Exploiting Polarization Diversity in Massive MIMO Systems using Hyper-Complex Variables

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A thesis submitted in partial fulfillment of the requirements for the degree of Ph.D. Electrical Engineering

In

School of Electrical Engineering and Computer Science,

National University of Sciences and Technology (NUST),

Islamabad, Pakistan.

(2021)

Approval

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Abstract

Space time block coding (STBCs) improves the reliability of transmission by sending multiple copies of data through multiple antennas. Recently, the study of orthogonal designs in higher dimensions has supported higher diversity gains, i.e. combining STBCs with different forms of diversity. Also, designing codes with higher code rates is a desired aim for future communication networks but the complexity of the receiver has always limited this freedom. Quaternion orthogonal designs (QODs) have been derived mostly from complex orthogonal codes (CODs). This supports the idea and benefits of using QODs to achieve higher code rates but it remained limited in fully exploiting the use of dual-polarized antennas. The real essence of adding the polarization diversity to the coding designs still remains unexplored. This research targets this research gap and presents a thorough analysis of using higher dimensional variables not only to achieve efficient code designs with higher code rates but also to investigate mechanisms to optimize the receiver design. Based on these aims, this research takes two major paths. First, it studies the impact of using quaternion designs with dual-polarized antennas. The underlying channel between the dual-polarized transmit and receive antennas is discussed when the pure QODs are transmitted. These QODs provide promising diversity gains and shows comparative code rates similar to the state-of-the-art Alamouti codes. Secondly, this research work presents

linear and decoupled decoders for pure QODs, that was not possible before. As an application of the proposals in this work, quaternionic channel-based modulation has been discussed that fully exploits the polarization diversity without considerable limitations on the transmit and receiver dimensions. The design of wireless communication systems using pure QODs transmitted using dual-polarized antennas will open new horizons of research. It will support higher data rates and improved receiver efficiency, that are the two main targets of the future generations of wireless systems.

Dedication

I dedicate this research work to my mother, Tallat Jabeen (Late) and my father, Dr. Muhammad Shakil Qureshi.

List of Publications

Published Papers

- S. S. Qureshi, S. Ali and S. A. Hassan, "Optimal polarization diversity gain in dual-polarized antennas using quaternions," *IEEE Signal Process. Lett.*, vol. 25, no. 4, pp. 467-471, 2018, doi: 10.1109/LSP.2018.2799569. (Impact Factor: 3.105).

- S. S. Qureshi, S.A. Hassan and S. Ali, "Linear and Decoupled Decoders for Dual-Polarized Antenna-Based MIMO Systems," *Sensors*, vol. 20, no. 24, pp.7141, 2020, doi: 10.3390/s20247141. (Impact Factor: 3.275).

- S. Ali, S. S. Qureshi and S. A. Hassan, "Quaternion Codes in MIMO System of Dual-Polarized Antennas," *Applied Sciences*, vol. 11, no. 7, pp. 3131, 2021, doi: 10.3390/app11073131.(Impact Factor: 2.474).

- S. S. Qureshi, S. A. Hassan and S. Ali, "Quaternionic Channel-based Modulation For Dual-polarized Antennas," *2020 IEEE 91st Vehicular Technology Conference (VTC2020-Spring)*, Antwerp, Belgium, 2020, pp. 1-5, doi: 10.1109/VTC2020-Spring48590.2020.9129096.

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Acknowledgment

First and foremost I offer the gratitude to Allah Almighty who bestowed me with all the strength and ability to carry out this work. Then I am thankful to my supervisor Dr. Syed Ali Hassan for all his cooperation, guidance, encouragement and the confidence he showed in me during the course of this research work. Secondly, I want to express my sincerest gratitude to my co-supervisor Dr. Hassaan Khaliq Qureshi and other committee members, Dr. Fahd Ahmed Khan and Dr. Muhammad Omer Bin Saeed, for their help and guidance. I would especially thank Dr. Sajid Ali in taking all the pains in making the ideas turn to reality and adding his immense knowledge of deep mathematics and algebra to my thesis.

I am also grateful to Allah for bestowing me with amazing acquaintance and in such an abundance that I cannot even mention name of all of them but I want to express my thanks to them for encouraging, sharing the gray and bright days of PhD. I wish and pray for all of them to have wonderful people around them as they have been. It has been a whole journey and many people have been source of guidance, inspiration, support and encouragement. Last but not the least my deepest gratitude goes to my beloved parents, my husband and my kids for their unconditional support and prayers.

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Chapter 1

Introduction

Wireless communication systems have evolved from wired and guided communication to non wired and unguided communication beyond the geographical limitations of countries and continents. All this has been possible due to continuous efforts from different fields of science including communications, electronics and antenna theory. The main targets have always been to address the capacity and speed of the communication through code and infrastructural design improvement in the underlying network layout [1–3]. Communications between transmitter and receiver ends has also transitioned from stationary units to mobile units with portability and interference increasing the complexity of the wireless communication systems [4, 5]. Thus, there is a dire need to investigate mechanisms which can support rapid increase in data rate as well as speed influencing the baseline system in terms of economics as well as hardware equipment. This thesis addresses the issues of efficient spectrum utilization as well as data rates by working on efficient data communication techniques. These have the capacity to provide reliable data transmission between transmitter and receiver without compromising the data rates and the antenna dimensions.

In wireless communication systems, targeting higher data rates with the present resources in the network for communication has been considered as the most challenging concern in research [6]. The spectrum is limited in terms of bandwidth while the devices have power restrictions [7]. Base station (BS) as well as user equipment (UE) are both limited in terms of size and mobility flexibility [8]. In light of all these issues, the design and development of data communication techniques that can address the broader requirements of the future wireless communication systems with improved data rates and spectral efficiency is cumbersome and demands fine balancing with the issues of mobility, size and power of the equipment at transmitter as well as receiver. This work has a been done to address the data communication between multiple antennas at both transmitter and receiver considering the environment being extremely unpredictable due to scattering, reflections, refractions, etc. The prime aim is to achieve better data rates as well as diversity gains with least infrastructural alterations. This proposal is valuable for the future of next generation wireless communication systems which involve massive multiple-input multiple-output (MIMO) systems.

