Energy Efficient Neighbor Discovery for mmWave D2D Networks using Polya's Necklaces

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***Abstract*—Device-to-device (D2D) communication is a novel paradigm in cellular networks and is being considered as one of the primary technologies for the upcoming fifth generation (5G) standard for cellular communication. On a similar note, millimeter wave (mmWave) communication is also one of the very enablers for 5G. For D2D communication, neighbor or device discovery is a fundamental problem in mmWave networks because of the use of highly directional antennas. In this paper, we consider this fundamental neighbor discovery problem: How can nodes discover their neighbors quickly and efficiently for communication in decentralized networks, without any prior coordination and with heterogeneous antenna configurations. We propose a novel D2D neighbor discovery algorithm that uses the idea of necklaces to reduce the worst case discovery delay as compared to the previous approaches. The results from numerical simulations confirm that the proposed algorithm leads to faster and energy efficient neighbor discover as compared to the existing algorithms.**

***Keywords*– directional neighbor discovery, D2D communication, mmWave networks, Pólya necklaces.**

I. INTRODUCTION

With the recent exponential growth in wireless traffic which is likely to continue, it is anticipated that the demand for bandwidth will grow beyond what is presently assigned for this purpose [1]. Currently, in the fourth generation (4G) cellular networks, the frequency band up to 6 GHz is in use. As more devices requiring Internet connectivity arrive, the spectrum crunch would result in a loss of quality-of-service parameters such as slower data rates. Furthermore, envisioned media-rich mobile applications will require data rates that are difficult to be delivered with current networks [2], [3]. A promising solution to this problem is to utilize unused frequency bands over 6 GHz, such as the millimeter wave (mmWave) bands ranging from 30 to 300 GHz. 5G systems operating on mmWave band promise high data rates and reduce latency. Among the proposed technologies for 5G, we, in this study, focus on device-to-device (D2D) communications operating on mmWave bands.

Although mmWave communication promises high data rates due to availability of abundant bandwidth, however, poses several propagation challenges. First, mmWave signals do not penetrate through solid materials and are easily absorbed by plants and rain drops. For instance, at 40 GHz, the obstacles such as brick wall and painted board can cause attenuation of more than 178 dB and 20 dB, respectively. Second, the propagation loss is much higher at mmWave frequencies, e.g., at 60 GHz, this loss is 28 dB higher than at 2.4 GHz [4]. Third, at mmWave frequencies, wavelengths shrink as compared to microwave frequencies, and waves do not diffract around the obstacles. Hence, line-of-sight (LOS) is necessary for mmWaves to propagate. For lower frequencies, if LOS link is not possible, the signal can arrive at the destination by diffracting around the obstacles which is not possible in mmWave.

Device-to-device (D2D) communication provides a connection between two close-by devices to communicate without any infrastructure such as base station (BS) [2], [5], [6]. To improve the capacity and energy efficiency of the network, D2D communication is expected to be an essential feature for 5G networks [7], [8]. D2D communication is being heralded as a solution of challenging problems faced by cellular systems by mitigating overall network interference and by provision of coverage in regions otherwise in outage. Because of direct communication with the neighboring device, the link reliability between the devices improves, the energy consumption and the latency within the network reduces.

In this paper, we consider the problem of neighbor discovery in mmWave-based D2D communication, which is particularly challenging at the mmWave band (as compared to conventional sub 6 GHz D2D networks) because of the use of directional antennas at devices. Discovery algorithm should be carefully designed to ensure the neighbor discovery in minimum bounded time, and it should guarantee discovery in an environment where nodes antennas are heterogeneous in terms of beam width and should not require any prior coordination between nodes. Specifically, we undertake the problem of discovering the neighbor nodes in the context of completely decentralized and uncoordinated manner referred to as the *oblivious discovery* [9]. The primary metric of performance of a D2D neighbor discovery algorithm is the amount of delay experienced by the devices and the energy consumed to complete this procedure. We improve the prior discovery algorithms by reducing the worst case latency in discovery. Our proposed method uses Pólya's enumeration theorem and Fredricksen, Kessler and Maiorana (FKM) algorithm to find shorter and efficient scanning sequences for the nodes.

