Optimal Polarization Diversity Gain in Dual-Polarized Antennas Using Quaternions

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***Abstract*—Over the past few decades, wireless communication aims to support high data rates and reliability and thus several techniques exploiting space, time, and polarization diversities have been used to achieve large diversity gains. Orthogonal space time block codes (OSTBC) in combination with polarization diversity promise optimal diversity gains. For dual-polarized antennas, we propose a new system model based on the quaternionic structure of the channel, which offers a way to exploit polarization diversity quite independently of other forms of diversities. Moreover, such OSTPBC achieve better throughput and provide a linear decoupled decoding solution at the receiver, which significantly reduces computational complexity.**

***Index Terms*—Dual-polarized antennas, multiple antennas, polarization diversity, quaternion algebra, space-time codes.**

I. Introduction

Wireless prorogation channel experiences attenuation due to multipaths and multiuser interference which makes the detection of the transmitted signal difficult at the receiver. Transmit antenna diversity is a viable solution where multiple copies of the same transmitted signal incorporate different delays and create frequency selective fading. The receiver can process the received signals to achieve the diversity gain exploiting various forms such as time, space, frequency or polarization diversity. Increasing the number of transmit antennas enhances the diversity gain yet introduces problems of reduced code rates and increased receiver complexity [1]. In practice, the insufficient antenna spacings and the lack of scattering reduce the capacity, owing to an increased channel correlation and for closely spaced antennas, mutual coupling might not be negligible. Interestingly, the antenna coupling might be beneficial to multiple-input and multiple-output (MIMO) systems [2] contrary to what was believed earlier that it degrades the capacity. Mobile communication is approaching and aiming to its fifth generation which demands high data rates and reduced latency. In order to achieve that, different diversity schemes are integrated with polarization diversity [3], [4] as they can be used to mitigate the multipath effect to maintain a reliable communication link with an acceptable quality of service.

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Polarization diversity requires no extra bandwidth as well as physical antenna separation. Polarization diversity enables the simultaneous transmission and reception of information signals using the orthogonally polarized antennas [5].

Recently, the quaternion orthogonal designs (QODs) have been studied extensively [6]–[9]. In all such approaches, the quaternion designs were not directly employed, rather complex quasi-orthogonal space time block codes (STBCs) were obtained and different system models were constructed from them. Such nonorthogonal designs were capable to achieve higher diversity gains in MIMO systems. However, these designs adversely affect the performance for two main reasons. First, because of the nonorthogonal nature of these codes, it is impossible to obtain decoupled decoding at the receiver end. On the other hand, the theory of QODs was initially designed following the idea proposed in [10] to exploit polarization diversity which it failed to attain in general. In STBCs, the polarization diversity is attained from the cross-polar (CP) components in the wireless channel which indicates that it relies on the time diversity and cannot be exploited independently.

In this letter, we introduce a new system model based on quaternion channels which provides optimal solutions to the aforementioned problems. It is demonstrated that quaternions offer a better solution especially for dual-polarized antennas to attain polarization diversity gains without exploiting time diversity. Such designs achieve better throughput and linear decoupled decoding solution is an intrinsic feature of the approach. It is shown that QOD of order 1, which is suitable for a 1×1 configuration of dual-polarized antennas, provides more information than the standard Alamouti code for a 2×2 single-polarized system. Besides, the existence of more than one QOD of order 2 with a maximal rate provides them a sharp edge over an Alamouti scheme in which there is only a single code of "complex" rate 1.

II. Interpreting QODs

A quaternion is represented by two complex numbers as $q = z_1 + z_2 j$, such that $z_1, z_2 \in \mathcal{C}$. The quaternion space Q is fundamentally composed of a noncommutative basis $\{1, i, j, k\}$, such that $i^2 = j^2 = k^2 = ijk = -1$ and $ij = k = -ji, jk = i = -kj, ki = j = -ik$. As in the complex domain, the quaternion conjugate q^Q is defined as $q^Q = z_1^* - jz_2^*$, along with the property that $q^Q q = qq^Q = |q|^2$. Furthermore, the transpose of a quaternion matrix $\mathbf{Q} = [q_{mn}]$ is defined as $\mathbf{Q}^Q = [q_{nm}^Q]$ [11].