1.1 Wireless Communication Systems and its Challenges

In wireless communication systems, the research initiated with the design of communication models consisting of single set of transmit and receive antennas. This tracks back to the time when there was very less interference in the spectrum and communication was possible with such limited infrastructure. But soon, the communication channel started becoming populated and thus introduced the challenges of data reliability [9]. Consequently, space-time coding emerged as a solution where multiple copies of a data stream was transmitted through the communication channel using multiple antennas [10]. This provided multiple copies of data at the receiver which increased the level of reliability of the data transfer [11]. The main motivation behind the design of space-time coding has been the fact that, after passing through a communication medium which is affected by scattering, reflections and refraction with possible distortions in data due to imperfect receivers, some of the copies received might be much closer to the original signals due to the channel being completely random. Reception of multiple copies of the same signal at the receiver improves the chances of retrieving the original signal through efficient decoding mechanisms [12]. Thus, the reliability of the system increases by using multiple antennas and space-time coding. Effectively, space-time coding is the fine integration or selection from these copies of data received to decode the original message.

1.1.1 Space Time Block Codes

Space-time coding comprises of a wireless communication system which has multiple transmit antennas with the flexibility to have any number of receive antennas. There are two variants of space-time coding including space-time trellis codes (STTCs) and space-time block codes. Space-time trellis codes are efficient in promising coding gain as well as transmit diversity gains with a drawback of computationally complex decoders at the receiving end. On the other hand, space-time block codes do not ensure coding gains but they provide better transmit diversity gains with the advantage of low-complexity decoders [13]. In space-time block coding, the data to be transmitted through the data streams is divided into blocks where each block of data is transmitted using the space and time diversities. Each data block is transmitted through a set of transmit antennas in multiple timeslots [14], [15]. Such division of data across the data streams through multiple transmit antennas empowers the communication to eliminate the effects of fading and interference caused by the channel impairments at the receiving end. Alamouti was the first to propose the design of a complex orthogonal space-time block code (COSTBC) for two transmit antennas [1]. He explored that space-time block codes which satisfy the property of orthogonality ensures simplification of the maximum-likelihood (ML) decoding rule by promising linear and decoupled decoding at the receiver. The additional benefit of reduced receiver complexity by maintaining orthogonality introduced limitations on the code rates that can be achieved when the number of antennas are increased. Although, having ambiguities and limitations, this proposal paved the way for research and realization of next generation of wireless communication systems starting from 3G and beyond. Many studies have been done to address the issue of code designs with multiple antenna systems including design of quasi-orthogonal codes which can promise higher code rates at a nominal compromise on the orthogonality of the space-time block codes [16]. This compromise in orthogonality comes at the expense of computationally complex decoders which provides pair-wise or completely coupled decoding [17], [18]. The quest for designing space-time block codes which can achieves better diversity gains with higher code rates remains as an open area of research in literature.

1.1.2 Exploiting Polarization Diversity: Quaternion Orthogonal Designs

Wireless communication systems have reached to a bottleneck of the code rate versus the maximum number of antennas that can be used maintaining the orthogonality condition. The future generation of wireless communication systems target MIMO systems to achieve diversity gains. Thus, to address the barrier of the limited code rates in STBC designs, other forms of diversities, i.e. polarization diversity, in addition to space and time diversities are studied [19]. The advancements in antenna theory has also moved hand-in hand from single polarized antennas towards dual-polarized (DP) and multi-polarized designs. A single DP antenna is equipped to transmit and receive two signals in orthogonal polarizations, simultaneously. The emergence of DP antenna opened new dimensions of research where studies worked new models to target better diversity gains, code rates and decoder designs with hardly any significant change in the antenna dimensions. QODs have been presented in [20], which opened new directions for research and introduced the concepts of orthogonal space-time polarization block codes $\big(OSTPBCs\big)$. Integration of STBCs with combinations of different forms of diversities, i.e. space, time and polarization, can help in surpassing the bottleneck of capcity in wireless communication systems. Working with STBCs over the foundations of polarization diversity provides gains in performance and diversity which directly maps to efficient cost, time and space solutions [21], [22]. This can be achieved without any effect and increase in the antenna dimensions [20].

Quaternion algebra has been used in the design of STBCs since more than a decade now, in an urge to address the limitations of code rate and the number of antennas, to stimulate the future needs of massive MIMO systems. Studies done considering the design of STBCs using polarization diversity and DP antennas have laid their foundations on the complex channel model which fails to completely exploit the advantages of pure quaternions. The designs presented consider quaternions consisting of two complex numbers transmitted through the two orthogonal polarizations of the DP antennas. Different studies exist which address the design of QODs using

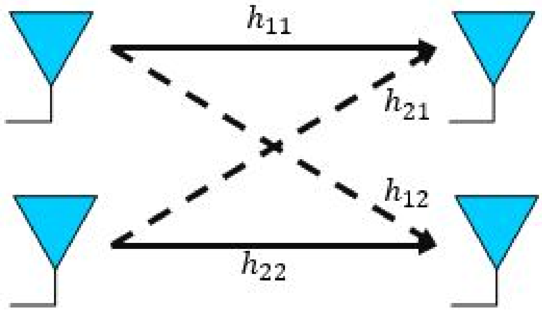

Figure 1.1: Two-input two-output (TITO) antenna configuration of single-polarized antennas.

different construction techniques [20]. All these designs were built by designing quasi-orthogonal STBCs in the complex domain and none of them directly employed the quaternion algebra in its pure form. Today, we are available with more variety in terms of hardware, where the STBCs has to be addressed again in combination with appropriate forms of diversities, i.e. space, time, polarization, etc., by fine integration of developments in antenna theory and algebra to tailor the need of higher data rates, reduced latency and reliability in next generation wireless communication systems.

1.2 Motivation

Last few decades have changed the way communication takes place, targeting higher data rates, reduced latency, improved spectral efficiency and hardware optimization. A lot of work has been done for 5G systems, e.g. see [23–56]. Figure 1.1 describes antenna configuration for a two-input two-output (TITO) single-polarized antennas. the communication channel comprises of complex channel gains between each pair of transmit and receive single-polarized antennas, i.e. $h_{ij} \in \mathbf{C}$ where $i.j \in [1, 2]$. Alamouti presented

