A New Approach to Cooperative NOMA Using Distributed Space Time Block Coding

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***Abstract*—This paper presents a novel approach to cooperative non orthogonal multiple access (NOMA) using distributed space time block coding (STBC) known as STBC-NOMA. In conventional NOMA, the strong users detect the messages of weak users through successive interference cancellation (SIC). In cooperative NOMA, these copies are then forwarded by strong users to weak users at the expense of extra time slots. However, the proposed scheme exploits this feature of cooperation using STBCs to enable cooperation among the users. The STBC-NOMA renders less complexity as lesser number of SICs are performed at each user. To this end, we derive the outage probability of the STBC-NOMA scheme which involves finding the distribution of signal-to-interference ratio at the receiving terminals. The numerical results show that STBC-NOMA outperforms conventional NOMA and conventional cooperative NOMA in terms of outage probability and average sum rate.**

***Index Terms*—Non orthogonal multiple access (NOMA), space time block coding (STBC), 5G networks.**

I. INTRODUCTION

With the evolution of fifth generation (5G) networks and increasing demand of high data rates and higher connectivity, new solutions are being investigated in both academia and industry. Recently the technology that has received considerable attention is non orthogonal multiple access (NOMA), which is considered to be one of the key 5G enabling technologies. NOMA is the new addition to the class of multiple access techniques, in which the user multiplexing is done in the power domain, such that the strong users having better channel conditions transmit with low power and the weak users having worse channels transmit with high power, while sharing the same spectral resources. NOMA increases the spectral efficiency further as compared to orthogonal multiple access (OMA) schemes by allowing different users to use the same spectral resources but with different power levels [1]. The strong users apply successive interference cancellation (SIC) to retrieve their signals while the weak users can directly decode their message with small interference from strong users [2], [3].

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The authors in [4] studied NOMA with random user deployment in a downlink scenario. Their analysis demonstrated that NOMA shows better performance as compared to OMA if the individual user data rates and power allocation coefficients are carefully selected. But the analysis also showed that there is lesser gain of NOMA at low signal-to-noise ratio (SNR) values, and the use of SIC adds more complexity to the system. In [5], the authors studied the effects of user pairing on the overall performance of NOMA and showed that the user pairing has a great effect on the overall gain. NOMA can achieve greater performance gains by selecting the users with certain channel conditions for pairing, i.e., the strong user should be paired with the weak user to fully achieve the NOMA gain. In [6], the authors proposed a cooperative NOMA scheme, in which the strong users cooperate with weak users by sending a new NOMA-based superimposed signal composed of the decoded signals of other users. The analysis showed that the cooperative NOMA has a better outage performance than the conventional NOMA. However the drawback of this scheme is the increased complexity of the system because of higher number of SICs performed on multiple received NOMA signals.

In this paper, a distributed space time block coding based downlink cooperative NOMA scheme is introduced. In the proposed STBC-NOMA scheme, the strong users cooperate with the weak users, however, unlike conventional cooperation, the strong users do not send a new superimposed signal to the weak users. Instead the strong users employ distributed Alamouti STBC scheme [7] to cooperate with the weak users. The advantages of using STBC-NOMA against the conventional cooperative NOMA scheme are lesser number of SICs performed overall in the system, implying less complexity and error.

The main contributions of this paper are the following

- We propose to use distributed STBC in cooperative NOMA and evaluate its performance in terms of outage probability and sum rates.
- To find the outage probability, we derive the expres-

- sion of the probability distribution function (PDF) of SIR, where SIR turns out to be the sum of the ratio of exponential and hypoexponential random variables (RVs) with a gamma RV.
- The analysis shows that the proposed scheme outperforms conventional cooperative NOMA scheme in terms of outage probability and sum rates.

In the following sections, the system model of STBC-NOMA and its performance analysis is discussed followed by the simulation results and conclusion.