For dual-polarized transmit antennas, we propose a relatively modified system model where it is assumed that two colocated single-polarized antennas (horizontal and vertical) of a particular dual-polarized antenna will be considered as a single unit, symbolically $T^D = T_H + T_V j$. We use superscript

D to indicate a dual-polarized antenna, where the symbols H and V denote the horizontal and vertical polarization, respectively. Therefore, the transmission of complex symbols through a single dual-polarized unit can be modeled by a quaternion $q = z_1 + z_2 j$, which ensures that symbols z_1 and z_2 are transmitted instantaneously through T_H and T_V, respectively. Note that the coupling with j ensures that the symbol z_2 is transmitted through an orthogonal polarization. Each complex symbol z is obtained from standard modulation schemes, e.g., quadrature phase shift key. In order to exploit diversity gains from space and time, the orthogonal space-time polarization block codes (OSTPBC) can be defined in the quaternion domain [6].

Definition II.1 (QOD): A QOD $\mathbf{Q}$, on pure quaternion elements $\{q_1, q_2, \ldots, q_n\}$ of type $\{s_1, s_2, \ldots, s_n\}$ is an $m \times n$ matrix with entries from set $\{0, q_1, q_1^*, q_2, q_2^*, \ldots, q_n, q_n^*\}$ including possible multiplications on the left and/or right by quaternion elements $q \in Q$ and satisfying the condition

$$\mathbf{Q}^Q \mathbf{Q} = \sum_{h=1}^{n} (s_h(|q_h|)^2)\mathbf{I}_{n\times n} = \lambda \mathbf{I}_{n\times n} \tag{1}$$

where $\mathbf{I}_{n\times n}$ is the $n \times n$ identity matrix and λ is a positive real number.

Based on the above realization, it is now natural to redefine code rates for QODs.

Definition II.2: The quaternion code rate r_q of a QOD $\mathbf{Q}$ is the ratio of number of transmitted quaternions to the number of time slots through a combination of dual-polarized antennas.

For example, for a single dual-polarized antenna T^D, a QOD $\mathbf{Q}_1 = [q]$ where $q^Q q = 1$, has rate $r_q = 1$, transmitting one quaternion in one time slot. It is emphasized that the above construction is in sharp contrast to the standard procedure given in [6]– [8], [13], where $\mathbf{Q}_1$ is thought to be made of two complex numbers and it yields a quasi-code $\mathbf{C}_q = [z_1\ z_2]$, which has “complex” rate 2. We will not be using such quasi-codes, $\mathbf{C}_q$, throughout this letter and they will only be indicated for a brief comparison.

In order to develop quaternion designs over an arbitrary number of transmitting dual-polarized antennas, we proceed as follows. First, it is pertinent to mention that in all previous approaches [6], [8], the QODs were obtained by employing conditions on underlying CODs $\mathbf{A}$ and $\mathbf{B}$, namely, they must form a symmetric-pair and satisfy the amicable condition. It was shown that by permuting columns of one of the CODs, another COD can be constructed which satisfy both of these conditions. Furthermore, the matrix $\mathbf{B}$ needs not necessarily be a permuted version of $\mathbf{A}$ [12]. In [6, Th. 9], Gu and Wu proved that two CODs which form a symmetric pair design generate a QOD. It was then required to see whether all QODs arise from symmetric pair designs. In the following theorem, we show that it is indeed a case.