The rest of the paper is organized as follows: Sect. II gives a brief overview of related work. System model and algorithm design are discussed in Sect. III. Sect. IV presents simulation results, whereas Sect. V concludes the paper.

II. RELATED WORK

As discussed in Section I, designing neighbor discovery algorithm for mmWave networks is more challenging due to the use of directional antennas. Neighbor discovery algorithm using directional antennas are usually categorized into two main classes, deterministic and probabilistic. Probabilistic approach is memoryless and is suitable for decentralized environments where no prior coordination is possible. In probabilistic algorithms, nodes randomly choose the directions to steer their antenna beams for neighbor discovery [10]–[12]. Usually, probabilistic approaches provide better results in terms of average discovery delay. The main problem of using probabilistic approach is that it does not guarantee the discovery and, at times, discovery delay becomes extremely large, which is known as the long-tail discovery latency problem.

In deterministic algorithms, the devices steer their beams in accordance with certain predefined sequences [13]–[15]. With deterministic strategy, we can guarantee the neighbor discovery in bounded time but the average discovery delays can be longer as compared to the probabilistic approaches. State-of-the-art deterministic algorithm such as [15] and pseudo-deterministic algorithms such as [9], [16] can perform neighbor discovery in the absence of time synchronization between devices. In [9] and [16], the authors have proposed pseudo-deterministic schemes such that the process of generation of scanning sequences includes both a deterministic and a random component. Nodes can generate their antenna scan sequences from extended addresses in a largely deterministic manner with some random bits involved. The deterministic part ensures the neighbor discovery in a bounded time, while the random part is there to reduce the average neighbor discovery delay.

In [15], the authors proposed a deterministic scheme in which each node continuously rotates its antenna beams either in clockwise or anti-clockwise direction. The nodes are not divided into sectors, however, they rotate their beams with some angular velocity. Each node operates either in transmission mode or in reception mode. Each node has its unique ID, and each node generates scan sequences according to the procedure as in [17]. Performance of hunting-based directional neighbor discovery (HDND) is dependent on length of the sequence, so, proposed scheme is also applicable for HDND.

III. SYSTEM MODEL

In oblivious neighbor discovery framework introduced in [9], the nodes are not synchronized and have no prior information about antenna beam widths of one another. The nodes are heterogeneous in the sense that they have different number of sectors depending upon their directional antenna beam width θ_i, where $(0 < \theta_i < 2\pi)$, i refers to the device index, $i \in \{0, 1, ..., M-1\}$ and M is the number of devices in the network. Each node has $N_i = 2\pi/\theta_i$ number of sectors. For example, in Fig. 1, node a has $N_a = 4$ and node b has $N_b = 6$ sectors. The sectors for node a are labeled from 0 to $N_a - 1$, and similarly for the other nodes in the network. Each node has a scanning sequence, which is used during the discovery phase to direct its antenna towards a specific

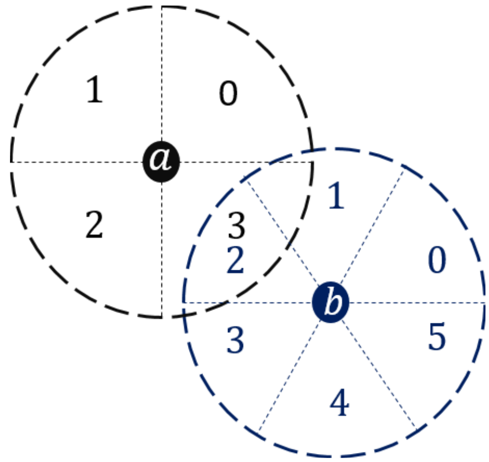

Fig. 1: Antenna configuration of two nodes a and b. Node a has 4 antenna sectors and node b has 6 antenna sectors ($N_a = 4$, $N_b = 6$).

Slot Index:	0	1	2	3	4	5	6	7	8	9	10	11	12	...
Node a:	0	1	2	3	0	1	2	3	0	1	2	3	0	...
Node b:	5	4	3	2	1	0	5	4	3	2	1	0	5	...