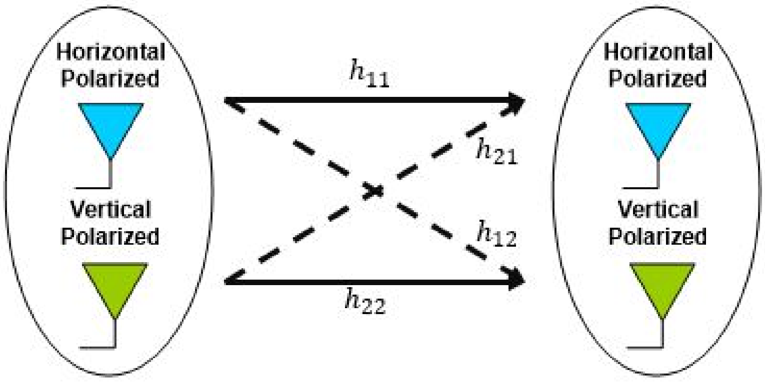

Figure 1.2: Single-input single-output (SISO) antenna configuration of DP antennas.

a full rate orthogonal STBC for this configuration, exploiting space and time diversities, opening numerous dimensions for design and research in orthogonal code designs [1]. In order to achieve considerable diversity gains and bit error rate (BER) performance at higher signal-to-noise ratio (SNR), this design is constrained in terms of maintaining antenna spacing and decoder complexity to maintain spatially independent streams across the medium of transmission. All the work till date fails to exploit the use of quaternions in its pure form. Rather, complex representation of quaternions is used by compromising the orthogonality and contributing to the computational complexity of the decoder at the receiving end.

With the demands of the future wireless communication systems to support higher data rates and capacity, the solution lies entirely in increasing the number of antennas at the transmitter and receiver ends. The design of DP antennas has initiated research which targets polarization diversity. Polarization diversity exploits the decorrelation of the data received at orthogonal polarizations of the DP antenna to attain the diversity gains. Quaternion algebra has been integrated with DP antennas to address limiting issues of code rate, diversity gain and spectral efficiency. Figure 1.2 provides antenna configuration for single-input single-output (SISO) DP antennas. With the status of research at present, the underlying channel is complex. This becomes the major bottleneck in exploiting the real benefits behind using polarization diversity through DP antennas. This work will explore the nature of channel and code design when DP antennas are used for the design of orthogonal space-time polarizational block codes. Contrary to what has been done in the past, the motivation behind this research is to study the baseline contributions made when DP antennas are integrated into the wireless communication system and pure quaternions are considered as basic algebraic representation of signals and channel. Effects of the transmission of STBCs through orthogonal polarizations will be studied. Code design in light of this changing paradigm will be explored with a composite study of how it affects the receiver side and the decoder complexity.

1.3 Contributions

The main contribution of this research work have been summarized below.

1. Quaternions offer a better solution especially for DP antennas to attain polarization diversity gains without exploiting time diversity. Such designs achieve better throughput and linear decoupled decoding solution is an intrinsic feature of the approach. A new system model has been proposed based on pure quaternion channel, i.e. quaternionic channel, which provides optimal solutions to the aforementioned problems. This channel model provides better opportunity to exploit the polarization diversity independently. It turned out that the above construction leads to following major achievements.

    (a) Natural realization of quaternions where a natural path has been taken from the complex $z = x + iy$, to quaternions $q = z_1 + z_2 j$ and $h = h_1 + h_2 j$, with analogous structure.

(b) Quaternionic form of channel with one possible explanation that it inherits effects in two orthogonal polarization planes after reflections, scattering etc.

(c) An effective way to generate space time codes for DP antennas.

(d) An incredibly high throughput in all space time codes in comparison to standard Almaouti analogues.

(e) Built-in linear and decoupled decoders.

(f) One possible way to attain polarization diversity without relying on time diversity.

(g) An efficient procedure to deal with MIMO systems.

A QOD of order one using a 1×1 DP antenna confuguration has been proposed that provides more information as compared to the standard Alamouti code for a 2×2 single-polarized antenna configuration [57]. The details of the complete system with its outstanding diversity gains have been detailed in Chapter 4.

2. In the presence of fully quaternion-valued channel model, design of linear and decoupled decoder for QODs based on Adam-Lax-Phillips approach has been presented in Chapter 5 and published in [58].

3. Quaternion modulation using the quaternionic channel model has been developed that is independent of the cross-polar scattering effects in Chapter 6. This defines the application of the proposed quaternionic channel model and the pure quaternion space-time polarization block codes (STPBCs) that fully exploits polarization using the DP antennas

in addition to space and time diversities to achieve maximum performance. This quaternion modulation has been discussed for its application in future massive MIMO systems by extending the work to higher dimensions and providing simulation results [59].

4. Generalized construction techniques have been presented in Chapter 7. They are designed using Liang construction approach [60]. This not only proposes linear and decoupled decoding solution for all the STBCs obtained from the QODs but also presents the decoupled decoding of quasi-orthogonal QODs constructed using the Liang's approach. This also supports feasibility of QODs for future massive MIMO systems.

1.4 Thesis Organization

This thesis is organised as follows:

In Chapter 2, massive MIMO have been discussed in terms of their prospects related to the future generations of wireless communication systems. The major challenges and salient research dimensions are discussed with an insight into how this work contributes towards the current and future developments.

In Chapter 3, the focus is made on STBC. A brief literature review on the space-time block coding using DP antennas is given. Then, a short review of the coding and decoding techniques of the QODs are studied.

In Chapter 4, emphasis is laid on the design of a new channel model based on pure quaternions using DP antennas. After a brief literature review on QODs using DP antennas, the detailed quaternionic model is discussed and different codes are developed for more than one DP antennas. Finally, this channel model is investigated for different QODs using simulation results.

In Chapter 5, the focus is made on quasi-orthogonal codes. A generic review of quasi-orthogonal codes is presented, followed by the investigation of these codes considering the new channel model. The decoding of these quasi-orthogonal codes, that has remained a research issue, has been optimized using the new channel model based on pure quaternion through simulation results.

In Chapter 6, the proposed channel model has been investigated for its application in modulation. To deal with it, the quaternion modulation using DP antennas has been studied, that embeds the new channel model. The gains have been evaluated through simulation results.

In Chapter 7, the design of the QODs have been explored using non-iterative techniques. The decoding and code rates of these codes have been discussed. The design of the quasi-orthogonal codes from this proposal presents optimal decoding complexity. The simulation results have been presented and discussed to support the contributions.

Lastly, in Chapter 8, the conclusions and prospects of the work are discussed. A list of possible future developments and recommendations has also been presented.

