Impacts of K-fading on the performance of massive MIMO systems

M.T. Mushtaq✉, S.A. Hassan, S. Saleem and D.N.K. Jayakody

Massive multiple-input multiple-output (MIMO) has been identified as a key technology for next generation cellular systems. The performance of a multi-cellular system with large antenna arrays at the base station and single antenna user terminals, operating under a composite fading-shadowing environment are considered. The outage probability of the considered system is evaluated for the uplink transmission that uses a maximum ratio combining receiver.

Introduction: Urge for higher data rates inevitably renders the use of multiple antennas in wireless systems [1]. Multiple antennas benefit from spatial diversity and scattering leading to improved system performance. However, channel impairments such as multipath fading and shadowing affect the performance badly. It has been established that small-scale fading effects average out using a large array at the base station (BS), however, the large-scale shadowing effects remain and their quantification is the main motivation behind this study. In this Letter, we consider a composite fading-shadowing channel, modelled by K-distribution, in a massive multiple-input multiple-output (MIMO) multi-cellular system with perfect channel state information (CSI) at the BS. Specifically, a closed-form expression of outage probability for the uplink of this system is derived where the desired as well as interfering channels are modelled as K-distributed. Simulation results are provided to validate the theoretical models.

System model: Consider a multi-cellular system consisting of L cells, each equipped with M antennas at the BS. Single antennas users are randomly present in all the cells. The received signal vector $\boldsymbol{y}_j \in \mathbb{C}^{M\times 1}$ at serving BS j is given by

$$\boldsymbol{y}_j = \sum_{l=1}^{L} \boldsymbol{H}_{jl}\boldsymbol{x}_l + \boldsymbol{n}_j, \tag{1}$$

where $\boldsymbol{H}_{jl} = [\boldsymbol{h}_{jl1}, \ldots, \boldsymbol{h}_{jlK}] \in \mathbb{C}^{M\times K}$ is the channel matrix of K users from lth cell to jth cell, and where for any kth user, $\boldsymbol{h}_{jlk} \in \mathbb{C}^{M\times 1}$ is the channel vector in cell l to BS j. The transmitted symbols and noise vectors are denoted by $\boldsymbol{x}_l$ and $\boldsymbol{n}_j$, respectively. For the uplink transmission, the considered channel has composite shadowing-fading characteristics. Each element of $\boldsymbol{h}_{jlk}$ is a product of independent and identically distributed zero-mean complex Gaussian random variable (RV) and square root of a gamma distributed RV given by [2]

$$h = \sqrt{z}(g_R + jg_I); \quad h \in \boldsymbol{h}_{jlk} \tag{2}$$

where g_R, g_I are real and imaginary parts of a complex Gaussian RV, respectively, and z denotes a gamma RV. The envelope of h follows K distribution with probability density function (PDF)

$$f_{|h|}(|h|) = \frac{2}{\alpha\Gamma(\nu+1)}\left(\frac{|h|}{2\alpha}\right)^{\nu+1} K_\nu\left(\frac{|h|}{\alpha}\right), \tag{3}$$

where α is the scale parameter and ν is the shaping parameter. The severity of shadowing varies by controlling ν. Here, large value of ν implies a low shadowing and vice versa. The $K_\nu(.)$ is the modified Bessel function of second kind. Considering an interference-limited scenario and assuming perfect CSI at the receiver with maximum ratio combining, the uplink signal-to-interference ratio (SIR) of a desired user k of jth cell is given as

$$\mathrm{SIR}_k = \frac{\left|\boldsymbol{h}_{jjk}^{\mathrm{H}}\boldsymbol{h}_{jjk}\right|^2}{\sum_{l\neq j}^{L}\left|\boldsymbol{h}_{jlk}^{\mathrm{H}}\boldsymbol{h}_{jlk}\right|^2}, \tag{4}$$

where $(.)^{\mathrm{H}}$ represents the Hermitian operator. We now use (4) as a general expression of SIR to find the outage probability.

Outage probability: Redefining (4), we obtain

$$\mathrm{SIR}_k = Z = \frac{U}{V}, \tag{5}$$

where U is the sum of M squared-K RVs and V is the sum of $(L-1)M$ squared-K RVs. The PDF of a RV Q, with squared-K distribution is given as [3]

$$f_Q(q) = 2\left(\frac{1}{\alpha}\right)^{\nu+1} q^{(\nu-1/2)} K_{\nu-1}\left[2\left(\frac{q}{\alpha}\right)^{1/2}\right]. \tag{6}$$

