocation (FPA). The FSPA performs well but it has high computational complexity. FPA has worse performance but least complexity. FTPA is a balance between the two but it does not distribute power among the multiplexed users in an optimum way to maximize sum rate. However, none of these works discuss power allocation for an uplink OFDMA-based NOMA system. Some other existing works on power allocation in NOMA are implemented in [7], [8], [11]. The combination of NOMA with mmWave and OFDMA is proposed in [12], [13].

Motivated by these ideas, for the uplink of an OFDMA-based NOMA system, we propose a suboptimal algorithm known as successive bandwidth division (SBD)-NOMA, to enhance the spectral efficiency of the existing OFDMA-based NOMA system. By orthogonally dividing the available spectrum into several identical sub-band, power allocation can be done in an optimal way to maximize the overall system utility with fairness. This is because the choice of the power allocation coefficients is a key to achieve a favorable throughput-fairness tradeoff in NOMA systems. The main contributions of our proposed algorithm are (1) to divide the users among identical orthogonal sub-bands, (2) power allocation among the paired users to maximize the sum rate of their sub-band, (3) to maximize the overall sum rate of the system by allocating equal power across all sub-bands and to compare it with other contemporary schemes without power allocation, (4) to ensure fairness in the system by imposing an upper limit on each user's individual data rate, and (5) to reduce the receiver complexity to a great extent. We derive the closed-form solutions, which are used in calculating the optimal power for SBD-NOMA with P users in each sub-band. The proposed scheme works together with the uplink power control scheme used for multi-user MIMO transmission.

The rest of this paper is organized as follows. Section II presents the system model of uplink SBD-NOMA scheme with multiple antennas. In Section III, we formulate the sum rate maximization problem and discuss the optimal solutions. Simulation results and discussions are provided in Section IV, whereas the concluding remarks are drawn in Section V.

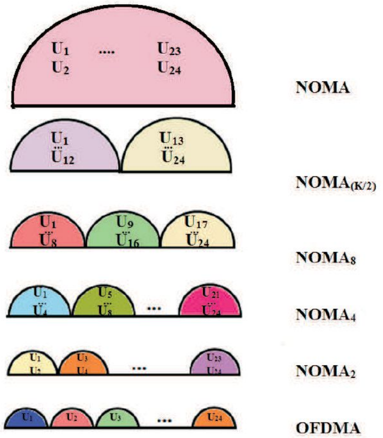

Fig. 1. OFDMA vs. NOMA with proposed SBD scheme, K=24 for this particular example.

II. System Model

Consider the uplink of a multi-user MIMO system, where a base station (BS), equipped with N antennas, communicates with K users, where $K \geq 2N$. The transmission occurs in the presence of additive white Gaussian noise (AWGN) over Rayleigh flat fading channels with path loss. For an OFDMA system with multiple antennas at the BS, the entire available spectrum is divided into K sub-bands and all K users can be supported simultaneously without any interference. For the proposed NOMA system, let ξ denotes the set of all K users, where the transmit power of any k^{th} user is denoted by $\alpha_{i,k} > 0$. Here $i \in \{1, 2\}$ represents the identity of strong and weak users, respectively while k signifies the user index. As multiple users are admitted simultaneously at the same frequency, the interference offered to them by other users within the allocated band becomes high, which leads to high outage probability. Therefore, to efficiently manage the intra cell interference in NOMA-based networks, the proposed technique works as follows. The available spectrum of bandwidth BW is divided into K/P identical orthogonal sub-bands, each having a bandwidth of BW_{sb} and the phenomena is termed as SBD-NOMA. Here P is the maximum number of users that can share a sub-band and $P \in \mathbb{N}$, where $\mathbb{N}$ is a set of natural numbers. Note that when $P = K$, the SBD scheme specializes to conventional NOMA and when $P = 1$, the SBD scheme specializes to conventional OFDMA. The superposition of relatively less number of users in each sub-band (than conventional NOMA) reduces the receiver complexity to a great extent and optimum MUD techniques with moderate complexity can be applied at the receiver side. The complexity of the proposed optimum MUD scheme will be $O(|\chi|^P)$, which is significantly less than the complexity of the conventional NOMA with K users i.e., $O(|\chi|^K)$, where χ denotes the constellation alphabet. However, dividing the entire bandwidth into sub-bands cannot exploit the full potential of NOMA, where all users can occupy the entire bandwidth.