Chapter 2

Massive Multiple-Input Multiple-Output (MIMO) Systems

2.1 Introduction

Massive MIMO is a system that uses very large number of antennas, i.e. hundreds or thousands, at either or both the transmitter or receiver ends of a wireless communication network. The conventional SISO systems used single antenna at both the transmitter and receiver. The main aim behind this is to provide additional efficiency to the 5G and future wireless communication generations in terms of energy and respective characteristics including the higher data rates due to large number of antennas capable of transmitting independent data streams at the transmitter [61]. MIMO outperforms SISO in terms of enhanced spectral efficiency, increased throughput, improved capacity, higher bit-rate, reduced BER and reliability with less or no impact on the underlying transmit power and bandwidth [62]. Due to these attractive features, MIMO has been researched and numerous new techniques have been developed for future generation of wireless communication networks.

2.1.1 Benefits of Massive MIMO

Concept of massive MIMO emerged in last decade but it gained huge attention in a very short period of time due to its benefits in 5G. It emerged as one of the most rigorously investigated areas of wireless communication networks and systems. The MIMO systems present today lack in the ability to meet the drastic data traffic requirements and spectral efficiency demands due to the introduction of applications i.e. Internet of Things (IoT), machine to machine (M2M) communciation and augmented reality [63]. Recent research studies have shown positive improvements in achievement of spectral efficiency with massive MIMO testbeds, in [64], that has never been possible before. Also, the hardware testing of these massive MIMO systems were carried out confirming the possibility of using low-cost and low complexity hardware for both digital and analog RF chains [63]. Further reduction in the power and cost has been targeted through design of efficient algorithms for detection, precoding, scheduling and equalization. Testing and studies have

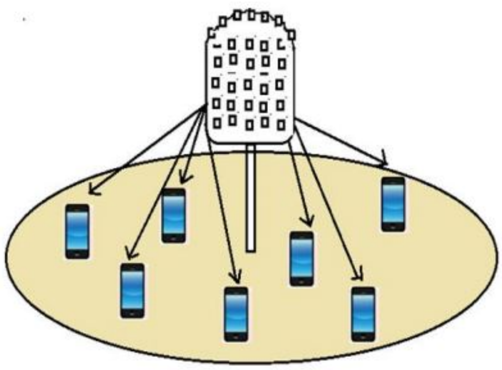

Figure 2.1: Multiple-input multiple-output (MIMO) system [61].

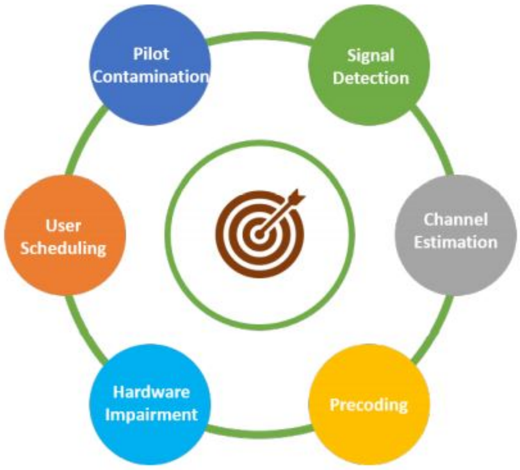

Figure 2.2: Challenges in massive MIMO systems [63].

been done to investigate the possibility of deploying near to infinite number of antennas at the BS. However, the practical testing has confirmed using 64 to 128 antennas at the BS [63]. This brings forth the salient advantage of massive MIMO where hardware complexity and sophistication increases only at the BS while the UEs can enjoy having only a single antennas with simplest antenna design. All these improvements make massive MIMO an attractive candidate for 5G and future wireless networks.

Some of the other benefits of massive MIMO technology has been listed below:

- Energy efficiency
- Spectral efficiency
- Data rate improvement
- Power efficiency
- Reduced latency

- Increased reliability [65, 66]
- Low Bit-error rates
- Linear processing modules
- Robust and resilient against fading, interference and jamming [67, 68]
- Greater security [69]

2.1.2 Challenges of Massive MIMO

With all the competitive advantages of massive MIMO, it still falls short of into different areas resulting in some major challenges, as shown in Figure 2.2. Each of them is discussed briefly below.

- **Pilot contamination**: The BS in a wireless communication system is able to accurately estimate the channel through the reception of pilot signals from the UEs in the home cell, where the pilot signals of the home and the neighbouring cells are orthogonal. Due to limited number of orthogonal pilot signals available for use with given period and bandwidth, the neighbouring cells might reuse the same orthogonal pilot signals. This might result in the BS receiving a linear combination of the channel response from the home and neighbouring cell. This mechanism is known as pilot contamination and has serious impact on the throughput. Research has been done to estimate the channel response to target accurately the user in the home cell using efficient beamforming techniques. It has been identified that increase in the number of antennas at the BS has a positive impact on the overall estimation process [63]. Also, studies have been done to develop pilot reuse techniques and conservative use of the available orthogonal pilot signals [70–73].

- **Channel estimation**: In massive MIMO systems, for the receiver to detect and decode the signal accurately, the information about the state of the communication link (i.e. effect of fading, scattering, etc.) between the transmitter and receiver is critical. This is known as Channel State Information (CSI). If the receiver is able to acquire the precise CSI, the performance of the massive MIMO system increases linearly with the increase in number of transmit or receiver antennas (whichever is less). Thus, channel estimation remains as one of the most investigated areas in massive MIMO systems [74–76].

- **Signal detection**: Due to the large number of antennas at the BS as well as the interference because of the numerous UEs sending the signals to the same BS, signal detection becomes cumbersome. This domain has attracted researchers to investigate and develop optimal signal detection methods with higher throughput [77, 78]. Increased number of antennas in the massive MIMO systems adversely influences the computational complexity of the decoder, that remains as a great challenge.

- **Precoding**: The concept of manipulating the signals to transmit them as multiple streams using multiple antennas. Precoding helps in achieving promising throughput by minimizing interference and path loss. Precoding is used by the BS in combination with the estimated CSI to overcome the effects of interference and fading [63]. The benefits of precoding are prominent in massive MIMO systems but the computational complexity also increases with the increased number of antennas.

This has influenced studies to design less complex precoders [79–81].

- **Hardware impairments**: To achieve the benefits of massive MIMO systems, the number of antennas need to be increased considerably. This increase in the number of antennas provide positive support towards handling of issues arising from interference, fading and noise. However, this also raises concerns in terms of increased hardware costs and requirements [63]. For a massive MIMO system, the huge number of antennas demand cost and computationally efficient hardware designs. While, on the other end, the use of such low cost equipment introduces hardware imperfections i.e. amplifier distortion, magnetization noise and phase noise [82]. Such large number of antenna elements introduces mutual coupling and increased power requirements that has great impact on the overall system performance. Investigation of compensation techniques to mitigate the impact of these hardware impairments has shown supportive results [83, 84].