Therefore, the PDF of U is given as

$$f_U(u) = \sum_{i=0}^{M}\binom{M}{i}\Gamma(1-\nu)^{(M-i)}(-1)^i \times\left[\sum_{n=0}^{\infty} c_n\left(\frac{\nu}{\alpha M}\right)^{(1+G/2)} u^{(G-1/2)} I_{G-1}\left(2\sqrt{\frac{\nu M}{\alpha}}u^{1/2}\right)\right], \tag{7}$$

where $c_o = a_o^i$, $c_n = (1/(na_o))[\sum_{t=1}^{h}(ti-h+t)a_t c_{h-t}]$ for $h\geq 1$, $a_o = (1/(1-\nu))$, $a_t = ((-1)^t/(t!(1-\nu+t)))$, with $G = M\nu + h + (1-\nu)i$. The $I_\nu(.)$ denotes the modified Bessel function of the first kind of order ν. Similarly, the PDF of V is given as

$$f_V(v) = \sum_{j=0}^{(L-1)M}\binom{(L-1)M}{j}\Gamma(1-\nu)^{(L-1)M-j)}(-1)^j \times\left[\sum_{k=0}^{\infty} d_k\left(\frac{\nu}{\alpha(L-1)M}\right)^{(1+G/2)} u^{(P-1/2)} I_{P-1}\left(2\sqrt{\frac{\nu(L-1)M}{\alpha}}v^{1/2}\right)\right], \tag{8}$$

where $d_o = b_o^j$, $d_k = (1/(nb_o))[\sum_{t=1}^{h}(tj-h+t)b_t d_{h-t}]$ for $h\geq 1$, $b_o = (1/(1-\nu))$, $b_t = ((-1)^t/(t!(1-\nu+t)))$, with $P = (L-1)M\nu + h + (1-\nu)j$.

To derive the outage probability, i.e. $\mathbb{P}\{Z<\tau\}$, where τ is the threshold, we compute the cumulative distribution function (CDF) of the ratio of two RVs, given as

$$F_Z(\tau) = \int_0^\infty F_U(\tau v) f_V(v)\,\mathrm{d}v, \tag{9}$$

where $F_U(\tau v)$ is the CDF of RV U, and is given by

$$F_U(\tau v) = \sum_{i=0}^{M}\binom{M}{i}\Gamma(1-\nu)^{(M-i)}(-1)^i \times\left[\sum_{n=0}^{\infty} c_n\left(\frac{\nu}{\alpha M}\right)^{G/2}(\tau v)^{(G-1/2)} I_G\left(2\sqrt{\frac{\nu M}{\alpha}}(\tau v)^{1/2}\right)\right]. \tag{10}$$

Hence, the outage probability is given by putting (10) and (7) in (9), as

$$\begin{aligned}\mathbb{P}(\mathrm{SIR}<\tau) = &\int_0^\infty \sum_{i=0}^{M}\binom{M}{i}\Gamma(1-\nu)^{M-i}(-1)^i \\ &\times\left[\sum_{n=0}^{\infty} c_n\left(\frac{\nu}{\alpha M}\right)^{(1+G/2)}(\tau v)^{(G-1/2)} I_{G-1}\left(2\sqrt{\frac{\nu M}{\alpha}}(\tau v)^{1/2}\right)\right] \\ &\times\sum_{j=0}^{(L-1)M}\binom{(L-1)M}{j}\Gamma(1-\nu)^{(L-1)M-j)}(-1)^j \\ &\times\left[\sum_{k=0}^{\infty} d_k\left(\frac{\nu}{\alpha(L-1)M}\right)^{(1+P/2)} v^{(P-1/2)} I_{P-1}\left(2\sqrt{\frac{\nu(L-1)M}{\alpha}}v^{1/2}\right)\right]\mathrm{d}v.\end{aligned} \tag{11}$$

We obtain the following expression after considerable algebraic manipulation:

$$\mathbb{P}(\text{SIR} < \tau) = \sum_{i=0}^{M}\binom{M}{i}\Gamma(1-\nu)^{M-i}(-1)^{i} \times \sum_{n=0}^{\infty} c_n\left(\frac{\nu}{\alpha M}\right)^{(1+G/2)} \times \sum_{j=0}^{(L-1)M}\binom{(L-1)M}{j}\Gamma(1-\nu)^{(L-1)M-j)}(-1)^{j} \times \sum_{k=0}^{\infty} d_k\left(\frac{\nu}{\alpha(L-1)M}\right)^{(1+P/2)}\tau^{G/2} \times \int_0^{\infty} \nu^{((G+P-1)/2)} I_G\left(2\sqrt{\frac{\nu M}{\alpha}}(\tau\nu)^{1/2}\right) \times I_{P-1}\left(2\sqrt{\frac{\nu(L-1)M}{\alpha}}\nu^{1/2}\right)\mathrm{d}\nu. \tag{12}$$

The intergral expression in (12) can be denoted as $\mathbb{I}_o$, and is given as

$$\mathbb{I}_o = \int_0^{\infty} \nu^{((G+P-1)/2)} I_G\left(2\sqrt{\frac{\nu M}{\alpha}}(\tau\nu)^{1/2}\right) I_{P-1}\left(2\sqrt{\frac{\nu(L-1)M}{\alpha}}\nu^{1/2}\right)\mathrm{d}\nu.$$