To further reduce the inter set interference, the P users paired within the same sub-band are chosen from two distinct sets A and B. The sets A, $B \subseteq \xi$ such that $A \cup B = \xi$ and $A \cap B = \phi$. The sets A and B are defined on the basis of the channel gains that the users experience. The users with relatively high channel gains are considered as strong and are members of set A while the ones with relatively weak channel gains are considered to constitute the weak set, B. The number of users in the two sets A and B are assumed to be equal and even.

The conceptual block diagram of the proposed SBD-NOMA is depicted in Fig. 1. Suppose that the number of users is $K = 24$. For $P = 2$, the access scheme is NOMA$_2$, where the number of users in each sub-band is 2 while the total sub-bands are 12. The number P divides the strong and weak sets A and B into K/P smaller sets such that $\sum_{i=1}^{K/P} A_i \cup B_i = \xi$. Each A_i, B_i from the sets A and B has a cardinality of 1 for $P = 2$ while there is a single interferer in this case. Similarly, for $P = 4$, SBD-NOMA has 4 users in each sub-band. Each A_i, B_i from the sets A and B has cardinality of 2, whereas the number of interferers is 3 in this case. $P = K$ specializes to the case of conventional uplink NOMA, with only one sub-band and $(K - 1)$ interferers. In other words, we define NOMA$_P$ to be the access scheme where K/P sub-bands are formed with P users in each sub-band and a conventional NOMA scheme works in each of sub-band.

A. Received Signal Model

Considering P users are multiplexed on the m^{th} sub-band, where $1 \leq m \leq K/P$, the superimposed signal received at the BS from P users is given by

$$\mathbf{y} = \sum_{i=1}^{2} \mathbf{H}_i \mathbf{s}_i + \mathbf{n}, \quad (1)$$

where $\mathbf{y}$ is an $(N \times 1)$ uplink received signal vector, $\mathbf{H}_1$ and $\mathbf{H}_2$ are $(N \times P/2)$ channel matrices of strong and weak sets, respectively, and $\mathbf{n}$ is an $(N \times 1)$ AWGN vector with zero mean and unit variance. The channel matrices $\mathbf{H}_1$, $\mathbf{H}_2$ of strong and weak sets are given by

$$\mathbf{H}_i = \begin{bmatrix} \mathbf{h}_{i,1} & \mathbf{h}_{i,2} & \dots & \mathbf{h}_{i,P/2} \end{bmatrix}, \quad (2)$$

where $i \in \{1, 2\}$. The $\mathbf{h}_{i,j}$ is the $(N \times 1)$ uplink channel vector of the j^{th} user and is drawn from the complex Gaussian distribution with zero mean and unit variance. It is chosen from the set J, where $J = \{1, 2, ..., P/2\}$. The transmitted symbols from each user can be given as

$$\mathbf{s}_1 = \left[\sqrt{\alpha_{1,1}}x_{1,1} \quad \cdots \quad \sqrt{\alpha_{1,P/2}}x_{1,P/2}\right]^{\mathsf{T}}, \tag{3}$$

$$\mathbf{s}_2 = \left[\sqrt{\alpha_{2,1}}x_{2,1} \quad \cdots \quad \sqrt{\alpha_{2,P/2}}x_{2,P/2}\right]^{\mathsf{T}}, \tag{4}$$

where $(\cdot)^{\mathsf{T}}$ denotes the transpose operator, $\alpha_{i,j}$ represents the NOMA power allocation coefficient of strong and weak users and $x_{i,j}$ represents the symbol transmitted by the j^{th} user to the BS with N antennas. The symbols $\mathbf{s}_1$ and $\mathbf{s}_2$ represent the $(P/2 \times 1)$ signal vector of strong and weak sets, respectively. The total number of signals received by the BS is K/P and it depends on the number of users within a particular sub-band.