- **User scheduling**: Due to the support of multiple antennas in massive MIMO systems, the BS can connect and communicate with multiple users at the same time. This provides valuable performance boost to the massive MIMO system but brings in the issues of multi-user interference and reduced throughput [63]. As the number of antennas is higher in the massive MIMO system, only precoding remains sufficient. But this demands an extra module of special user scheduling algorithm when the number of users exceed the number of antennas at the BS. Efficient user scheduling algorithms have been studied for the massive MIMO systems to address the concerns of computational complexity and throughput [85–87].

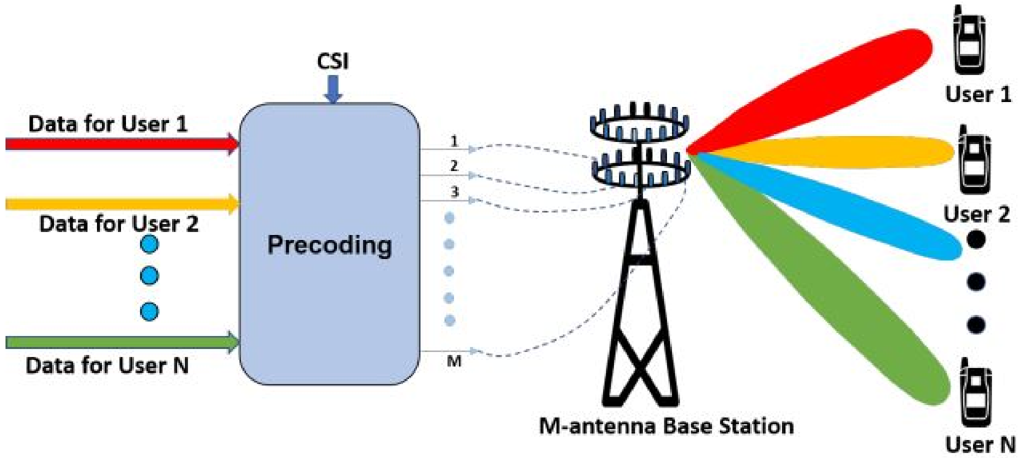

Figure 2.3: Precoding in Massive MIMO system [63].

2.1.3 Precoding in Massive MIMO

Investigation of all the above challenging areas define numerous dimensions that can be independently or collectively studied to contribute to the developments in massive MIMO systems. MIMO has been classified in [88] into three main categories including Beam Forming Techniques, Spatial Multiplexing Techniques and Antenna Diversity Techniques. This research work addresses the issue of precoding where the specific dimension of spatial diversity is explored.

- **Beamforming**: In beamforming technique, the concept of optimizing the beam size and width is utilized through the use of smart antennas. This is done under the consideration of the environmental variations and location of the receiver. Multiple antennas are manipulated to form beams directed to specific users for better signal-to-interference ratio (SINR) and thus gains better throughput. The transmitter is equipped with efficient signal processing modules capable of adapting to the changing conditions of the receiver and environment in the wireless system. Also, precoded tables are used that resemble hard coded beams that are decided in advance. Beam Forming helps to generate improved and high output signals for the mobile users [89]. Research is being done in generating efficient beam forming techniques in analog, digital and hybrid domains to target higher capacity requirements (i.e. thousand times higher) as well as maintaining energy constraints of the 5G networks [90].

- **Spatial multiplexing methods**: Multiple data streams are used to increase the system throughput. Spatial multiplexing exploits the existence of multiple channels between the transmitter and receiver pairs to support independent data streams. This forms parallel data streams between respective transmiter-receiver pairs. For a massive MIMO system with N transmit antennas, N sub streams can be created between the transmitter and receivers. The receivers are equipped with adequate combining schemes to collect the incoming signals and form the signal [91].

- **Antenna diversity techniques**: Also known as spatial diversity techniques, the presence of multiple antennas is manipulated to generate a signal at the receiver with less fading. This has two main forms namely transmit diversity and receiver diversity. Copies of the same signal is transmitted using the multiple antennas that are spaced at least half a wavelength apart from each other. This is known as transmit diversity where precoding of the transmit antennas is done to ensure that the signals received at the receiver remain uncorrelated. These signals travel across the channel and are received at the receiver with different energies. The receiver uses efficient combining schemes to recover the original signal with considerable SNR [63]. The use of multiple antennas at the receiver to successfully recover the signal is known as receiver diversity. Combination of transmit and receiver diversity has been targeted to achieve recovery of a single data stream between each transmitter and receiver pair.

Considering the spatial multiplexing and antenna diversity techniques, both have their own benefits in supporting better performance. The former has the advantage of higher data rates while the latter supports optimized bit-error rates. As in [63], both benefits of higher data rates and reduced bit-error rates can not be achieved together and a compromise has to be made in selecting one of these techniques, i.e. spatial multiplexing or antenna diversity. To target better bit-error rate, redundancy can be introduced using the multiple antennas and the same information sequence. The most common type of this technique utilized space and time to transmit redundant data signals and is known as space-time coding. The receiver uses appropriate signal combining techniques to recover the signal [91]. The benefit of using antenna diversity schemes is better diversity gains with considerable coding gain without compromise on the bit-error rate [63]. Space-time coding has been used to implement spatial diversity techniques where STTCs and STBCs have been studied as the two main variants [62].

2.1.4 Research Motivation

It is evident from the above discussion that precoding is a major concern in massive MIMO as well as future generation of wireless communication networks. This research work targets the design of efficient STBCs to address the issue of precoding in the massive MIMO systems. A novel system model is designed and presented based on hyper-complex variables using DP antennas.

This work addresses the problem of limited data rates when it comes to using multiple antennas elements and increased transmit symbols. Novel coding paradigms are proposed that consider the concerns about throughput in massive MIMO and future wireless systems. Both transmit and receiver diversities are studied for their influence in optimizing the performance of the massive MIMO systems for 5G networks and beyond.