Fig. 2: Antenna scan sequences, the highlighted box shows where the two beams are aligned.

sector in a given time slot. For the scenario shown in Fig. 1, Fig. 2 describes the discovery process for the two nodes using an example of scanning sequences for the two nodes. The two nodes can discover each other when their antennas are pointed towards each other in any time slot. From Fig. 2, it is clear that the nodes discover each other in 3rd time slot when the node a directs its beam in its sector number 3 and similarly node b directs its beam in its sector number 2. We are considering a pseudo-deterministic strategy as a probabilistic strategy cannot guarantee the neighbor discovery in a bounded time and sometimes a pair of nodes could experience excessive discovery delay.

Each node has its own unique ID, henceforth referred to as the device ID, which can be taken as the binary representation of the device index. The minimum number of bits in device ID are determined by the number of devices M deployed in the network. For example, for a network consisting of 250 nodes, each node may be assigned a unique ID of at least 8 bits as $250 < 2^8$. In general, the number of bits must be at least $\lceil log_2(M) \rceil$, where $\lceil . \rceil$ denotes the ceiling operator. By using its device ID, each device can construct its extended address, and the choice of the extended address of the node is made such that the extended addresses of nodes are mutually cyclically rotationally distinct to each other, i.e., no valid extended address can be obtained by cyclic rotation of any other extended address. This condition is enforced to ensure that different devices have scanning sequences that allow neighbor discovery for arbitrary orientations of devices that may not be synchronized with each other. The length of the extended address is very significant as it is an important factor in determining the neighbor discovery latency. How to make extended addresses cyclically rotationally distinct while keeping their lengths small is a challenge. Different methods of constructing extended address sequences that are cyclically rotationally distinct to each other are proposed in [9], [15], [16], and [17]. From extended address, each node constructs its scan sequence which the node employs during neighbor discovery. For oblivious neighbor discovery, the method of constructing extended address sequence is given in [9] with an improved version introduced in [16]. The latter approach is more efficient and results in a shorter extended address of nodes in the network.

The method for constructing extended ID sequences proposed in [16] is summarized as follows:

- Each node has its globally unique ID of length l_0 bits denoted by $\boldsymbol{\alpha}$. Typically, l_0 is chosen such that $l_0 = \lceil log_2(M) \rceil$. Let α_k denote the k-th bit of $\boldsymbol{\alpha}$ such that
$$\boldsymbol{\alpha} = [\alpha_1, \alpha_2, ..., \alpha_{l_0}] . \quad (1)$$
- The device ID $\boldsymbol{\alpha}$ is partitioned into two subsequences $\boldsymbol{\alpha}_l$ and $\boldsymbol{\alpha}_r$ such that
$$\boldsymbol{\alpha}_l = [\alpha_1, \alpha_2, ..., \alpha_{k-1}] , \quad (2)$$
and
$$\boldsymbol{\alpha}_r = [\alpha_k, \alpha_{k+1}, ..., \alpha_{l_0}] , \quad (3)$$
where $k := \lfloor l_0/2 \rfloor$.
- The extended ID sequence (I) is now constructed as
$$I = [\mathbf{0}(l_1), \boldsymbol{\alpha}_l, 1, 0, \boldsymbol{\alpha}_r, \mathbf{1}(l_2)] . \quad (4)$$
Here $l_1 := \lceil k/2 \rceil + 1$, $l_2 := \lceil k/2 \rceil$, $\mathbf{0}(l_1)$ is a string of all zeros of length l_1 and, similarly, $\mathbf{1}(l_2)$ is a string of all ones of length l_2 .
- The length in bits of the extended ID sequence L is given as
$$L = l_0 + l_1 + l_2 + 2. \quad (5)$$

