Received: 28 July 2017 | Revised: 18 September 2017 | Accepted: 25 October 2017

DOI: 10.1002/itl2.13

LETTER

WILEY

Outage probability for a decode-and-forward SWIPT relaying system in Nakagami fading

Farhan Nawaz[1] | **Syed Ali Hassan**[1] | **Sonia Aissa**[2] | **Sajid Saleem**[1]

[1]School of Electrical Engineering & Computer Science (SEECS), National University of Sciences & Technology (NUST), Islamabad, Pakistan
[2]INRS-EMT, University of Quebec, Montreal QC, Canada

Correspondence
Farhan Nawaz, School of Electrical Engineering & Computer Science (SEECS), National University of Sciences & Technology (NUST), Islamabad, Pakistan.
Email: fnawaz.msee16seecs@seecs.edu.pk

The outage probability of a decode-and-forward (DF) relaying system based on simultaneous wireless information and power transfer (SWIPT) in the presence of Nakagami fading is investigated. The relaying model considers both the source-destination direct link in addition to the source-relay-destination link. The power splitter at the relaying device provides energy to the relay by splitting the received signal power into energy harvesting and information transfer parts. The derived outage expression is verified using simulation results. The results show an impressive amount of percentage decrease in outage probability for Nakagami fading with different values of shape parameter in comparison to Rayleigh fading.

KEYWORDS

decode-and-forward, energy harvesting, Nakagami fading, outage probability, power splitting, relaying, SWIPT

1 | INTRODUCTION

Energy harvesting (EH) has recently attracted a lot of attention by the research community.[1–7] In fact, the concept of simultaneous wireless information and power transfer (SWIPT) is gaining momentum among other EH approaches. In wireless systems, SWIPT is an EH-based wireless technology where signals targeted to a particular receiver are not only used to decode the information message but also to harvest energy. This concept was first proposed by Varshney et al.,[1] after which, a lot of work has been done in this domain for the past half decade.

In Reference 4, the basic architectural designs for the receiver of SWIPT are presented and the rate-energy tradeoffs for these architectures are discussed. In Reference 5, different techniques were proposed for the practical implementation of SWIPT, involving time switching, power splitting (PS), spatial multiplexing, and antenna selection. The basic idea behind these techniques is to split the received signal into 2 orthogonal portions; one for the decoding of information and the other for EH. In Reference 6, a SWIPT-based 3 node relay model is considered, where the ergodic capacity as well as the outage probability are derived analytically using the time switching and PS techniques at the relay node in the absence of direct link between the source and the destination. Similarly, in Reference 7, the same SWIPT-based relay model is adopted and the outage probability is investigated assuming the decode-and-forward (DF) protocol and PS technique at the relay where no direct link between the source and the destination exists.

In Reference 8, the outage performance of a SWIPT network with full duplex relay channels is investigated by employing PS at the relay node and using digital network coding. Similarly, in Reference 9, for a SWIPT relaying system, the outage probability and energy shortage probability are investigated subject to Weibull fading using the time switching and PS at the relay. The work in Reference 5 is further carried forward in Reference 10, where the outage probability is analyzed by considering a relaying system implementing PS. In that study, the direct link as well as the relay links to the destination are considered, the relay uses the amplify-and-forward protocol, and channels are assumed to be subject to Rayleigh fading. A DF multihop device-to-device network under SWIPT assumption has recently been studied in References 11 and 12. Other works related to EH in DF relaying systems have been considered in References 13 and 14, where an interference-based EH scheme is proposed. The relay node transmits the message to the destination by harvesting energy from the source signal as well as from the co-channel interference signals. The relay uses DF protocol and the results for outage probability and ergodic capacity are derived.

In this paper, taking as starting point the work in References 7 and 10, a 3-node DF relaying model is considered, where communication also takes place through the direct link between the source and the destination. The relay uses no external power source and is fully dependent on the energy harvested from the source signal. The EH technique used in this paper is PS. The direct link provides both diversity gain as well as multiplexing gain.[15] We derive the outage probability at the destination node when all channels are subject to Nakagami fading. The derived results are discussed with respect to different parameters, such as the PS factor, efficiency of PS, and signal-to-noise ratio (SNR). The simulation results verify the analytical results and prove that Nakagami channels provide better coverage over Rayleigh channels with less probability of outage.