Chapter 3

Literature Review

The future of wireless communications demands high data rates, better coverage, negligible latency, enhanced reliability and ultra-high quality of audio as well as video transmissions. The modern wireless systems have incorporated mobility and communication between distant places where the communication can experience scattering, reflection, diffraction and multipath fading [1]. The underlying environments are also diverse including indoor and outdoor; rural, urban and suburban; and, micro, macro and picocellular. Dealing with such diverse environments demand power and bandwidth efficient BS as well as remote devices yet they are aimed to be small in their size. Considering all these requirements, studies have been evaluated to target the spectral efficiency for achieving greater data rates by enhancing the antenna design and incorporating efficient data communication schemes. The main research challenge is the achievement of a cost and performance efficient solution within the restricted antenna designs and resources of the wireless communication systems. Following sections will appreciate the studies which highlight the significance of multiple antennas at transmitter and receiver ends of the communication link followed by an analysis of how antennas and data communication schemes can be integrated together for efficient performance results. In this regime, space-time coding will be studied, which is one of the data communication techniques.

3.1 Multiple Transmit and Receive Antennas

Early wireless communications, i.e. until early 1990s, comprised of infrastructure with only the receiver equipped with antenna arrays while the transmitter had a single antenna. This paradigm shifted when studies articulated the importance of antenna arrays at both ends of the wireless communication channel [92–94]. These emphasized on the fact that for higher scattering environments, use of antennas arrays at both the transmitter and receiver ends can generate gains in terms of better channel efficiency and data rates. Different variants of these antenna arrays at the two ends of the wireless communication channel have been studied including antenna arrays at the receiver only, antenna arrays at both the ends of the link and finally, antenna arrays only at the transmitter. The reliability of the wireless communication channel is mostly influenced to deteriorate due to the multipath fading, which incorporates bigger challenges as compared to wired and even satellite communication cases. Factors like transmitter power control and sacrificing the channel bandwidth provide a direct increase in the SNR of the wireless communication link. The first factor is the most effective scheme to address the issue of multipath fading yet it exposes the wireless communication system to two major issues. First, the transmitter needs to increase its transmit power which has constraints in terms of cost and size of the underlying equipment. Second, the transmitter can gain knowledge about channel's state through a feedback channel from the receiver which is exhaustive in terms of link throughput and complexity at both ends of the communication link. Yet, these changes are in direct contrast to the demands of the future generations of wireless communication systems. Thus, addressing the multipath fading at both the remote UE and BS without increasing transmit power or sacrificing channel bandwidth seems a very optimistic goal.

Diversity techniques has been put forth in research studies as alternative techniques which are effective in fighting the multipath fading. Known forms of diversity techniques include frequency diversity, time diversity, space diversity and polarization diversity [95]. Time interleaving and spread spectrum has been studied in addressing the issue of multipath fading in the paradigm of time and frequency diversity, respectively. The limitation of the delays in the slowly varying channel and restrictions on the coherence bandwidth being significantly different as compared to the spreading bandwidth due to the channel having small delay spread. Antenna diversity is the practical approach to this mitigate the effects of multipath fading [1]. The classical method used multiple antennas at the receiver and used selection or combining techniques to enhance the signal power and achieve better signal quality. This approach brings in the concerns about cost, power and size of the remote UEs. Thus, in order to preserve the simplicity of the remote UEs, diversity techniques have been used at the BSs to get better signal quality and system performance.

Hundreds and thousands of remote units are served by a single BS. Thus, any additional equipment and circuitry installed at a single BS proves much more economical as compared to the cost of the installation of the same equipment at so many remote units. Thus, the best solution is to use transmit diversity techniques where additional antennas and transmit chains are added to the BS improving the quality of the signal received at the receiver and optimizing the infrastructural expenses to a minimum level. In literature, different techniques have been used to integrate transmit diversity into the wireless communication infrastructure. one of the technique sent multiple copies of the same signal using multiple transmit antennas at different time intervals. At the receiver, maximum-likelihood sequence estimator (MLSE) or a minimum mean-squared error (MMSE) equalizer is used to obtain the original signal with considerable diversity gains [96, 97]. Other techniques have also been proposed to exploit the use of transmit diversity with coding techniques to combat multipath fading and achieve better diversity gains. STTCs have been used in combination with transmit antenna diversity to achieve coding and diversity gains, where maximum likelihood decoder is used at the receiver to decode the symbols [98]. Although, this solution did proved beneficial, it is computationally expensive. Thus, it cannot be deployed for systems which demand efficiency in terms of cost and computations. The cost of processing in the STTCs based transmit diversity scheme increases exponentially with increasing diversity order and bandwidth efficiency (bits / s/ Hz). At this point, in 1998, Alamouti presented the solution to multipath fading through a simple transmit diversity scheme [1]. In this scheme, the transmission symbols are processed for transmission through two transmit antennas to achieve better signal quality at the receiving end. This transmitted symbols were decoded using maximum likelihood decoding rules and the achieved diversity order matched that of maximum-ratio receiver combining (MRRC), where two antennas were used at the receiving end. The proposed scheme by Alamouti proved to be the classical work where transmit diversity was exploited to achieve better performance. The diversity order of this scheme could be increased to 2M, if two transmit and M receiver antennas are used. Alamouti scheme became a foundation in the area of space-time block coding for addressing multipath fading. It overcame the need of computational complexity, redundancy of transmit symbols and any receiver feedback, as it exploits the use of space through multiple antennas and not time or frequency for its symbol transmission.

Use of multiple transmit and receive antennas have been extensively adopted in the areas of communication systems and information theory to generate diversity gains with improved system reliability and promising data rates. Such systems can be deployed in diverse environments and applications. Integration of receive antenna diversity with transmit diversity adds to the diversity and performance gains at the cost of added equipment at the remote UEs. Yet, this surely becomes a beneficial approach for applications where quality of the received signal, reliability and data rate is the priority. Recently, studies have shown that MIMO increases the capacity of the channel significantly in comparison to the single-transmit single receive antenna systems with constraints of maintaining similar power and bandwidth requirements [99–101]. Thus, next generation wireless communication systems demand efficient data communication techniques deployed over MIMO systems to meet the challenges of higher data rates, reduced latency and promising diversity gains.

3.2 Orthogonal Code Designs

In wireless communication, the signal travels through diverse environments with reflection, refraction, scattering and other effects like thermal noise. These influence the quality of the signal while it moves from the transmitter to the receiver. In space-time block coding, multiple copies of data are transmitted through multiple spaced antennas forming independent data streams

$$\text{time-slots} \downarrow \overset{\text{transmit antennas} \rightarrow}{\begin{bmatrix} s_{11} & s_{12} & \cdots & s_{1N} \\ s_{21} & s_{22} & \cdots & s_{2N} \\ \vdots & \vdots & & \vdots \\ s_{T1} & s_{T2} & \cdots & s_{TN} \end{bmatrix}}$$

Figure 3.1: Matrix representation for an STBC

generating many data paths to the receiver. In presence of the above mentioned factors, some of these copies might get severely effected while other copies might reach to the receiver matching closely to the transmitted signal quality [98]. This has been made possible due to the redundancy produced through space-time block coding, which enables fine combination of all the copies of the received signal through efficient decoding techniques. Alamouti, for the first time, presented a transmit diversity scheme using block codes.. Later, several studies were done based on construction and benefits of STBCs [102].