Since the BS receives a superimposed signal, the signals of the strong set users are decoded first, they are corrupted with the interference of weak set users while the users in the weak set are decoded without interference after successive interference cancellation (SIC). As there are only P users in each sub-band hence, the interference comes only from the users with relatively weak channel gains. The remaining users are orthogonal and offer no inter set interference.

Assuming perfect channel state information (CSI), a zero-forcing (ZF) postcoded or detection matrix is used to decode the signals of strong and weak sets. The corresponding postcoded matrices $\mathbf{Z}_i$ are given by

$$\mathbf{Z}_i = \left[\mathbf{z}_{i,1}^{\mathsf{T}} \quad \mathbf{z}_{i,2}^{\mathsf{T}} \quad \cdots \quad \mathbf{z}_{i,P/2}^{\mathsf{T}}\right]^{\mathsf{T}} = (\mathbf{H}_i)^*((\mathbf{H}_i)(\mathbf{H}_i)^*)^{-1}, \tag{5}$$

where $(.)^{-1}$ and $(.)^*$ denotes the inverse and complex conjugate operators. In the above equation, $\mathbf{Z}_1$ is the $(P/2 \times N)$ postcoded matrix of users in the strong set and $\mathbf{z}_{i,j}$ is the $(1 \times N)$ vector corresponding to the uplink channel vector of the j^{th} user, respectively. After applying the postcoded matrix $\mathbf{Z}_1$ to decode the users of strong set, the received signal becomes

$$\mathbf{r}^{(1)} = \mathbf{Z}_1\mathbf{y}, \tag{6}$$

where $\mathbf{r}^{(1)}$ is a $(P/2 \times 1)$ received signal vector. The corresponding SINR of an n^{th} strong user, chosen from the set J, where $n \in J$ is given by

$$SINR_{1,n} = \frac{||\mathbf{z}_{1,n} \cdot \mathbf{h}_{1,n}||^2\alpha_{1,n}}{I + \sigma^2}, \tag{7}$$

where I represents the interference and is given by

$$I = \sum_{j=1, j\neq n}^{P/2} ||\mathbf{z}_{1,n} \cdot \mathbf{h}_{1,j}||^2\alpha_{1,j} + \sum_{j=1}^{P/2} ||\mathbf{z}_{1,n} \cdot \mathbf{h}_{2,j}||^2\alpha_{2,j} \tag{8}$$

where $||.||$ denotes the modulus and $(\mathbf{a} \cdot \mathbf{b})$ represents the point to point multiplication between any two vectors $\mathbf{a}$ and $\mathbf{b}$. The $||\mathbf{z}_{1,n} \cdot \mathbf{h}_{1,n}||^2\alpha_{1,n}$ represents the desired signal of n^{th} strong user from the set J while the signal $\sum_{j=1,j\neq n}^{P/2} ||\mathbf{z}_{1,n} \cdot \mathbf{h}_{1,j}||^2\alpha_{1,j} + \sum_{j=1}^{P/2} ||\mathbf{z}_{1,n} \cdot \mathbf{h}_{2,j}||^2\alpha_{2,j}$ represents the intra set interference from the remaining strong and weak users. However, if $P/2 = N$, i.e., the half of the number of users in any sub-band in SBD-NOMA is equal to the number of BS antennas, the resultant channel matrices formed at the BS become square, which removes the intra set interference coming from the other strong and weak users.