Theorem II.1: The necessary and sufficient condition for an STBC in the quaternion domain $\mathbf{Q} = \mathbf{A} + \mathbf{B}j$ to be a QOD is that both $\mathbf{A}$ and $\mathbf{B}$ are CODs and satisfy the symmetric property, i.e., $(\mathbf{A}^H\mathbf{B})^T = \mathbf{A}^H\mathbf{B}$, where $(.)^H$ and $(.)^T$ denotes the Hermitian and transpose operators, respectively.

Proof: A QOD $\mathbf{Q}$ has a unique decomposition in the complex domain, i.e., $\mathbf{Q} = \mathbf{A} + \mathbf{B}j$, where both $\mathbf{A}$ and $\mathbf{B}$ are two complex matrices of the same order as of $\mathbf{Q}$. We first note that $\mathbf{A}^Q = \mathbf{A}^H$ and following identity holds. Suppose $\mathbf{A} = [a_{mn}]$, such that $a_{mn} \in \mathcal{C}$, then $\mathbf{A}j = j\mathbf{A}^*$ where $*$ denotes the conjugation operation. Consequently, $\mathbf{A}^H j = j\mathbf{A}^T$ and $j\mathbf{A}^H = \mathbf{A}^T j$. Note that the multiplication with quaternion j eats up the conjugation in Hermitian. It turns out that

$$\mathbf{Q}^Q\mathbf{Q} = (\mathbf{A}^H - j\mathbf{B}^H)(\mathbf{A} + \mathbf{B}j), \tag{2}$$

$$= \mathbf{A}^H\mathbf{A} - j\mathbf{B}^H\mathbf{B}j + \mathbf{A}^H\mathbf{B}j - j\mathbf{B}^H\mathbf{A}, \tag{3}$$

$$= \mathbf{A}^H\mathbf{A} - j\mathbf{B}^H\mathbf{B}j + \left(\mathbf{A}^H\mathbf{B} - (\mathbf{A}^H\mathbf{B})^T\right) j \tag{4}$$

therefore the condition (1) is true if and only if

$$\mathbf{A}^H\mathbf{A} = \lambda_1\mathbf{I},\ \mathbf{B}^H\mathbf{B} = \lambda_2\mathbf{I},\ \mathbf{A}^H\mathbf{B} - (\mathbf{A}^H\mathbf{B})^T = \mathbf{0}.$$

Hence, proved.

There were four ways indicated in [6] which can be used to obtain viable QODs of order 2. Theorem 2.1 indicates that the construction of QODs entirely depends on finding two CODs which form symmetric-pair design, therefore, it can be shown that all those QODs arise from symmetric-pairs. We now consider the problem of generating QODs of order 2 from a geometrical point of view. Suppose we have a quaternion matrix

$$\mathbf{Q}_1 = \begin{bmatrix} q_1 & q_2 \\ q_3 & q_4 \end{bmatrix} = \begin{bmatrix} z_1 + z_2 j & z_3 + z_4 j \\ z_5 + z_6 j & z_7 + z_8 j \end{bmatrix} \tag{5}$$

which can provide a coding matrix of maximum rate $r_q = 2$ for two dual-polarized antennas provided $\mathbf{Q}_1$ satisfies orthogonality (1). Geometrically, the requirement (1) can be interpreted as follows. The quaternion vectors $\mathbf{q}_1 = [q_1\ q_3]^T, \mathbf{q}_2 = [q_2\ q_4]^T$, reside in an eight-dimensional space $\mathbf{q}_1, \mathbf{q}_2 \in Q \times Q \simeq \mathbb{R}^8$, containing eight linearly independent real vectors. The problem now is to choose two such orthogonal vectors from $Q \times Q$, resulting in $\binom{8}{2} = 28$ combinations to choose from, such that the resulting pair ensures (1). Since the diagonal entries of $\mathbf{Q}_1^Q\mathbf{Q}_1$ are same, i.e., $|\mathbf{q}_1|^2 + |\mathbf{q}_2|^2$. Therefore, to meet the orthogonality condition (1), the off-diagonal terms must vanish necessarily, i.e.,

$$\begin{aligned} q_1^Q q_2 + q_3^Q q_4 &= 0, \\ q_2^Q q_1 + q_4^Q q_3 &= 0. \end{aligned} \tag{6}$$