If we define p_i as the smallest odd prime number that is not smaller than N_i and also co-prime to L, and q_i as the smallest power of 2 that is not smaller than N_i, then the scanning sequence is given as

$$S_t = \begin{cases} t \bmod p_i & E_t^i = 0 \text{ and } t \bmod p_i < N_i, \\ t \bmod q_i & E_t^i = 1 \text{ and } t \bmod q_i < N_i, \\ \text{rand}(N_i - 1) & \text{otherwise.} \end{cases} \quad (6)$$

where E_t^i denotes a bit in extended ID sequence at some specific time instant and rand$(N_i - 1)$ denotes a uniform random integer in $[0, N_i - 1]$. As shown in [16], the upper bound of the worst case discovery delay is directly proportional to the length of extended address L and is given as

$$W = L \max\{p_a q_b, p_b q_a\}. \quad (7)$$

Among the existing techniques for oblivious neighbor discovery, the ODND [16] provides the least latency by choosing an efficient way to construct extended addresses such that all of them are mutually distinct with respect to cyclic rotations. However, the approximate length of the extended address is still approximately $1.5l_0$. In the sequel, we present a new algorithm to obtain extended addresses such that they are shorter in length and result in faster and energy efficient discovery of neighbors in a directional mmWave D2D network.

IV. Proposed Algorithm

The latency in discovering a neighbor is a very serious obstacle towards justification of the viability of mmWave D2D communication as an energy efficient solution. In this work, we focus on further reduction of the worst case latency of the neighbor discovery so that energy efficiency of the discovery process can be improved. Since the length of the extended address is an important determining factor of the worst case discovery delay, the basic idea of the proposed neighbor discovery approach is to reduce the length of the address by careful construction of the extended addresses using ideas from abstract algebra and combinatorics.

As presented in the last section, the extended address is defined from the device ID $\boldsymbol{\alpha}$, which is either issued by a central station to all the devices in the network or specified at the time of device configuration. In either case, the device ID's are decided in a non-distributed manner such that all the device ID's have odd number of bits and are mutually distinct from each other under cycle rotations. In the proposed approach, we find the largest set of strings such that it contains all binary strings of fixed length that are cyclically distinct from each other. The binary strings in this set are arranged lexicographically and are indexed by the device ID's. Now, using the device ID as the index of the set, we assign the corresponding binary string as the extended address of the device. The length of the strings is chosen to be the smallest odd number such that there is a cyclically distinct sequence for every device in the network. Now, by the definition of the set, the extended address should have the minimal length, in particular, it is expected that the length is shorter than (5).

To formalize this process and determine the length of the strings in the above defined set as a function of the number of devices, M, we define the notion of a necklace as employed in combinatorics.

1) Pólya's Necklaces: Necklace is a representation of n circularly connected beads with c different colors and they are invariant to rotation [18].

To explain the concept of necklaces, we present the illustration in Fig. 3. Here we consider binary ($c = 2$) necklaces, i.e., the beads are of two different colors, e.g., black or white. If we are allowed to use only one bead ($n = 1$) of either color, then, it is clear that only two distinct necklaces can be formed. Similarly, it is shown that by using 2, 3 and 4 beads, we get 3, 4 and 6 binary necklaces, respectively.

An alternative way of representing binary necklaces is by using binary strings, which are constructed as follows. First, binary digits are assigned to colors, e.g., a black bead is represented by 1 and a white bead by 0. Then, the equivalent binary representation is obtained by using the bits corresponding to the colors of beads in a necklace, e.g., in Fig. 3, the binary

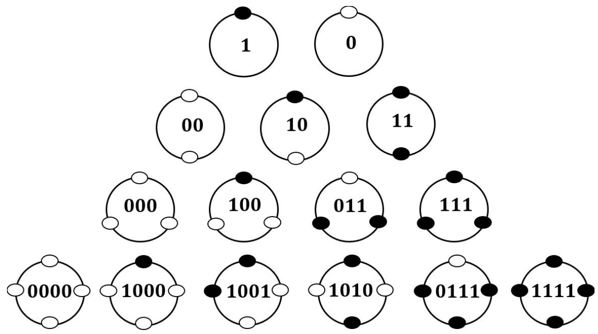

Fig. 3: The set of unique necklaces formed by using different number of binary colored beads (bits).

strings are obtained by clockwise mapping of the colors of beads to their respective bit assignment. A collection of beads form two different necklaces only if one of them cannot be obtained by rotating the first one. For example, for $n = 3$, the binary strings 100 and 010 represent the same necklaces since 010 can be obtained by right circular shift of 100. Thus, two binary strings represent the same necklace if one can be obtained by circular shift of the other.