After applying the postcoded matrix to weak users, the corresponding received signal and SINR of n^{th} weak user, chosen from the set J, where $J = \{1, 2, ..., P/2\}$ becomes

$$\mathbf{r}^{(2)} = \mathbf{Z}_2\mathbf{H}_2\mathbf{s}_2 + \mathbf{n}, \tag{9}$$

$$SINR_{2,n} = \frac{||\mathbf{z}_{2,n} \cdot \mathbf{h}_{2,n}||^2\alpha_{2,n}}{\sum_{j=1,j\neq n}^{P/2} ||\mathbf{z}_{2,n} \cdot \mathbf{h}_{2,j}||^2\alpha_{2,j} + \sigma^2}. \tag{10}$$

The rate of strong and weak set in SBD-NOMA across any sub-band is given by

$$R_i = BW_{sb}\sum_{j=1}^{P/2} log_2\left(1 + SINR_{i,j}\right); \quad i \in \{1, 2\}. \tag{11}$$

The summation of P users over all sub-bands m is K, where K is the total number of users and it belongs to the set ξ. In other words, we can say that $n \in J \wedge J \in \xi$. The total sum rate of the system is given by

$$\begin{aligned} R_{sum} = BW_{sb}\sum_{k=1}^{K/2} log_2\left(1 + SINR_{i,k}\right) \quad i \in \{1, 2\}, \\ k \in \{1, ..., K/2\}, \end{aligned} \tag{12}$$

From (12), it can be seen that the user power has a direct effect not only on individual user data rate but also on the SINR of other users, which shows the importance of power allocation in SBD-NOMA system.

III. POWER ALLOCATION

One of the motivation behind this work is to maximize the sum rate across all sub-bands by allocating optimal power and also to provide some degree of fairness to each user. The resource allocation problem considers the number of users in each sub-band, the choice of user pairing and the power allocated to each sub-band. The framed optimization problem maximizes the sum rate while also ensures fairness in the system. Instead of allocating power values to each sub-band through iterative water filling, the power allocated across each sub-band is assumed to be equally allocated, which would reduced the signaling overhead and complexity of the proposed numerical solutions. However, in general, the sum rate is not convex in nature for more than two transmitting sources, hence, the solution is proposed here for NOMA$_2$. The numerical solution for other schemes is not proposed here due to duality gap in transformation from non-convex to convex function. However, a novel approach is proposed to allocate power to the users in SBD-NOMA with more than two users in each sub-band.

Mathematically, the problem is formulated as

$$
\begin{aligned}
\underset{\alpha_{i,j}}{\text{maximize}} \quad & R_i \\
\text{subject to} \quad & \alpha_{1,j} + \alpha_{2,j} \leq P_T, \; \forall j \\
& \alpha_{i,j} \geq 0, \forall i, \forall j \\
& R_{i,j} \geq r_{min}, \text{ where } j \in J,
\end{aligned} \tag{13}
$$

where (13) represent the constraints of the framed optimization problem, P_T is the total power allocated to users within a particular sub-band, R_i represents the data rate across each sub-band, $R_{i,j}$ represents the individual data rate of j^{th} user and r_{min} represents the minimum target rate to maintain fairness among the users. The users are already paired and are chosen from two distinct sets to reduce the inter set interference. Taking into account the objective function described above and solving the optimization problem through Lagrange multipliers leads to the following formulation of the objective function denoted by F, i.e.,

$$
\begin{aligned}
F = R_i &+ \sum_{j=1}^{P/2} \lambda_j \left(P_T - \alpha_{1,j} - \alpha_{2,j} \right) \\
&+ \sum_{j=1}^{P/2} \kappa_j \left(R_{1,j} - r_{min} \right) + \sum_{j=1}^{P/2} \tau_j \left(R_{2,j} - r_{min} \right),
\end{aligned} \tag{14}
$$

where λ_j, κ_j and τ_j represent the Lagrange multipliers. Using the standard procedure, by differentiating F with respect to all $\alpha_{1,j}$ and $\alpha_{2,j}$ and by setting them equal to zero, we obtain a non-linear system of $5|J|$ equations having $5|J|$ unknowns, where $|J|$ denotes the cardinality of set J. For the sake of simplicity, the derivatives here are taken only for $\alpha_{1,j}$, $\alpha_{2,j}$, τ_j, λ_j and κ_j. A similar procedure can be used for other variables as well. The entire procedure is repeated over all sub-bands to maximize the overall sum rate. We obtain