As we have two algebraic equations in four unknown quaternions, therefore, the quaternionic rate is bounded above by 1, i.e., $|r_q| \leq 1$ for QODs of order 2. This result seems analogous to an Alamouti code which has a maximum rate 1 for two single-polarized transmit antennas. However, in QODs the quaternionic rate $r_q = 1$ corresponds to an STBC with *complex* rate 2 which implies transmission of four complex symbols in two time slots. As is known, there exists no such COD of order 2 which has “complex” rate 2. More concretely, for four single-polarized antennas corresponding to two dual-polarized antennas, there does not exist any COD which has maximal rate equal to 1 and it is a unique code of order 2 which exists only for two transmit antennas [17]. Therefore, the theory of QODs offers us better ways to deal with the problem of generating codes especially for dual-polarized antennas. In addition to this, there is a class of QODs of order 2 with maximal rate $r_q = 1$, thereby, violating the uniqueness condition of Alamouti schemes. Another essential feature of such QODs is that they provide linear decoupled decoding solution which is briefly discussed in the subsequent section.

The conditions in (6) can be decomposed into a system of eight real algebraic equations, which can be solved using any computer algebra system. We consider one such solution of a

QOD of order 2 with maximal rate $r_q = 1$, i.e.,

$$\mathbf{Q}_2 = \begin{bmatrix} z_1 + z_2 j & z_4 + z_3 j \\ z_2^* - z_1^* j & -z_3^* + z_4^* j \end{bmatrix} \tag{7}$$

in which two dual-polarized antennas transmit two quaternions $z_1 + z_2 j$ and $z_4 + z_3 j$ in the first time slot, respectively. Subsequently, $z_2^* - z_1^* j = -j(z_1 + z_2 j)$ and $-z_3^* + z_4^* j = j(z_4 + z_3 j)$ are sent in the second time slot. For completeness, we construct another QOD of a maximal rate. Suppose we have two quaternions $q_1 = z_1 + z_2 j$ and $q_2 = z_3 + z_4 j$, such that $q_1 q_2^Q$ does not contain kth component. Then, it is easy to generate a QOD such that

$$\mathbf{Q}_3 = \begin{bmatrix} i(z_1 + z_2 j) & -j(z_4 + z_3 j) \\ j(z_4 + z_3 j) & -i(z_1 + z_2 j) \end{bmatrix}.$$

Similarly, another QOD of rate $r_q = 1/2$ is given as

$$\mathbf{Q}_4 = \begin{bmatrix} z_1 + z_2 j & j(z_1 + z_2 j) \\ i(z_1 + z_2 j) & -k(z_1 + z_2 j) \end{bmatrix}. \tag{8}$$

Likewise, a QOD of order 3 of "complex" rate 3/4 is,

$$\mathbf{Q}_5 = \begin{bmatrix} z_1 + z_2 j & z_2 + z_1 j & z_3 + z_3 j \\ -z_2^* + z_1^* j & z_1^* - z_2^* j & 0 \\ -z_3^* & -z_3^* j & z_1^* + z_1^* j \\ -z_3^* j & -z_3^* & z_2^* + z_2^* j \end{bmatrix}. \tag{9}$$

All such codes have decoupled decoding as shown in the subsequent section.