The above integral containing two Bessel functions is solved using integration by parts and other mathematical manipulation [4]. The details are not provided because of space limitations. Finally, we get the closed-form expression of outage probability as

$$\mathbb{P}(\text{SIR} < \tau) = \sum_{i=0}^{M}\binom{M}{i}\Gamma(1-\nu)^{M-i}(-1)^{i} \times \sum_{n=0}^{\infty} c_n\left(\frac{\nu}{\alpha M}\right)^{(1+G/2)} \times \sum_{j=0}^{(L-1)M}\binom{(L-1)M}{j}\Gamma(1-\nu)^{(L-1)M-j)}(-1)^{j} \times \sum_{k=0}^{\infty} d_k\left(\frac{\nu}{\alpha(L-1)M}\right)^{(1+P/2)}\tau^{G/2} \times \left[-\left(\frac{\tau\nu M}{\alpha}\right)^{G/2}\left(1-\frac{\nu}{L-1}\right)^{-G-P}\right] \times \left[\left(\frac{\nu(L-1)M}{\alpha}\right)^{C_1/2}\frac{\cos((C_2)\pi)\Gamma(C_2)\sin(G\pi)}{\pi}\right], \tag{13}$$

where $C_1 = (-1-2G-P)$ and $C_2 = G+P$.

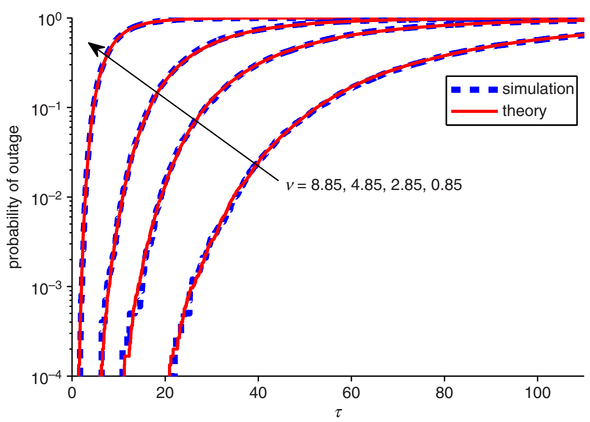

Fig. 1 *Outage probability for M = 50, L = 7 and varying τ (dBm)*

Results: In Fig. 1, we present the outage probability curves of the considered massive MIMO system. We assume $M = 50$, $L = 7$, $\alpha = 0.65$ and vary the value of ν. Note that small values of ν represent severe shadowing and vice versa. It can be seen that as we increase the value of threshold τ, the probability of outage increases. Also, as we decrease the shadowing intensity by increasing ν, the probability of outage decreases. For example, at $\tau = 40$ dBm and $\nu = 0.85$, the outage probability approaches to unity. However, for the same threshold with $\nu = 8.85$, the probability of outage becomes much smaller.

In Fig. 2, we fix the value of τ and vary the number of antennas at the BS. It is evident from this figure that as we increase the number of antennas, the probability of outage for a fixed ν decreases. It can, however, be noticed that even at 200 BS antennas, the effects of shadowing are prominent which show that shadowing is not averaged out by increasing the massive MIMO BS antennas. This observation is contrary to multipath fading which generally averages out by increasing the BS antennas. Therefore, the design of efficient receivers becomes inevitable for reliable massive MIMO operation in heavily shadowed environments.

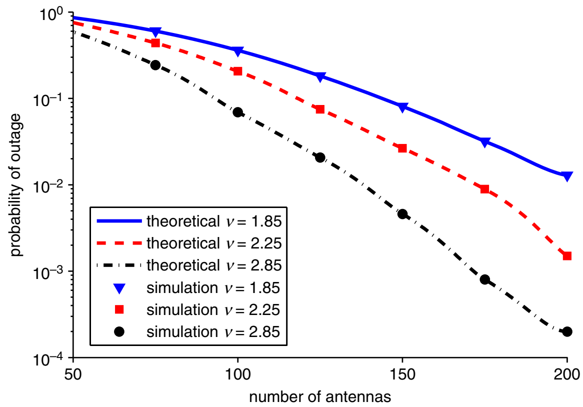

Fig. 2 *Outage probability for varying BS antennas, $\tau = 30$ dBm, $L = 7$*

Conclusion: This Letter provided the performance of a massive MIMO system in the presence of *K*-fading. Closed-form expression of the outage probability has been derived and it has been shown that the shadowing affects the performance of massive MIMO system badly. Although, an increase in the number of BS antennas improves the performance, however, unlike fading, the shadowing cannot be completely eliminated by only increasing the BS antennas.

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One or more of the Figures in this Letter are available in colour online.

M.T. Mushtaq, S.A. Hassan and S. Saleem (*School of Electrical Engineering and Computer Science (SEECS), National University of Sciences and Technology (NUST), Islamabad, Pakistan*)

✉ E-mail: tauseef06cse13@rcms.nust.edu.pk

D.N.K. Jayakody (*Department of Control System Optimization, Institute of Cybernetics, National Research Tomsk Polytechnic University (TPU), Tomsk, Russia*)

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