$$
\begin{aligned}
\partial F / \partial \alpha_{1,j} = & \frac{BW_{sb} ||\mathbf{z}_{1,j} \cdot \mathbf{h}_{1,j}||^2}{||\mathbf{z}_{1,j} \cdot \mathbf{h}_{2,j}||^2 \alpha_{2,j} + \sigma^2 + ||\mathbf{z}_{1,j} \cdot \mathbf{h}_{1,j}||^2 \alpha_{1,j}} \\
& - \lambda_j - \kappa_j \frac{BW_{sb} ||\mathbf{z}_{1,j} \cdot \mathbf{h}_{1,j}||^2}{||\mathbf{z}_{1,j} \cdot \mathbf{h}_{2,j}||^2 \alpha_{2,j} + \sigma^2 + ||\mathbf{z}_{1,j} \cdot \mathbf{h}_{1,j}||^2 \alpha_{1,j}},
\end{aligned} \tag{15}
$$

$$
\begin{aligned}
\partial F / \partial \alpha_{2,j} = & -\frac{BW_{sb} ||\mathbf{z}_{1,j} \cdot \mathbf{h}_{1,j}||^2 ||\mathbf{z}_{1,j} \cdot \mathbf{h}_{2,j}||^2 \alpha_{1,j}}{||\mathbf{z}_{1,j} \cdot \mathbf{h}_{2,j}||^2 \alpha_{2,j} + \sigma 2 + ||\mathbf{z}_{1,j} \cdot \mathbf{h}_{1,j}||^2 \alpha_{1,j} \varphi} \\
& + \frac{||\mathbf{z}_{2,j} \cdot \mathbf{h}_{2,j}||^2}{||\mathbf{z}_{2,j} \cdot \mathbf{h}_{2,j}||^2 \alpha_{2,j} + \sigma^2} \\
& - \lambda_j + \kappa_j \frac{BW_{sb} ||\mathbf{z}_{1,j} \cdot \mathbf{h}_{1,j}||^2 ||\mathbf{z}_{1,j} \cdot \mathbf{h}_{2,j}||^2 \alpha_{1,j}}{||\mathbf{z}_{1,j} \cdot \mathbf{h}_{2,j}||^2 \alpha_{2,j} + \sigma^2 + ||\mathbf{z}_{1,j} \cdot \mathbf{h}_{1,j}||^2 \alpha_{1,j} \varphi} \\
& + \tau_j \frac{BW_{sb} ||\mathbf{z}_{2,j} \cdot \mathbf{h}_{2,j}||^2}{\sigma^2},
\end{aligned} \tag{16}
$$

where $\varphi = ||\mathbf{z}_{1,j} \cdot \mathbf{h}_{2,j}||^2 \alpha_{2,j} + \sigma^2$.

$$
\partial F / \partial \lambda_j = P_T - \alpha_{1,j} - \alpha_{2,j}, \tag{17}
$$

$$
\partial F / \partial \kappa_j = BW_{sb} log_2 \left(1 + \frac{||\mathbf{z}_{1,j} \cdot \mathbf{h}_{1,j}||^2 \alpha_{1,j}}{||\mathbf{z}_{1,j} \cdot \mathbf{h}_{2,j}||^2 \alpha_{2,j} + \sigma^2} \right) - r_{min}, \tag{18}
$$

$$
\partial F / \partial \tau_j = BW_{sb} log_2 \left(1 + \frac{||\mathbf{z}_{2,j} \cdot \mathbf{h}_{2,j}||^2 \alpha_{2,j}}{\sigma^2} \right) - r_{min}, \tag{19}
$$