II. SYSTEM MODEL

We consider the downlink of a communication system with a single base station (BS) and M users, where each user is equipped with a single antenna. Let the set of all users be denoted by $\mathcal{M} = \{1, 2, ..., M\}$ where without the loss in generality, we assume that the first user is the strongest whereas M-th user is the weakest user. The conventional cooperative NOMA has two phases; in the first phase, i.e., first time slot, the BS sends a superimposed signal to all the M users. Each user decodes its signal either by treating other users' signal as noise (in case of a weak user) or by using SIC (in case of a strong user). The second phase is the cooperation phase and consists of $M-1$ time slots. In each time slot, the n-th user, where $n \in \mathcal{M}$ and $n \neq M$, broadcasts a new superimposed signal which is composed of the $M-n$ decoded messages. For instance, for $n = 1$, the first user transmits a superimposed signal to the rest of $M-1$ users and so on. After M time slots, each n-th user receives n superimposed signals, where SIC is applied on each superimposed signal and different observations of the same message are combined using maximum ratio combining (MRC). A two user scenario is depicted in Fig. 1(a).

In the proposed STBC-NOMA scheme, the strong users instead of sending superimposed signal, perform distributed Alamouti coding to cooperate with weak users. Just like the conventional cooperative NOMA, there are two phases in the STBC-based cooperative NOMA, as described below.

The first phase is the conventional NOMA phase, where the BS sends a superimposed signal, X_S to all the M users, such that $X_S = \sum_{i=0}^{M} \sqrt{u_i P} x_i$, where u_i is the power allocation coefficient of the i-th user while x_i is the message for the i-th user, P is the total transmission power of BS and $\sqrt{u_i P} \triangleq \sqrt{P_i}$ represents the portion of the power given to the i-th user from the total transmission power P. The signal received by the k-th user is given by

$$y_k = h_k \sum_{i=0}^{M} \sqrt{P_i} x_i + n_k, \tag{1}$$

where h_k represents the Rayleigh fading channel coefficient from BS to the k-th user and n_k represents the Gaussian noise. As mentioned previously, the channel coefficients of M users are assumed in descending order as, $|h_1|^2 \geq |h_2|^2 \cdots \geq |h_M|^2$, thereby assuming user 1 as the strongest while M-th user as the weakest. The power coefficients are allocated based on the principle of NOMA, i.e., the user with the lowest channel gain gets the highest power allocation coefficient, such that, $u_1 \leq u_2 \cdots \leq u_M$, where $\sum_{i=0}^{M} u_i = 1$.

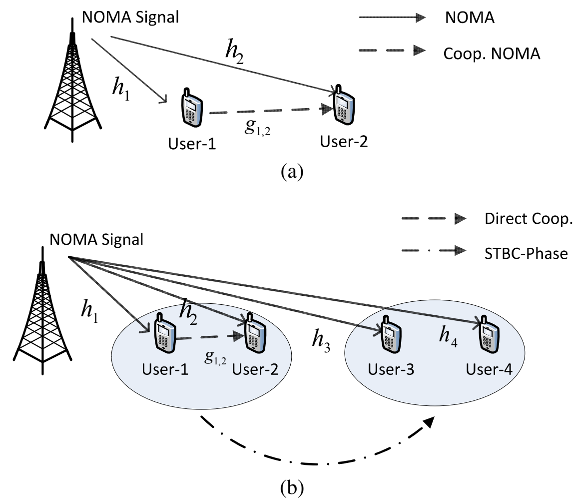

Fig. 1. Illustration of the system model for (a) M=2, (b) M=4.

The k-th user, such that $k \neq M$, detects j-th user's message signal, where $j < k$, and then performs SIC and subtracts it from the superimposed signal. The SINR at the k-th user after the conventional NOMA phase is given by

$$SINR_k = \frac{|h_k|^2 P_k}{\sum_{i=1}^{I} |h_k|^2 P_i + \sigma^2} , \tag{2}$$

where I is the number of interfering signals, such that, $1 \leq I \leq (k-1)$ and σ^2 is the noise variance assumed identical for all users.