The rest of this paper is organized as follows. Section 2 details the system model. Analysis of the outage probability is carried out in Section 3, followed by derivation of the closed-form expression for the outage probability in Section 4. Simulation results verifying the analytical results are discussed in Section 5, and finally the conclusion is presented in Section 6.

2 | SYSTEM MODEL

The system consists of 3 nodes, that is, the source, the destination, and an intermediate relaying node. The source communicates with the destination node through direct link as well as through the relay. The relay uses half-duplex DF protocol and operates with the energy harvested from the signal transmitted by the source node. The source-destination, source-relay, and relay-destination links are subject to Nakagami fading.

Communication occurs in 2 time slots due to the half-duplex characteristic of the relay node. The signal received during the first time slot at the destination node is given by

$$y_D{}^{(1)} = \sqrt{P_t} x h_0 + n_0, \tag{1}$$

whereas the signal received at the relay is given by

$$y_R = \sqrt{P_t} x h_1 + n_1. \tag{2}$$

In Equations (1) and (2), x is the transmitted signal, $n_0 \sim CN(0, \sigma_0^2)$ and $n_1 \sim CN(0, \sigma_1^2)$ are the white noise random variables, which are complex Gaussian with zero mean and variance σ_0^2 and σ_1^2, respectively. The envelopes of h_0 and h_1 are Nakagami distributed, that is, $|h_i| \sim Nakagami(m, k_i)$ where $i = \{0, 1\}$, with identical shape parameter m and distinct scale parameter k_i, whereas their squared envelopes are Gamma distributed.

The PS module at the relay splits the signal into 2 parts using PS factor, ρ, that is,

$$y_{RE} = \sqrt{\rho} y_R, \tag{3}$$

and

$$y_{RI} = \sqrt{1-\rho} y_R, \tag{4}$$

where y_{RE} is the amount of energy that the relay node harvests and y_{RI} is the message part that is transmitted by the relay to the destination node.

In the second time slot, the relay transmits the decoded signal $\hat{x}$ to the destination with power P_r. The signal received at the destination during the second time slot is given by

$$y_D{}^{(2)} = \sqrt{P_r}\sqrt{1-\rho} h_2 \hat{x} + n_2, \tag{5}$$

where $P_r = \eta\sqrt{\rho} P_t |h_1|^2$ is the total relay power in the second time slot, $n_2 \sim CN(0, \sigma_2^2)$ is the additive white Gaussian noise, and η is the energy conversion efficiency of PS. The destination node uses maximum ratio combing (MRC) to combine both copies of the received data signal, that is, the one obtained through the direct link and the one through the relaying link.

3 | ANALYSIS OF OUTAGE PROBABILITY

The outage probability at the destination node is given by

$$P_{out} = \mathbb{P}_r[R < R_\tau], \tag{6}$$

where R_τ is the minimum rate threshold and

$$R = \frac{1}{2}\log_2(1 + \gamma_1 + \gamma_2), \tag{7}$$

with γ_1 and γ_2 denoting the source-destination link SNR and the relay-destination link SNR, respectively, given by

$$\gamma_1 = \frac{P_t|h_0|^2}{\sigma_0^2}, \tag{8}$$

$$\gamma_2 = \frac{P_r\sqrt{1-\rho}|h_2|^2}{\sigma_2^2}. \tag{9}$$

We now present an expression for the outage probability of the system, as detailed in the next theorem.

Theorem 1. *For a DF cooperative SWIPT relaying system, the outage probability is given by*

$$P_{out} = \frac{1}{\Gamma(m)^3 k_0^m k_1^m} \int_0^{b(c-1)} \int_0^{\infty} \gamma\left(m, \frac{z}{k_2}\right) x^{m-1} y^{m-1}\, e^{\frac{-x}{k_0}} e^{\frac{-y}{k_1}}\, dydx, \tag{10}$$

where $z = \frac{b(c-1)-x}{ay}$, with $a = \eta\rho(1-\rho)$, $b = \frac{\sigma^2}{P_t}$, $c = 2^{2R_\tau}$, and $\gamma\left(m, \frac{z}{k_2}\right)$ is the lower incomplete Gamma function.