$$
\alpha_{1,j} \cdot \partial F / \partial \alpha_{1,j} = 0, \tag{20}
$$

$$
\alpha_{2,j} \cdot \partial F / \partial \alpha_{2,j} = 0, \tag{21}
$$

$$
\lambda_j \cdot \partial F / \partial \lambda_j = 0, \tag{22}
$$

$$
\kappa_j \cdot \partial F / \partial \kappa_j = 0, \tag{23}
$$

$$
\tau_j \cdot \partial F / \partial \tau_j = 0, \tag{24}
$$

where $\alpha_{1,j}, \alpha_{2,j}, \lambda_j, \kappa_j, \tau_j \geq 0$. Solving the above Equation (15) for the Lagrange variable λ_j, and substituting it in (16) and the value of $\alpha_{2,j}$ from (17) gives the value of $\alpha_{1,j}$. The $\alpha_{1,j}$ turns out to be

$$
\begin{aligned}
\alpha_{1,j} = & \frac{\sqrt{\varphi_b}(-2\varphi_c(r_{min}+\tau_j)+\varphi_a(2r_{min}+\kappa_j+\tau_j)(\varphi_c P_T+\sigma^2)}{2\sqrt{(\varphi_b)(\varphi_a-\varphi_c)\varphi_c(r_{min}+\tau_j)}} \\
& - \frac{\sqrt{\varphi_a}\sqrt{(\varphi_c P_T+\sigma^2)}\sqrt{\varphi_a\varphi_b\varphi_c P_T(\kappa_j-\tau_j)^2+\Psi_1+\Psi_2}}{2\sqrt{(\varphi_b)(\varphi_a-\varphi_c)\varphi_c(r_{min}+\tau_j)}},
\end{aligned} \tag{25}
$$

where $\Psi_1 = \varphi_b(-4\varphi_c(r_{min} + \kappa_j)(r_{min} + \tau_j) + \varphi_a(2r_{min} + \kappa_j + \tau_j)^2)\sigma^2$ and $\Psi_2 = 4\varphi_c(-\varphi_a + \varphi_c)(r_{min} + \kappa_j)(r_{min} + \tau_j)\sigma^2$. The φ_a, φ_b and φ_c are given by $\varphi_a = ||\mathbf{z}_{1,j} \cdot \mathbf{h}_{1,j}||^2$, $\varphi_b = ||\mathbf{z}_{2,j} \cdot \mathbf{h}_{2,j}||^2$, $\varphi_c = ||\mathbf{z}_{1,j} \cdot \mathbf{h}_{2,j}||^2$.

The value of $\alpha_{2,j}$ is derived by substituting the value of $\alpha_{1,j}$ in (17) and is given by

$$
\alpha_{2,j} = P_T - \alpha_{1,j}. \tag{26}
$$

A. Extension to SBD NOMA with P users in each sub-band

To investigate the scenario of more than two users in each sub-band, multistage power allocation is proposed. In the first stage, the users form two distinct sets A and B on the basis of the CSI, after this it forms two groups having equal number of users. The members of the two groups are chosen from two distinct sets A and B. The power allocated to each sub-band is equal to half of the total transmission power. In this scenario, the channel gains become the sum of all channel powers in the two groups. The same procedure for SBD-NOMA$_2$ is repeated for power allocation within the two groups. In the second stage, the two groups further divide themselves into smaller groups i.e., two subgroups are formed in each group of the previous stage with the total power per group as determined in the first stage. In the third stage, these subgroups further divide into smaller groups to allocate power to all users. The procedure is repeated until all users are allocated with an optimal transmission power.

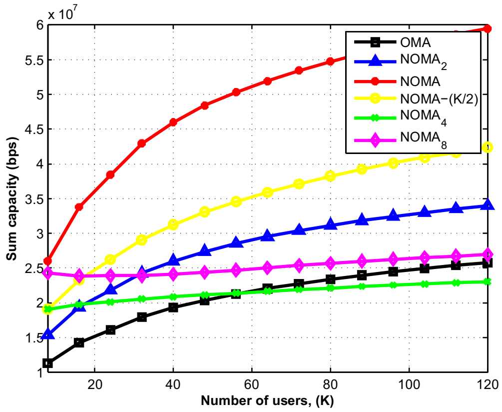

Fig. 2. Sum capacity comparison between SBD-NOMA, conventional OMA and NOMA techniques.