III. Quaternionic System Model

A. Channel Realization

Suppose we have a system with a 1×1 configuration of dual-polarized antennas. As described above, the transmission through a dual-polarized antenna T^D is modeled by a single quaternion $q = z_1 + z_2 j$, where z_1 and z_2 are simultaneously transmitted through T_H and T_V, respectively. We now propose a quaternionic channel gain between transmit and receive dual-polarized antennas, given by $h = h_1 + h_2 j$. Therefore, the received vector is a quaternion and has the form

$$r = qh + n \tag{10}$$

where n is a quaternionic noise, i.e., $n = n_1 + n_2 j$. It should be pointed out that in [6] and [8], the same design was implemented in a different way where a quasi-code was constructed, i.e., $\mathbf{C}_q = [z_1 \;\; z_2]$, using an operator $\mathbb{C}$ [14]. The necessity of such an operation is due to the channel which was based on complex numbers. In our proposed model, the channel is quaternionic where the aim is to exploit quaternion domain and its intrinsic operations in full spirit. More briefly, the quaternionic form of channel is also studied in a different context in [19]. For a fair comparison, this situation may be compared with a 2×2 single polarized antennas where the system model is

$$\mathbf{R} = \begin{bmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{bmatrix} = \begin{bmatrix} z_1 & z_2 \\ -z_2^* & z_1^* \end{bmatrix} \begin{bmatrix} h_{HH} & h_{HV} \\ h_{VH} & h_{VV} \end{bmatrix} + \begin{bmatrix} n_{11} & n_{12} \\ n_{21} & n_{22} \end{bmatrix}. \tag{11}$$

It is clear from above that there are four complex channel coefficients owing to the fact that there must be four complex channel gains between two dual-polarized antennas. Apparently, our proposed quaternion channel seems to contain two complex numbers. However, it is important to note that in our analysis the quaternionic product "qh" is very crucial. It is only after its decomposition, we begin to see the underlying operations behind quaternions. For example, we have $qh = z_1 h_1 - z_2 h_2^* + (z_1 h_2 + z_2 h_1^*)j$, which indicates that the first complex symbol $z_1 h_1 - z_2 h_2^*$, which is received at $H-$ polarized antenna is a combination of two complex channel gains. Similarly, the other part $z_1 h_2 + z_2 h_1^*$, received at $V-$ polarized antenna, again contains two complex channel gains producing a total of four gains as is expected in the standard approach of which two are related with the remaining two through conjugation. Consequently, our model inherently assumes the quaternionic form of the channel which can be further confirmed by performing practical implementations on a test-bed and is left as a future work.

The CP components h_{HV} and h_{VH} in (11) are responsible for incorporating the polarization twists by reflection, scattering, and other means [18]. However, their use entirely depends on sending the same copies of signals in different time slots due to the nature of Alamouti code in (11). Therefore, the exploitation of polarization diversity relies on time diversity. On the other hand, quaternionic model (10) does not need time diversity at all as both signals z_1 and z_2 are instantaneously sent through orthogonal polarization planes. This characteristic of quaternion designs for dual-polarized antennas gives them edge over almost all known complex *orthogonal* designs and to the best of knowledge of authors, it cannot be replicated in the complex domain. Also note that in the system model (10), there is no coding/decoding delay while in (11) there is a delay at both ends. Besides the performance of Alamouti code, (11) depends on maintaining spatial distance of at least half a wavelength at the receiver side while 10 wavelengths at the base station in order to achieve maximum diversity gains [4].

Following above lines, it is now natural to propose a general quaternionic system model for $N_T \times 1$ configuration of dual-polarized antennas which transmits symbols in T time slots

$$\mathbf{R}_{T\times 1} = \mathbf{Q}_{T\times N_T} \mathbf{H}_{N_T\times 1} + \mathbf{N}_{T\times 1} \tag{12}$$