An STBC can be represented using a matrix where the columns represent the transmission from each transmit antenna over time while each row defines the timeslot, as shown in Figure 3.1. The entries of this matrix are the modulated symbols, $s_{ij}, i = 1 \dots T, j = 1 \dots N$, where T is the number of timeslots required to transmit a single code block and N is the number of transmit antennas, being transmitted from j^{th} transmit antenna in the i^{th} timeslot. The code rate, r, of a STBC with k number of symbols is the ratio of the number of symbols and the timeslot [102], given in Equation (3.1). The Alamouti code is defined for two transmit one receive antenna $\big(2 \times 1\big)$ and achieves full rate $\big(r = 1\big)$.

$$r = \frac{k}{T} \tag{3.1}$$

$$\mathbf{C} = \begin{bmatrix} c_1 & c_2 \\ -c_2^* & c_1^* \end{bmatrix}, \tag{3.2}$$

where $c_i, i = 1, 2$ can be real or complex numbers. Alamouti's scheme considers the complex channel between the transmitter and receiver. Is defined by three functions including the encoding and transmission sequence of the data symbols at the transmitter, the combining scheme at the receiver, and the decision rule to recover the original signal from the encoded data symbol at the receiver [1]. The code developed by Alamouti was a COD that supported decoupled decoding at the receiver.

Definition 2.1: *A generalize COD* $\mathbf{A}$*, on commuting complex number* $\{z_1, z_2, \ldots, z_n\}$ *of type* $\{s_1, s_2, \ldots, s_n\}$ *is an* $m \times n$ *matrix with entries from the set* $\{\ 0, \pm z_1, \pm z_1^*, \pm z_2, \pm z_2^*, \ldots, \pm z_n, \pm z_n^*\}$ *including possible multiplications by complex imaginary unit* i *and satisfies the following condition*

$$\mathbf{C}^H\mathbf{C} = \sum_{h=1}^{n}(s_h(|z_h|)^2)\mathbf{I}_{n\times n} = \lambda\mathbf{I}_{n\times n}\,, \tag{3.3}$$

where $\boldsymbol{I}_{n\times n}$ *is the* $n \times n$ *identity matrix,* $\boldsymbol{H}$ *is the Hermitian tranform,* λ *is a positive real number and so the columns of* A *are formally orthogonal.*

Alamouti presented this scheme for two transmit antennas with one and two receive antenna combinations. This resulted in exploiting the space and time diversities to ensure orthogonality of the code with increased diversity gains. The results showed significant diversity gains due to increased receiver diversity while this has been traded with the reduced code rate with increased number of transmit antennas.

STBCs, introduced as a generalization of the Alamouti Scheme, are orthogonal. Orthogonality is seen when any two pair of columns of the STBC are orthogonal. This reflects onto the receiver side in terms of simplicity and linearity at the decoder. Such an STBC is supposed to comply to the conditions of the code rate. Also, there is a specific class of STBCs which supports greater data rates by compromising orthogonality and experiencing inter-symbol interference (ISI). These STBCs are known as quasi-othrogonal STBCs.

Orthogonal designs have been used for wireless communication with multiple transmitters and receivers as STBCs. These are known to have maximum transmit diversity with simple decoupled decoding at the receiver. The aim is to achieve maximum rate to by increasing the bandwidth efficiency through the use of orthogonal designs [16]. In this quest, the struggles have been made to design codes which maximize rate considering real as well as complex symbols.

Definition 2.2: *A real orthogonal design* $\mathbf{O_x}$ *is a* $m \times n$ *rectangular matrix with entries from the set* $\{\ 0, \pm x_1, \pm x_2, \pm x_3, \ldots, \pm z_n\}$ *and satisfies the following condition*

$$\mathbf{O_x}^T \mathbf{O_x} = \sum_{h=1}^{n} (x_h)^2 \mathbf{I}_{n \times n} = \lambda \mathbf{I}_{n \times n}\,, \tag{3.4}$$

where $\boldsymbol{I}_{n\times n}$ *is the* $n \times n$ *identity matrix,* $\boldsymbol{T}$ *is the transpose of a matrix tranform,* λ *is a positive real number and so the columns of* A *are formally orthogonal.*

3.2.1 QODs

With advancements in the underlying technology and communication paradigms, the need to increase the capacity of the wireless communication system has become a goal for researchers. Thus, investigation to develop newer and more efficient means of coding of the data symbols increased [13, 102]. This initiated the need to consider better antenna designs. Also, considering the flexibility and benefits of STBCs, which are adopting well with multiple antennas and promising performance gains, they have been studied as a valuable candidate for next generation MIMO systems [13, 98, 102–108].

DP antennas have been adopted in literature as the infrastructural reform in the point-to-point wireless communication. This is presented in the realm of having two orthogonal antennas installed together on a single antenna base. DP antenna designs have been studied as they exhibit the flexibility to transmit and receive two symbols simultaneously. This has been adopted due to the increased demands for higher channel capacity and data rates. DP antennas has been used to exploit the polarization diversity. In [109,110], DP antennas were used to integrate the wireless channel model with quaternion codes. These codes were defined over quaternion algebra concepts over the complex domain. The designs presented using the quaternion algebra are known as QODs. They were claimed to integrate another form of diversity to the STBC designs, i.e. polarization diversity. QODs provide a valuable platform to exploit the polarization diversity in addition to space and time diversities. However, this has its own trade offs in terms of constructing orthogonal codes from the noncommutative nature of the quaternions.

In order to develop orthogonal codes using quaternion algebra and DP antennas, in [20], authors developed a system model where the STBCs are used to integrate polarization diversity for the development of orthogonal code designs. To investigate several methods to build QODs, Seberry and his team developed different construction techniques. First comprised of designs based on existing orthogonal and COD designs. These were obtained using quaternion permutation matrices or the symmetric paired designs (i.e.