Now, a natural question arises as to how many necklaces $N(n, c)$ (cyclically distinct sequences) can be obtained for a given number of bits (beads) n with c distinct colors. For example, from Fig. 3, it is clear that $N(3, 2) = 4$. This question is answered, in general, by Pólya's Enumeration theorem, which is a powerful method for enumerating number of orbits of a group on a particular configuration [19].

2) Pólya's Enumeration Theorem: The number of unique necklaces that can be made using n beads of up to c distinct colors $N(n, c)$ is given as

$$N(n,c) = \frac{1}{n}\sum_{i=1}^{\nu} \varphi(d_i) c^{n/d_i}, \tag{8}$$

where φ represents Euler's totient function and d_i are the ν divisors of n. Euler's totient function counts the positive integers that are relatively prime to any given number d [20] and can be expressed as

$$\varphi(d) = d\prod_{i=1}^{\nu}(1-\frac{1}{\rho_i}), \tag{9}$$

where ρ_i represents the ν distinct prime factors of d. For example, if $d = 10$, then there are four numbers that are co-prime to d, i.e., are 1, 3, 7, and 9, and thus $\varphi(10)$ = 4.

Now, the concept of Pólya's necklaces can be used to choose the smallest odd length extended addresses for all the devices. Given the number of devices M, we can select the length of extended address L^* as follows:

$$\begin{aligned} L^* &= \min\ n \\ \text{subject to } N(n,2) &\geq M, \text{ and } \mathrm{mod}(n,2) = 1. \end{aligned} \tag{10}$$

The values of L^* thus obtained are given in Table I corresponding to a given number of devices. Once the number of bits of the extended address, i.e., L^* is selected, we can assign the extended addresses to the devices by using the device ID's as the index of the set containing necklaces ordered lexicographically. Such a set of binary necklaces can be obtained by using FKM method [21] [22], which is presented as *Algorithm 1*.

Let us consider a network with $M = 8$ devices. Now, using Table I, the required number of bits for the extended address is $L^* = 5$. The FKM algorithm can then be used to generate all the binary necklaces of length 5. All the iterations of the FKM Algorithm along with intermediate steps are described in detail in Table II.

TABLE I: Number of devices supported by given number of bits in the extended address.

No. of Devices (M)	No. of Bits (L^*)	N
$1 \leq M \leq 2$	1	2
$3 \leq M \leq 4$	3	4
$5 \leq M \leq 8$	5	8
$9 \leq M \leq 20$	7	20
$21 \leq M \leq 60$	9	60
$61 \leq M \leq 188$	11	188
$189 \leq M \leq 632$	13	632
$633 \leq M \leq 2192$	15	2192
$2193 \leq M \leq 7712$	17	7712
...	...	...

Algorithm 1: FKM Algorithm

Input: By knowing the number of devices, determine L^* and c from (10).

Output: All necklaces ($\boldsymbol{\eta}^1$ to $\boldsymbol{\eta}^N$).

1 $\boldsymbol{\eta}^1 \leftarrow \mathbf{0}(L^*)$; where zero is repeated L^* times
2 $\boldsymbol{\gamma} = \boldsymbol{\eta}^1$;
3 $i \leftarrow L^*$;
4 **if** $\gamma_i < (c-1)$ **then**
5 $\quad \boldsymbol{\delta} = [\gamma_1, \gamma_2, \gamma_3 ...., \gamma_{i-1}, (\gamma_i + 1)\ ]$;
6 $\quad \gamma(m) = \delta((m-1)_i + 1)$ for all $m \in [1, 2, 3, ..., L^*]$;
7 $\quad$ **if** *mod*$(L^*, i) = 0$ **then**
8 $\quad\quad \boldsymbol{\eta}^{j+1} = \boldsymbol{\gamma}$; $j = j + 1$
9 $\quad\quad$ Go to step 3;
10 $\quad$ **else**
11 $\quad\quad$ Go to step 3;
12 $\quad$ **end**
13 **else**
14 $\quad i - -$;
15 $\quad$ **if** $i > 0$ **then**
16 $\quad\quad$ Go to step 4;
17 $\quad$ **else**
18 $\quad\quad \boldsymbol{\eta}^N = \mathbf{1}(L^*)$;
19 $\quad\quad$ **break**;
20 $\quad$ **end**
21 **end**