where $\mathbf{H} = [h_1, h_2, \ldots, h_{N_T}]$, such that each entry is a quaternion $h_a = h_{a1} + h_{a2} j$, for all $a \in \{1, 2, \ldots, N_T\}$. The complex channel gains, h_{a1} and h_{a2}, incorporate the effects of CP scattering and the channel is assumed to be Rayleigh fading, which implies that each element of channel gain matrix is a complex Gaussian random variable (RV) with zero mean and unit variance. Moreover, the noise $\mathbf{N} = [n_1, n_2, \ldots, n_T]$ and $n_b = n_{b1} + n_{b2} j$, such that $n_{b1}, n_{b2} \, \forall \, b = \{1, 2, \ldots, T\}$ represent the entries of white noise as two-dimensional independent and identically distributed complex Gaussian RVs with zero mean and identical variance per dimension. It is emphasized that the above approach is different from previous attempts due to the quaternionic nature of the channel. For clarity, we include a complex "quasi-code" [16]

$$\mathbf{C}_q = \begin{bmatrix} z_1 & z_2 & z_2 & z_1 \\ -z_2^* & z_1^* & z_1^* & -z_2^* \end{bmatrix} \tag{13}$$

constructed from QOD (8) in [13], where odd columns refer to transmission from one polarization plane and even columns contain symbols that are transmitted through orthogonal polarization plane. It was shown in [13] that an OSTPBC for a 2×1 dual-polarized unit performs better than an Alamouti code for a 2×1 single-polarized unit. Unfortunately, this is not a fair comparison because the performance of a 2×1 dual-polarized unit may be compared with the performance of a 4×2 single-polarized unit as is also pointed out in [4]. Furthermore, the above quasi-code (13) does not provide a decoupled solution at the receiver end. On the other hand, our proposed model is fully decoupled as shown below.

B. Linear Decoupled Solution

In all previous attempts, the ML-decoding rule norm is equivalent to the minimum of either the norm $||\mathbf{R} - \mathbf{C}_q\mathbf{H}||$ or its square for finding the transmitted symbols where the channel $\mathbf{H}$ was assumed complex. Seberry *et al.* [6] proposed that a decoupled decoding can be obtained for any QOD even if the channel gain matrix of a dual-polarized transmission system is not modeled by a single quaternion gain. Later on, they corrected their decoding rule [14] and clarified that the decoupled decoding can be achieved for certain QODs only [15]. The main reason was assumption of the complex nature of the channel. The following theorem confirms a linear decoupled solution to our proposed model in (12).

Theorem III.1: For a given system model in (12), the ML-decoding rule assumes a linear decoupled form

$$\min_z ||\mathbf{R} - \mathbf{QH}||^2 = \min_z \Big(tr(\mathbf{R}^Q\mathbf{R}) + \lambda tr\big(\mathbf{H}^Q\mathbf{H}\big) - 2\Re\big(tr\big(\mathbf{R}^Q\mathbf{QH}\big)\big)\Big). \quad (14)$$

Proof: The proof is straight forward owing to the fact that $||\mathbf{R} - \mathbf{QH}||^2 = \text{tr}\left((\mathbf{R} - \mathbf{QH})^Q(\mathbf{R} - \mathbf{QH})\right)$, which is easy to expand. The term $\text{tr}(\mathbf{H}^Q\mathbf{Q}^Q\mathbf{QH})$, which was the main source of problems in all previous attempts, reduces to $\lambda\text{tr}(\mathbf{H}^Q\mathbf{H})$, using orthogonality condition (1) and does not contain the transmitted symbols. Consequently, there is only one term which is linear and contain transmitted symbols. Hence proved.

Since the first two terms in (14) are pure constants, therefore, the ML-decoding rule of minimizing the norm is equivalent to minimize $\Re\big(tr\big(\mathbf{R}^Q\mathbf{QH}\big)\big)$, where $\Re$ denotes the real part of a complex number.

IV. Simulation Results

To evaluate the performance and diversity gains, we employ QODs given in (7)–(10), corresponding to configurations 2×1, 3×1, and 1×1 of dual-polarized antennas, respectively. For simulations, quadrature phase shift keying is used and equal power distribution is ensured per antenna per polarization. The receivers are aware of the channel coefficients, and uniform white noise is added in each polarization.