Proof 1 By substituting Equation (7) into Equation (6) and solving it with the assumption that $\sigma_0^2 = \sigma_1^2 = \sigma_2^2 = \sigma^2$, we get the following expression

$$P_{out} = [\sigma^2 + P_t|h_0|^2 + \eta\rho(1-\rho)P_t|h_1|^2|h_2|^2 < \sigma^2 2^{2R_\tau}]. \tag{11}$$

Now consider 3 random variables X, Y, and Z, such that $X = |h_0|^2$, $Y = |h_1|^2$, and $Z = |h_2|^2$, with the probability density function (PDF) of each following the Gamma distribution given by

$$f_\psi(\phi) = \frac{\phi^{m-1} e^{\frac{-\phi}{k_0}}}{k_0^m \Gamma(m)}, \tag{12}$$

where $\psi = \{X, Y, Z\}$. By substitution, Equation (11) becomes

$$P_{out} = [\sigma^2 + P_t X + \eta\rho(1-\rho)P_t Y Z < \sigma^2 2^{2R_\tau}]. \tag{13}$$

By using simple algebra, we obtain

$$P_{out} = \mathbb{P}_r\left[Z < \frac{B}{A}\right], \tag{14}$$

where $B = b(c-1) - X$ and $A = aY$. From these definitions, we can see that A is always positive and that B can be positive or negative. As such,

$$P_{out} = \mathbb{P}_r\left[Z < \frac{B}{A}\right] = 0, \text{ for } B < 0, A > 0. \tag{15}$$

Hence, the outage probability is given as

$$P_{out} = \int_0^{b(c-1)} \int_0^{\infty} \mathbb{P}_r\left[Z < \frac{B}{A}\right] f_X(x) f_Y(y)\, dydx, \tag{16}$$

and by putting the values, we have

$$P_{out} = \int_0^{b(c-1)} \int_0^{\infty} \frac{1}{\Gamma(m)} \gamma\left(m, \frac{z}{k_2}\right) f_X(x) f_Y(y)\, dydx. \tag{17}$$

Now by substituting the expressions of $f_X(x)$ and $f_Y(y)$ shown in Equation (12) into Equation (17), we get the result in Equation (10), which completes the proof. ■

Since the probability expression shown in Equation 10 involves integrals, next we go toward finding a closed-form solution.

4 | CLOSED-FORM SOLUTION OF OUTAGE PROBABILITY

We start by rewriting Equation (10) as follows

$$P_{out} = \frac{1}{\Gamma(m)^3 k_0^m k_1^m} \int_0^{b(c-1)} x^{m-1} e^{\frac{-x}{k_0}} \left(\int_0^{\infty} \gamma\left(m, \frac{z}{k_2}\right) y^{m-1} e^{\frac{-y}{k_1}}\, dy \right) dx. \tag{18}$$

For convenience, let

$$I = \int_0^{\infty} \gamma\left(m, \frac{z}{k_2}\right) y^{m-1} e^{\frac{-y}{k_1}}\, dy. \tag{19}$$

Now by substituting $\frac{z}{k_2} = \frac{t-x}{ayk_2} = u$, with $t = b(c-1)$ and solving, we obtain

$$I = \frac{-(t-x)^m}{(ak_2)^m} \int_0^\infty \frac{\gamma(m,u)}{u^{m-2}} e^{\frac{-(t-x)}{auk_1k_2}} du, \tag{20}$$

Furthermore, with the substitution $a = 3-m$ and $c = \frac{(t-x)}{ak_1k_2}$ in Equation (20), the integral term can be written as

$$\int_0^\infty \gamma(m,u) u^{a-1} e^{\frac{-c}{u}} du. \tag{21}$$

The integral shown in (21) can be solved by opening the Taylor series of the exponential function as.

$$\int_0^\infty \gamma(m,u) u^{a-1} e^{\frac{-c}{u}} du = -\sum_{n=0}^{\infty} \frac{(-c)^n}{n!} \frac{\Gamma(g+m)}{g}, \tag{22}$$

where $g = a-n$. Therefore, Equation (20) becomes

$$I = \frac{(t-x)^m}{(ak_2)^m} \sum_{n=0}^{\infty} \frac{(-c)^n}{n!} \frac{\Gamma(g+m)}{g}. \tag{23}$$