V. Simulation Results

In this section, we present the results of numerical simulations to assess the performance of the proposed approach as

TABLE II: Necklaces generation for 8 devices ($L^* = 5, c = 2$) by using Algorithm 1.

j	i	δ	γ	Necklaces
1	5	-	00000	$\eta^1 = 00000$
2	5	00001	00001	$\eta^2 = 00001$
2	4	0001	00010	-
3	5	00011	00011	$\eta^3 = 00011$
3	3	001	00100	-
4	5	00101	00101	$\eta^4 = 00101$
4	4	0011	00110	-
5	5	00111	00111	$\eta^5 = 00111$
5	2	01	01010	-
6	5	01011	01011	$\eta^6 = 01011$
6	3	011	01101	-
6	4	0111	01110	-
7	5	01111	01111	$\eta^7 = 01111$
8	0	1	11111	$\eta^8 = 11111$

compared to the ODND algorithms presented in [9] and [16].

In Fig. 4, we present the number of bits required by the proposed algorithm, and the HDND and ODND algorithm in [15] and [16] for the extended address. It can be noticed that the ODND approach of [16] is always more efficient as compared to that of [15], however, our proposed approach outperforms both existing methods and it always chooses shorter extended address for devices irrespective of the size of the network. For example, for a network with 600 devices, our proposed algorithm assigns 13 bit extended addresses to all the devices, while the HDND and ODND algorithm in [15] and [16] assign 21-bit and 19-bit addresses, respectively. For brevity of discussion of further results, we skip the comparison with HDND [15].

Fig. 5 shows the upper bound on the discovery delay (given in (7)) as a function of the number of devices. The results are presented with $N_a = 6$ and for both $N_b = 6$ and $N_b = 9$. The latency bound increases with larger number of sectors and there is significant reduction for the proposed scheme. The increase in latency bound with larger number of sectors is due to the reason that latency bound is directly proportional to the value of p and q, and by increasing the number of sectors, the value of p_i and q_i also increases.

For additional results presented in Fig. 6, 7 and 8, we have assumed that there are up to 250 devices present in the network, so each of them are assigned 13 bit extended address as given in Table I. The node locations were generated randomly so that the neighbors are placed at various angles with respect to each other. We have performed the neighbor discovery experiment with different antenna configurations of nodes, i.e., different combinations of numbers of sectors (N_a, N_b). For this simulation, we also assume that the nodes in the network are not synchronized in time and have a clock drift among themselves. Figs. 6, 7 and 8 show the worst case discovery delays as well as the upper bound of the proposed scheme and the baseline scheme (ODND). From the results, we clearly see that the worst case delay increases as the number of sectors increases, i.e., for a given N_a, the worst case discovery delay increases by increasing N_b. From results, it is clear that with the proposed scheme, the nodes generally discover each other in less time than in ODND. Sometimes, e.g., $(N_a, N_b) = (6, 9)$

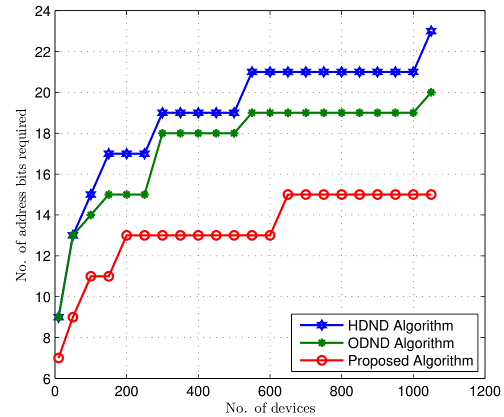

Fig. 4: Extended address length comparison of different neighbor discovery schemes.