The second phase of the proposed architecture is further divided into two sub phases, i.e., *direct cooperation phase* and the *STBC cooperation phase*. Note that there are $M-1$ time slots in the second phase, which are used by the strong users to cooperate with the weak users using a combination of direct cooperation and STBC cooperation. In the first time slot of the cooperation phase, user 1 cooperates directly with user 2 by sending a copy of user's 2 signal as shown in Fig. 1(b). Notice that user 2 has now two copies of its intended signal; one from first NOMA phase and second from this cooperative phase, which are combined using MRC. In the case where M is even, users 1 and 2 cooperate with users 3 and 4 using a 2×2 distributed Alamouti scheme. This happens in the second and third time slots of the cooperation phase. In the next two time slots, users 3 and 4 cooperate with the next two users again using STBC and this process continues till the data

TABLE I
PERCENTAGE REDUCTION IN THE TOTAL NUMBER OF SICS

Number of users, M	4	6	8	10
%age reduction in SIC	40%	57%	66.7%	72.7%

is received by $(M-1)$-th and M-th user. In case where M is odd, the process is similar until the $(M-1)$-th time slot in which $(M-1)$-th user cooperates with the M-th user directly. Note that for a fair comparison, the protocol is devised so that the time slots for conventional cooperative NOMA and STBC-NOMA remain equal.

The total transmission power, P_t, of both the phases is given by $P_t = P + P_c$, where P is the power transmitted in the conventional NOMA phase by the BS and P_c is the total power transmitted in the cooperation phase. The power transmitted in the cooperation phase is the sum of the power transmitted in the direct cooperation phase represented by $P_c^{(1)}$ and the STBC cooperation phase represented by $P_c^{(2)}$, such that $\sum_{i=1}^{2} P_c^{(i)} = P_c$.

In case of conventional cooperative NOMA, the total number of SICs performed for M users is given by $\sum_{j=0}^{M-2} \sum_{i=1}^{M-1-j} (M-(i+j))$, while for STBC-NOMA it is given by $\sum_{i=1}^{M-1} (M-i)$. For instance, for $M=4$, the total number of SICs performed in case of conventional cooperative NOMA is 10, while it is 6 for STBC-NOMA. The percentage reduction in the number of SICs performed overall in the system for STBC-NOMA as compared to conventional cooperative NOMA is given in Table I. It should however, be noticed that STBC-NOMA requires strict timing synchronization between user pairs to perform distributed STBC [8].

We now focus our attention on the SNR and SINR of the users. For $k=1$, the SNR of user 1, is given as[1]

$$SNR_1 = \frac{|h_1|^2 P_1}{\sigma^2} \ . \tag{3}$$

Whereas for $k=2$,

$$SINR_2 = \frac{|h_2|^2 P_2}{|h_2|^2 P_1 + \sigma^2} + \frac{|g_{1,2}|^2 \hat{P}}{\sigma^2} \ , \tag{4}$$

where $\hat{P}$ represents the portion of the power allocated to a single direct transmission from the total transmission power of direct cooperation phase, $P_c^{(1)}$. In (4), $|g_{1,2}|^2$ is the channel gain between user 1 and 2. For $2 < k \leq M$, the SINR is given by

$$\begin{aligned} SINR_k = & \frac{|h_k|^2 P_k}{\sum_{i=1}^{I} |h_k|^2 P_i + \sigma^2} \\ & + \frac{(|g_{k-n-1,k}|^2 + |g_{k-n-2,k}|^2)\tilde{P}}{\sigma^2} \ , \end{aligned} \tag{5}$$

where $\tilde{P}$ represents the portion of the power allocated to a single STBC transmission from the total transmission power of STBC cooperation phase, $P_c^{(2)}$, while $n = \{0,1\}$ represents the first and second receiver in the STBC receiver pair. The second term in (5) represents the gain due to the Alamouti scheme. For the case when M is odd, the SINR for $k = M$ is given by

$$SINR_k = \frac{|h_k|^2 P_k}{\sum_{i=1}^{I} |h_k|^2 P_i + \sigma^2} + \frac{|g_{k-1,k}|^2 \hat{P}}{\sigma^2} \ . \tag{6}$$

III. PERFORMANCE ANALYSIS

In this section, the outage probability and outage rate analysis is provided. We assume an interference-limited scenario and hence noise can be ignored.