Now putting Equation (23) back into Equation (18) and rearranging, we get

$$P_{out} = \frac{1}{\Gamma(m)^3 k_0^m k_1^m (ak_2)^m} \sum_{n=0}^{\infty} \frac{(-1)^n \Gamma(3-m-n)}{n!(ak_1k_2)^n (3-m-n)} \int_0^t (t-x)^{m+n} x^{m-1} e^{\frac{-x}{k_0}} dx. \tag{24}$$

By solving the integral in Equation (24), we obtain the generalized closed-form solution for the outage probability as

$$P_{out} = \frac{1}{\Gamma(m)^3 k_0^m k_1^m (ak_2)^m} \sum_{n=0}^{\infty} \frac{(-1)^n \Gamma(3-m-n)}{n!(ak_1k_2)^n (3-m-n)} t^{2m+n} \Gamma(m) \Gamma(1+m+n) {}_1\widetilde{F}_1\left(m, 1+2m+n, \frac{-t}{k_0}\right), \tag{25}$$

where ${}_1\widetilde{F}_1\left(m, 1+2m+n, \frac{-t}{k_0}\right)$ is the Hypergeometric regularized function.[16]

5 | NUMERICAL RESULTS AND DISCUSSION

Next, we compare numerical results based on the derived expression for the outage probability with simulations. Throughout the simulations, we use $k_0 = k_1 = k_2 = 1$, $\eta = .5$, $R_\tau = 2$.

In Figure 1, the outage probability for different values of Nakagami shape parameter m is plotted for different values of the PS factor ρ. The figure clearly shows that the outage probability decreases with increasing m, that is, for $\rho = .5$ there is a percentage decrease of almost 61.3% in outage probability when we go from $m = 1$ to $m = 2$, and a percentage decrease of 99% when we go from $m = 1$ to $m = 4$. As expected, the Rayleigh fading case which corresponds to $m = 1$ yields the highest outage probability. Moreover, it can be seen that the simulations results match closely with the analytical results.

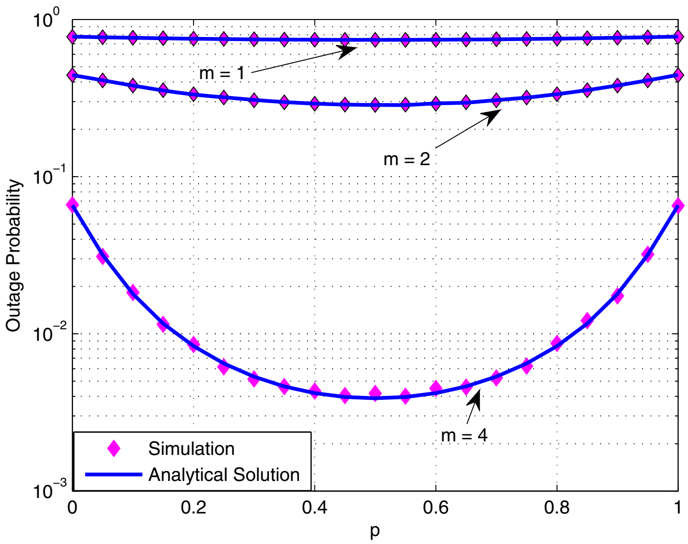

FIGURE 1 The outage probability as a function of the PS factor, ρ, with SNR = 10 dB, $\eta = .5$, and $R_\tau = 2$

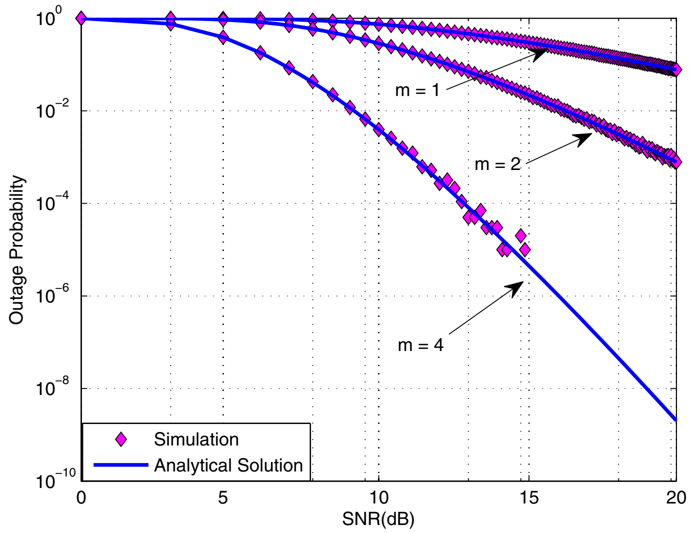

FIGURE 2 The outage probability as a function of SNR with $\rho = .5$, $\eta = .5$, and $R_{\tau} = 2$