From Fig. 1, it is clear that the performance of (10) for a 1×1 dual-polarized system matches with that of an Alamouti design (11) for 2×2 single-polarized antennas with non-CP (NCP) components. However, it is worth pointing out that the Alamouti design exploits time diversity to attain it whereas (10) achieves the same performance using polarization diversity. Contrary to what one would suspect that the performance match between (10) and (11) with NCP components will occur in case of higher order configurations. The QOD (8) for 2×1 dual-polarized antennas that has complex rate 1 performs significantly better in Fig. 2 than its counterpart which is an Alamouti design for 4×2 single-polarized antennas with NCP. An improvement in the performance also comes from the fact that (8) exploit time

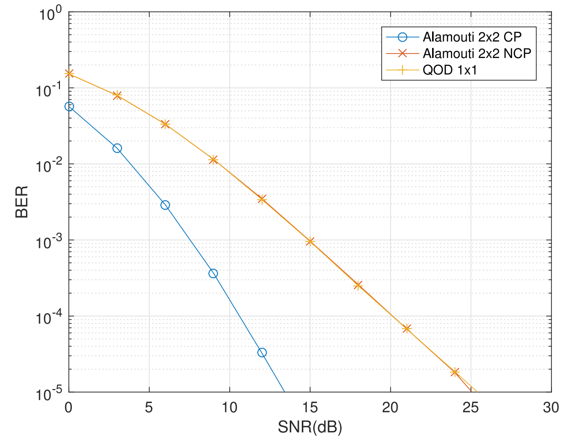

Fig. 1. BER versus SNR performance of (1× 1) QOD.

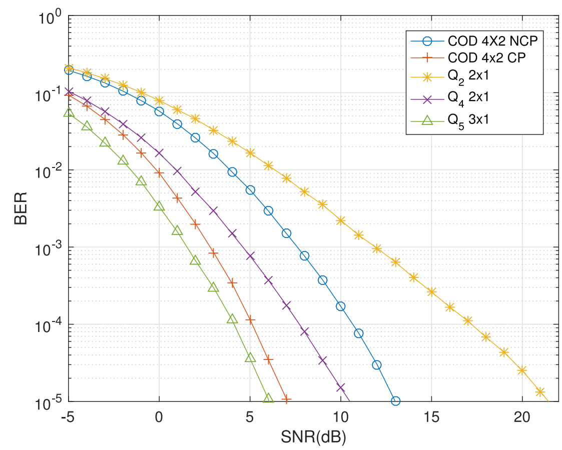

Fig. 2. BER versus SNR performance of $\mathbf{Q}_2$, $\mathbf{Q}_4$, $\mathbf{Q}_5$, and CODs.

diversity besides the polarization gain. In Fig. 2, we also include the performance of QOD (7) which has its unique characteristic of sending four complex symbols in two time slots. Finally, polarization diversity gain is attained by increasing the transmit antenna dimensions only by 1 and it is clear from bit error rate (BER) performance of Q_5 that the gain will become more pronounced in higher dimensions.

V. Conclusion

This letter was aimed to attain diversity gains due to space, time, and polarization diversities in the quaternion domain. A new system model based on the quaternion channel is introduced. The use of quaternions guarantees polarization diversity gain and does not necessarily requires the presence of time diversity. A remarkable feature of the proposed model is linear decoupled solution at the receiver end. The simulation graphs in Fig. 1 indicates that quaternions are more suitable candidate to exploit polarization diversity than complex codes. There are several potential areas of interest that we aim to explore in the future. For more than one dual-polarized antenna, the quaternion designs seem to rely on time diversity, e.g., QODs in (7) and (8) both exploit it. It will be interesting to find mathematical structures that remove this necessity. Intuitively, a natural jump from quaternions to designs with octonions may provide us a positive answer. Further study is required for higher order QODs and a mechanism which identifies the equivalent codes.

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