A. Outage Probability Analysis

The SIR in (5) can be written as

$$A = K + Z \ , \tag{7}$$

where $K = X/Y$ denotes the SIR at the k-th user after the conventional NOMA phase, while Z denotes the STBC gain after the cooperative phase. In the definition of K, X denotes the power of the desired signal, which is exponentially distributed, and its probability density function is given as, $f_X(x) = \frac{1}{\lambda}\ exp\left(-\frac{x}{\lambda}\right)$, where λ is the mean power of the desired signal X. On the other hand, Y is the sum of $(k-1)$ exponentially distributed RVs with distinct parameters. Hence the distribution of Y is hypoexponential and is given by

$$f_Y(y) = \sum_{i=1}^{I} C_i \ \frac{1}{\Omega_i}\ exp\left(-\frac{y}{\Omega_i}\right) \ , \tag{8}$$

where $C_i = \prod_{j \neq i} \frac{1/\Omega_j}{1/\Omega_j - 1/\Omega_i}$ and I is the total number of interferers such that, $1 \leq I \leq (k-1)$ and Ω_i is the mean power of the i-th interferer.

The distribution of Z in (7) is gamma and is given by, $f_Z(z) = \frac{z}{\eta^2}\ exp\left(-\frac{z}{\eta}\right)$, where η is the mean power of the signal received from a user transmitting in cooperation mode. The outage probability of a user in STBC-NOMA can be expressed as

$$\mathbb{P}\{SIR < \tau\} = 1 - \int_{\tau}^{\infty} f_A(a)\ da \ , \tag{9}$$

where τ is the SIR threshold. To find the outage probability, we require the PDF of A in (7), which is the sum of two RVs, namely K and Z. Since X and Y are mutually independent, the cumulative distribution function (CDF) of K is obtained as

$$\mathbb{P}\{K > k\} = \int_{x=0}^{\infty} \left(\int_{y=0}^{x/k} f_Y(y) dy \right) f_X(x) dx \ , \tag{10}$$

where

$$\int_{y=0}^{x/k} f_Y(y) dy = \sum_{i=1}^{I} C_i \left[1 - exp\left(-\frac{x}{k\Omega_i}\right) \right] \ . \tag{11}$$

[1]Note that the first user detects data for all $(M-1)$ users using $(M-1)$ SIC operations.

The expression in (11) is the CDF of the hypoexponential RV [9]. The expression in (10) can be written as

$$\mathbb{P}\{K > k\} = \int_{x=0}^{\infty} \sum_{i=1}^{I} C_i \left[1 - exp\left(-\frac{x}{k\Omega_i}\right)\right] \times \frac{1}{\lambda}\ exp\left(-\frac{x}{\lambda}\right) dx\ , \tag{12}$$

which after mathematical manipulations becomes

$$\mathbb{P}\{K > k\} = \sum_{i=1}^{I} C_i \left(\frac{\lambda}{\lambda + k\Omega_i}\right) . \tag{13}$$

The PDF of K is thus given as

$$f_K(k) = \sum_{i=1}^{I} C_i \Omega_i \left(\frac{\lambda}{(\lambda + k\Omega_i)^2}\right) . \tag{14}$$

Using the properties of random variables, the PDF of A in (7) can be obtained by convolving the PDF of K in (14) with the PDF of Z and is given as

$$f_A(a) = \sum_{i=1}^{I} \frac{C_i}{\Omega_i^2 \eta^3}\ exp\left(-\frac{\lambda + \Omega_i a}{\eta\Omega_i}\right) \left[exp\left(\frac{\lambda}{\eta\Omega_i}\right) \times \eta\Omega_i(\lambda - e^{\frac{a}{\eta}}\lambda - a\Omega_i) + \lambda(\lambda + \Omega_i(a - \eta)) \times \left(Ei\left[\frac{\lambda + a\Omega_i}{\eta\Omega_i}\right] - Ei\left[\frac{\lambda}{\eta\Omega_i}\right]\right)\right] , \tag{15}$$