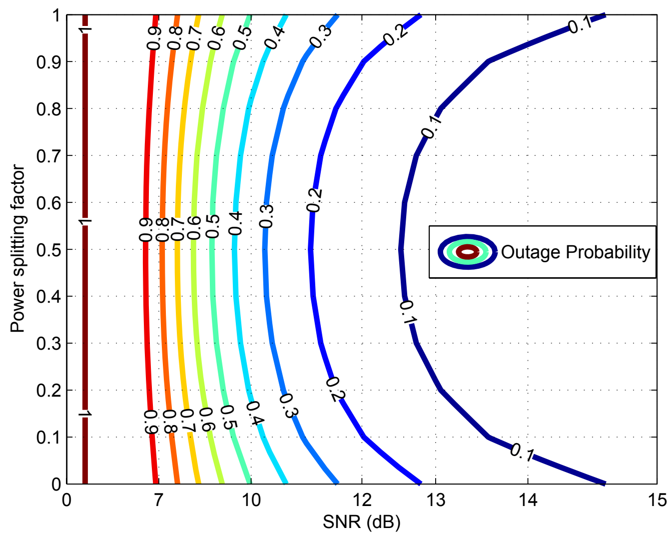

FIGURE 3 The contour plot of outage probability against SNR and PS factor, ρ

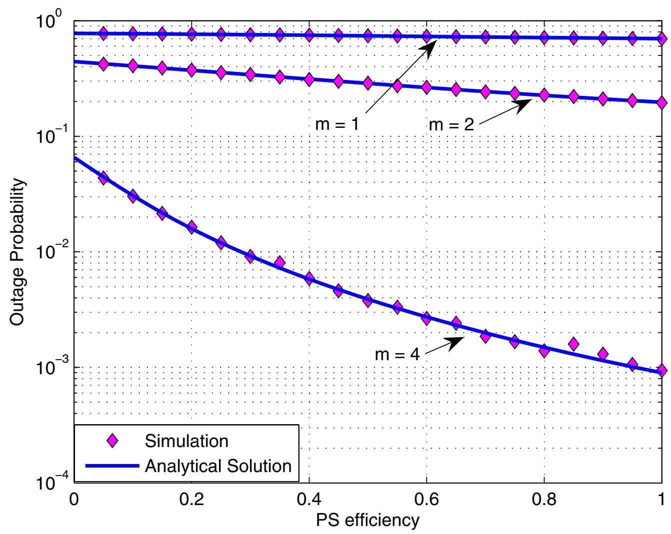

FIGURE 4 The outage probability as a function of PS efficiency, η, with $\rho = .5$, SNR = 10 dB, and $R_{\tau} = 2$

In Figure 2, the outage probability for different values of the Nakagami shape parameter m is plotted as a function of SNR. For a fixed SNR value of 10 dB, there is a percentage decrease of almost 61.3% in outage probability when we go from $m = 1$ to $m = 2$, that is, same as in the case of Figure 1. But when we go for higher SNR values that is, at SNR of 20 dB, then we get 99% decrease in outage probability while going from $m = 1$ to $m = 2$.

In Figure 3, the contour plot is shown which shows that the same outage probability can be obtained at different combinations of SNR and PS factor. It can be seen that the outage probability of .1 can be obtained for a range of $SNR = 12.5$ dB to $SNR = 14.5$ dB with the value of PS factor from $\rho = 0$ to 1, but the best value can be obtained for $\rho = .5$ at $SNR = 12.5$ dB. Figure 4 shows that the outage probability decreases with increasing efficiency of the power splitter employed at the relay. It can also be seen that the outage probability depends more on the efficiency of the power splitter as we move from $m = 1$ to $m = 4$, that is, from non line-of-sight to line-of-sight channel. To achieve good results, the efficiency of the power splitter should be greater than 70%.