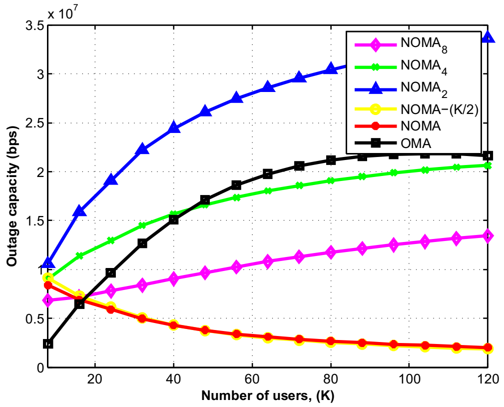

Fig. 3. Outage capacity analysis of SBD-NOMA, conventional OMA and NOMA techniques.

IV. NUMERICAL RESULTS

In this section, the performance of the proposed SBD-NOMA is evaluated through extensive Monte Carlo simulations. We consider an uplink scenario, with single cell having a cell radius of 1000m. The channel coefficients are assumed to be independent and identically distributed (i.i.d) Rayleigh flat faded. The optimal transmission powers are allocated by solving the above constrained optimization problem. The comparison of this power allocation scheme is evaluated with equal power allocation scheme (EPAS) and FPA for SBD-NOMA$_2$. The maximum number of available sub-bands is K/P, where K is the total number of users in a cell. The noise is assumed to be zero mean circular-symmetric complex Gaussian having a noise power spectral density of -174dBm/Hz. The overall system bandwidth is 4.32MHz. The path loss is calculated by using the following model

$$PL_{dB} = 30 + 10\beta \log_{10}(d),$$

where d is the distance between the BS and the user and β is the path loss exponent, which is kept at 3.71 in this study.

Fig. 2 compares the sum capacity of SBD-NOMA with conventional OMA and conventional NOMA techniques with equal power allocation. The primary observation is comparing the effect of number of users on sum capacity in SBD-NOMA. It can be easily seen from the figure that NOMA outperforms all other schemes in terms of sum capacity. Although, the increase in sum capacity is not substantial for NOMA$_4$ and NOMA$_8$ schemes with increasing number of users, because of significant interference, however, these schemes offer better coverage probability and hence their outage capacity is quite high and they offer less complexity than the conventional NOMA technique. The inconsistency in performance is because of the reason that SBD-NOMA can reduce the achievable rates as dividing the entire bandwidth into sub-bands cannot exploit the full potential of NOMA, where all users can occupy the entire bandwidth. As bandwidth is directly proportional to each users's data rate, the effect is visible in the sum capacity analysis. However, dividing the users into orthogonal groups will reduce the intra set interference. As a result, the rate offered by conventional OMA is minimum but its outage probability is low, which increases the overall outage capacity of the conventional OMA system. The major outcome of sharing is that the sum rate is expected to improve, which can be further enhanced by properly allocating power to weak users. The *outage capacity* is defined as the maximum rate that can be maintained by each user multiplied with the success probability of the users. It shows a clear picture of system performance for both data rates and success of each user. The outage capacity analysis of SBD-NOMA is evaluated in Fig. 3. SBD-NOMA performs well in terms of outage capacity as shown. We can derive an interesting result by combining Fig. 2 and 3 that although the conventional uplink NOMA achieves maximum sum capacity but it increases the receiver complexity and outage probability to a great extent, which is practically not desired especially if the number of users in a system is quite large. Hence, in the situations, where the user priority is reduced receiver complexity, cost and enhanced QoS, SBD-NOMA schemes should be preferred over conventional uplink NOMA that provide better rate and a fairly reliable transmission scheme.