where Ei represents the exponential integral function, i.e., $Ei(x) = \int_{-\infty}^{x} \frac{e^t}{t} dt$. The outage probability of a user in the STBC-NOMA scheme is thus given as

$$P_{out}(\tau) = \sum_{i=1}^{I} \frac{C_i}{\Omega_i^2 \eta^2}\ exp\left(-\frac{\lambda + \Omega_i \tau}{\eta\Omega_i}\right) \left[exp\left(\frac{\lambda}{\eta\Omega_i}\right) \times \eta\Omega_i(e^{\frac{\tau}{\eta}}(\lambda + \eta\Omega_i) - \Omega_i(\tau + \eta) - \lambda) + \lambda(\lambda + \tau\Omega_i) \times \left(Ei\left[\frac{\lambda}{\eta\Omega_i}\right] - Ei\left[\frac{\lambda + \tau\Omega_i}{\eta\Omega_i}\right]\right)\right] . \tag{16}$$

B. Outage Rate Analysis

The analytical expression of the outage probability has been derived in the above section, for users with SIR less than the threshold, τ. Here the expression for rate outage probability is provided, which is the probability that the rate of a certain user is less than some threshold, ζ. Given the SIR outage probability of a user is less than a threshold τ, $P_{out}(\tau)$, the outage rate probability is given by $P(\zeta) = P_{out}\ (2^{\zeta/B} - 1)$, where ζ is the rate threshold and B is the bandwidth [10].

IV. RESULTS & SYSTEM PERFORMANCE

In this section, the performance of the proposed scheme is investigated via numerical simulations. For analysis, the downlink of a communication system is considered with a single BS and M users, where the channel coefficients between BS and users are Rayleigh flat faded. The maximum transmit power of a user in cooperation phase is taken to be $1W$. A user takes part in cooperation only if its rate is greater than the rate threshold, ζ. For a fair comparison, the total transmission power of the three schemes is kept equal, i.e., $P_t^{NOMA} = P_t^{C-NOMA} = P_t^{STBC-NOMA}$.

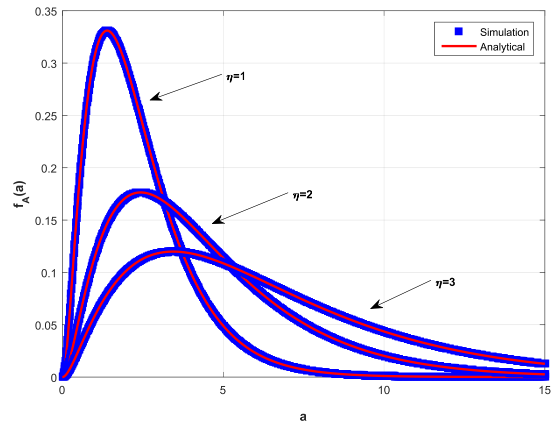

Fig. 2. Comparison of the simulated and analytical PDFs for $I = 3$, $\lambda = 3$, $\Omega_i = [2, 4, 6]$ and varying η.

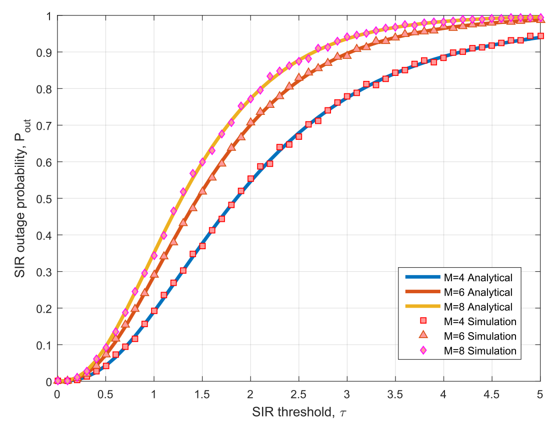

Fig. 3. SIR outage performance of STBC-NOMA for varying number of users.

Fig. 2 compares the PDF of SIR derived in Section III with the simulation results for $M = 4$ and different values of η. It can be seen that the simulation results closely match the analytical results. Figs. 3 and 4 show the SIR outage probability and rate outage probability as a function of τ and ζ, respectively, for $P = 20\ dB$. It can be observed from both the graphs that the outage performance of the system degrades with increasing the number of users. Furthermore, the analytical results obtained from the derived equations in Section III agree with the simulation results.