6 | CONCLUSION

In this paper, the outage probability of a DF SWIPT relaying system was analyzed in the presence of Nakagami fading. A closed-form solution for the outage probability was obtained. The derived results were discussed with respect to different parameters such as the PS factor, the efficiency of PS and the SNR, which provide a thorough comparison between the performance in Rayleigh fading and in Nakagami fading. A quantitative analysis showed the percentage decrease in outage probability when Nakagami shape parameter m is changed.

ORCID

Farhan Nawaz http://orcid.org/0000-0001-6006-918X
Syed Ali Hassan http://orcid.org/0000-0002-8572-7377

REFERENCES

1. Varshney LR. Transporting information and energy simultaneously. Paper presented at: IEEE International Symposium on Information Theory, 2008:1612–1616, Toronto, Canada.
2. Grover P, Sahai A. Shannon meets tesla: wireless information and power transfer. Paper presented at: Proceedings of IEEE International Symposium on Information Theory, June, 2010:2363–2367.
3. Liu L, Zhang R, Chua KC. Wireless information transfer with opportunistic energy harvesting. *IEEE Trans Wireless Commun*. Jan. 2013;*12*(1):288–300.
4. Zhou X, Zhang R, Ho CK. Wireless information and power transfer: architecture design and rate-energy tradeoff. *IEEE Trans Commun*. 2013;*61*(11):4754–4767.
5. Krikidis I, Timotheou S, Nikolaou S, Zheng G, Ng DWK, Schober R. Simultaneous wireless information and power transfer in modern communication systems. *IEEE Commun Mag*. Nov. 2014;*52*(11):104–110.
6. Nasir AA, Zhou X, Durrani S, Kennedy RA. Relaying protocols for wireless energy harvesting and information processing. *IEEE Trans Wireless Commun*. 2013;*12*(7):3622–3636.
7. Wang F, Xu W, Li S, Feng Z, Lin J. Outage probability analysis of DF relay networks with RF energy harvesting. Paper presented at: IEEE Global Communications Conference (GLOBECOM), 2015:1–5, San Diego, CA.
8. Jiang R, Xiong K, Fan P, Zhong Z. Outage performance of SWIPT-enabled two-way relay networks. Paper presented at: International Workshop on High Mobility Wireless Communications (HMWC), 2015:106–110, Xi'an, China.
9. Jameel F, Ali A, Khan R. Optimal time switching and power splitting in SWIPT. Paper presented at: 19th International Multi-*Topic Conference* (INMIC), 2016:1–5, Islamabad, Pakistan.
10. Lee H, Song C, Choi SH, Lee I. Outage probability analysis and power splitter designs for SWIPT relaying systems with direct link. *IEEE Commun Lett*. 2017;*21*(3):648–651.
11. Ansari RI, Hassan SA, Chrysostomou C. A SWIPT-based device-to-device cooperative network. Paper presented at: 24th International Conference on Telecommunications (ICT), May, 2017, Limassol, Cyprus.
12. Chen E, Xia M, da Costa DB, Aïssa S. Multi-hop cooperative relaying with energy harvesting from co-channel interferences. *IEEE Commun Lett*. May 2017;*21*(5):1199–1202.
13. Gu Y, Aissa S. RF-based energy harvesting in decode-and-forward relaying systems: ergodic and outage capacities. *IEEE Trans Wireless Commun*. Nov. 2015;*14*(11):6425–6434.
14. Gu Y, Aissa S. Interference aided energy harvesting in decode-and-forward relaying systems. Paper presented at: IEEE International Conference on Communications (ICC), 2014:5378–5382, Sydney, Australia.
15. Song C, Lee KJ, Lee I. MMSE-based MIMO cooperative relaying systems: closed-form designs and outage behavior. *IEEE J Select Areas Commun*. September 2012;*30*(8):1390–1401.
16. Abramowitz M. *Handbook of Mathematical Functions, with Formulas, Graphs, and Mathematical Tables*. Milwaukee, WI: Dover Publications; 1974.

How to cite this article: Nawaz F, Hassan SA, Aissa S, Saleem S. Outage probability for a decode-and-forward SWIPT relaying system in Nakagami fading, *Internet Technology Letters*, 2017;e13. https://doi.org/10.1001/itl2.13