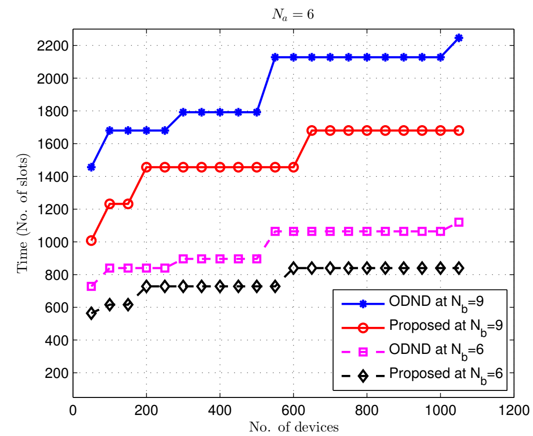

Fig. 5: Discovery delay upper bound comparison between ODND and proposed algorithm.

and $(N_a, N_b) = (6, 12)$, it happens that the delay of proposed algorithm is larger than that of ODND. This happens due to the choice of higher values of p_a or p_b (to ensure they are co-prime with L^*). This can however be avoided by choosing $L^* + 2$ as the length of the extended address.

VI. Conclusions

In this paper, a novel energy efficient scheme for constructing extended addresses for neighbor discovery in D2D communication is proposed in the context of millimeter wave communication in 5G networks. According to the proposed algorithm, the nodes discover their neighbors quickly in an efficient and decentralized manner without any prior coordination. The idea of Pólya's necklaces is applied here in order to reduce the worst case discovery delay. The results are simulated and are compared with the previously proposed schemes, which clearly show that the proposed algorithm in this paper leads to faster and energy efficient neighbor discovery.

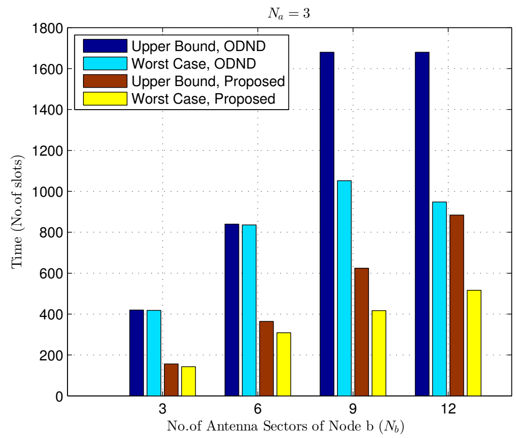

Fig. 6: Discovery delay comparison between proposed algorithm and ODND at $N_a = 3$.

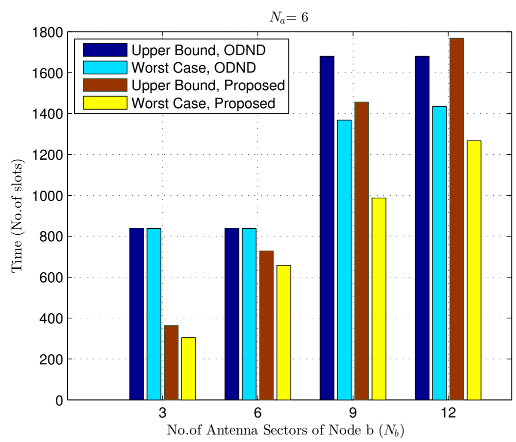

Fig. 7: Discovery delay comparison between proposed algorithm and ODND at $N_a = 6$.

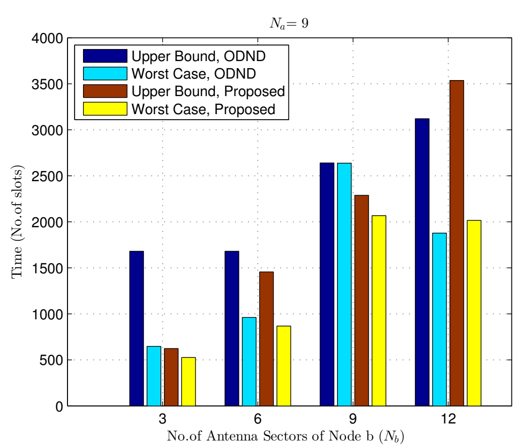

Fig. 8: Discovery delay comparison between proposed algorithm and ODND at $N_a = 9$.

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