Fig. 5 compares the sum rates of non-cooperative

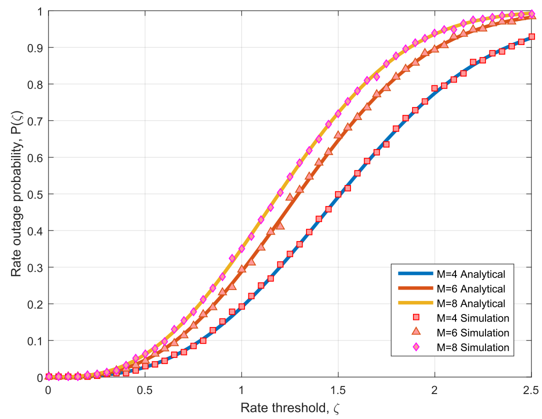

Fig. 4. Rate outage performance of STBC-NOMA for varying number of users.

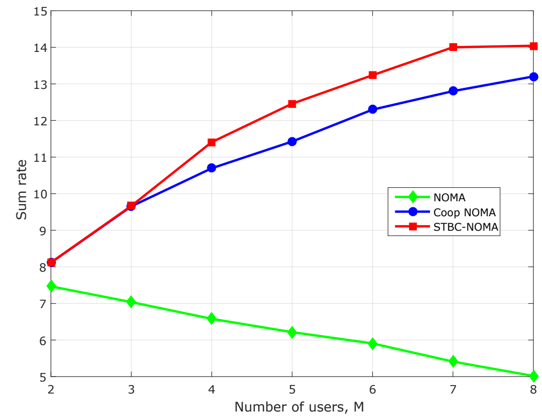

Fig. 6. Sum rates comparison of NOMA, cooperative NOMA and STBC-NOMA, for M=8.

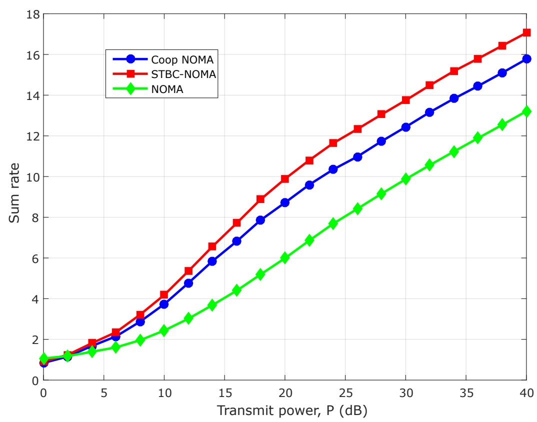

Fig. 5. Comparison of sum rates of NOMA, cooperative NOMA and STBC-NOMA, for M=8 and ζ=1.5.

NOMA, cooperative NOMA and STBC-NOMA for $M = 8$ and $\zeta = 1.5$. The simulation results show that STBC-NOMA has better performance in terms of sum rate. It can be seen that for low BS transmit power, P, there is no visible performance gain of cooperative NOMA and STBC-NOMA as compared to NOMA. But as the total transmit power, P, increases, a large performance gain is observed. Fig. 6 shows the comparison of sum rates with number of users, M for $\zeta = 1.5$ and $P = 20\ dB$. It can be observed that STBC-NOMA exhibits better performance. The performance of NOMA degrades with increasing M, which is due to the fact that with large number of users, the allocated power to each user in NOMA becomes small and the gain becomes insignificant.

V. CONCLUSION

In this paper, we studied an STBC-based cooperative NOMA scheme, where the strong users cooperate with weak users using distributed Alamouti STBC. The SIR and rate outage probability expressions are obtained based on the received SIR. The simulation results show that STBC-NOMA can significantly outperform both NOMA and conventional cooperative NOMA in terms of outage probability and sum rates. As a future work, an optimization scheme for the power allocation for STBC-NOMA can be studied.

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