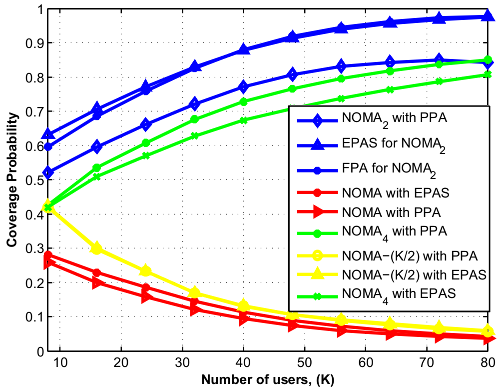

Fig. 4. Coverage probability comparison of NOMA$_2$, NOMA$_4$, NOMA$_{(K/2)}$ with different power allocation schemes and NOMA technique.

Fig. 4 compares the coverage probability of conventional NOMA and SBD-NOMA with different power allocation schemes. Coverage probability is defined as the probability

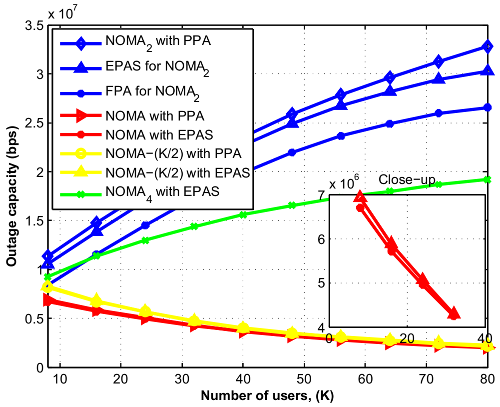

Fig. 5. Outage capacity comparison of NOMA$_2$, NOMA$_4$, NOMA$_{(K/2)}$ with different power allocation schemes and NOMA technique.

that a certain number of users achieves a data rate greater than the target data rate, which is taken as 3.34 Mbps in this study. It can be easily seen that SBD-NOMA performs well as compared to NOMA in terms of coverage probability. The coverage probability of FPA and EPAS is even better in this case as compared to the proposed approach, however, EPAS being the simplest criteria of allocating power to users does not perform well, as it allocates equal power to all users and therefore treats the weak users in the same manner as the strong ones. The SBD-NOMA provides every pair a power that suits their channel gain, which allows a balance between the required power, fairness and the target rate with total power allocation constraint across each sub-band. Although, the optimal NOMA scheme shows greater sum rate but its coverage probability in this scenario is less than the optimal solution even after proposed power allocation. The same trend is depicted in the Fig.4.

Fig. 5 shows the outage capacity of SBD-NOMA with conventional NOMA scheme. The outage capacity is larger for SBD-NOMA$_2$ with the proposed power allocation scheme. The proposed power allocation in NOMA increases the outage capacity as compared to the NOMA with EPAS but the increase is quite substantial. However, with equal power allocation, the system treats the weak users in the same way as strong users. As strong users get significant interference from weak users, hence, the capacity of strong users gets degraded. The optimal power allocation in SBD-NOMA ensures fairness in the system by imposing an upper limit on each user's individual data rate as compared to EPAS and FPA schemes. The performance margin in NOMA and SBD-NOMA with and without power allocation is quite high in terms of outage capacity. From this figure, it is clear that the proposed power allocation scheme performs better than the existing ones.

V. CONCLUSION

This paper investigated the power allocation problem for SBD-NOMA, which is an OFDMA-based NOMA system and the paper also evaluated the performance gap between OMA, NOMA and SBD-NOMA. From a practical implementation point of view, a maximum of 2 users are allowed to share a resource block, however, in this case P users are allowed to share a resource block while the total number of users in the system is K. The performance comparison of this scheme is made with conventional NOMA scheme and other existing power allocation schemes. The results depicted that SBD-NOMA performs better than NOMA in terms of achievable rates. The proposed SBD scheme also offered better sum rates with reduced complexity especially in massive access scenarios. The proposed scheme with under different path loss and power allocation outperformed the EPAS and FPA schemes.

VI. ACKNOWLEDGEMENT

Part of this work is funded by the Russian Federal budget funds for research work (Fundamental research, applied research and experimental development) through the grant No. 3942 and performed in accordance with Russian Government Resolutions No. 2014/226 of 2016.

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