$A + Bj$). Work done in this area has been promising in creating entirely a new dimension for research. However, it remained inefficient in achieving the real goal of exploiting the polarization diversity for higher code rates. Also, the QODs developed using the existing CODs are expected to face the same limitation as with CODs in terms of increased decoding complexity and limited code rates. The inherent constraints on the CODs propagates to the QODs designed from them. Thus, in [20], authors developed QODs and related theory that was independent of the existing CODs. These comprises of quaternion-commuting variables, amicable designs and size two QODs. First method was a simple generalization of the orthogonal and CODs in a constraint system model. Second and third QOD construction techniques did provided QOD designs in a unique way but the number of constraint to achieve this goal kept them unaccepted.

Considering these schemes, none of the proposal could explicitly show that a full QOD is transmitted using the DP antenna and its polarizations. Thus, this remained as an open area to study the future of QODs independent of the CODs. Use of complex OSTBCs has been shown to support better performance. Similar was studied in the need to develop a system that can exploit polarization diversity independently. Complex OSTBCs and OSTPBCs are the same in transmitting the signals using multiple antennas but here, in the case of OSTPBCs, the antennas used for communication are DP antennas.

3.3 Research Gaps

Work has been done in quaternion domain [13, 20, 98, 102, 109, 110] to design orthogonal codes that provide higher diversity gains. These codes has been designed using the complex orthogonal codes or complex quasi-orthogonal codes. This limits the true exploitation of the polarization diversity when the DP antennas are used. The explicit representation of the communication channel for DP antennas and the code structure has not been addressed using the pure quaternion algebra. This leaves the gains entirely dependent on exploiting together the space, time and polarization diversities. There is a requirement to investigate the effects of polarization diversity independently while integrated into the system of DP antennas using pure quaternions. This study will address these shortcomings by addressing the QODs based on pure quaternion algebra. Also the underlying channel characteristics will be studied to determine the impact of using DP antennas as the transmitter and receiver antennas.

Chapter 4

Optimal Polarization Diversity Gain in DP Antennas Using Quaternions

Over the past few decades, wireless communication aims to support high data rates and reliability and thus several techniques exploiting space, time and polarization diversities have been used to achieve large diversity gains [111–113]. Orthogonal space time block codes (OSTBC) in combination with polarization diversity promise optimal diversity gains [109]. For DP antennas, a new system model based on quaternionic structure of the channel is proposed in this thesis that offers a way to exploit polarization diversity independently of other forms of diversities. Moreover, such OSTPBC achieve better throughput and provide a linear decoupled decoding solution at the receiver, which significantly reduces computational complexity.

4.1 Introduction

Wireless propagation channel experiences attenuation due to multipaths and multiuser interference which makes the detection of transmitted signal difficult at the receiver. Transmit antenna diversity is a viable solution where multiple copies of the same transmitted signal incorporate different delays and create frequency selective fading. The receiver can process the received signals to achieve the diversity gain exploiting various forms such as time, space, frequency or polarization diversity. Increasing the number of transmit antennas enhances the diversity gain yet introduces problems of reduced code rates and increased receiver complexity [16]. In practice, the insufficient antenna spacing and the lack of scattering reduces the capacity, owing to increased channel correlation and for closely spaced antennas, mutual coupling might not be negligible. Interestingly, the antenna coupling might be beneficial to MIMO systems [114] contrary to what was believed earlier that it degrades the capacity. Future generations of mobile communication demands higher data rates and reduced latency. In order to achieve that different diversity schemes are integrated with polarization diversity [115, 116] as they can be used to mitigate the multipath effect to maintain a reliable communication link with an acceptable quality of service (QoS). Polarization diversity requires no extra bandwidth as well as physical antenna separation. Polarization diversity enables the simultaneous transmission and reception of information signals using the orthogonality of the polarized antennas [117].

Recently, the QODs have been studied extensively [20, 109, 118]. In all such approaches, the quaternion designs were not directly employed, rather complex quasi-orthogonal STBCs were obtained and different system models were constructed from them. Such non-orthogonal designs were capable to achieve higher diversity gains in MIMO systems. However, these designs adversely affect the performance for two main reasons. Firstly, because of the non-orthogonal nature of these codes it was impossible to obtain decoupled decoding at the receiver end. On the other hand, the theory of QODs was initially designed following the idea proposed in [19] to exploit polarization diversity which it failed to attain in general. In STBCs, the polarization diversity is attained from the cross-polar (CP) components in the wireless channel which indicates that it relies on the time diversity and cannot be exploited independently.

In this dissertation, a new system model based on quaternion channels is proposed that provides optimal solutions to the aforementioned problems. It is shown that quaternions offer a better solution especially for DP antennas to attain polarization diversity gains without exploiting time diversity. Such designs achieve better throughput and linear decoupled decoding solution is an intrinsic feature of the approach. It is demonstrated that QOD of order one which is suitable for a 1×1 configuration of DP antennas provides more information than the standard Alamouti code for a 2×2 single-polarized system. Besides, the existence of more than one QOD of order two with maximal rate provides them a sharp edge over an Alamouti scheme in which there is only a single code of "complex" rate 1.

4.2 Interpreting QODs

A quaternion is represented by two complex numbers as $q = z_1 + z_2 j$, such that $z_1, z_2 \in \mathcal{C}$. The quaternion space Q is fundamentally composed of a non-commutative basis $\{1, i, j, k\}$, such that $i^2 = j^2 = k^2 = ijk = -1$ and $ij = k = -ji, jk = i = -kj, ki = j = -ik$. As in the complex domain, the quaternion conjugate q^Q is defined as $q^Q = z_1^* - jz_2^*$, along with the property that $q^Q q = q q^Q = |q|^2$. Furthermore, the transpose of a quaternion matrix $\mathbf{Q} = [q_{mn}]$ is defined as $\mathbf{Q}^Q = [q^Q_{nm}]$ [119].

For DP transmit antennas, we propose a relatively modified system model where it is assumed that two co-located single-polarized (SP) antennas (horizontal and vertical) of a particular DP antenna are considered as a single unit, symbolically $T^D = T_H + T_V j$. We use superscript D to indicate a DP antenna, where the symbols H and V denote the horizontal and vertical polarization, respectively. Therefore, the transmission of complex symbols through a single DP unit can be modeled by a quaternion $q = z_1 + z_2 j$, which ensures that symbols z_1 and z_2 are transmitted instantaneously through T_H and T_V, respectively. Note that the coupling with j, ensures that the symbol z_2 is transmitted through an orthogonal polarization. Each complex symbol z is obtained from standard modulation schemes, e.g., quadrature phase shift key (QPSK). In order to exploit diversity gains from space and time, the OSTPBC can be defined in the quaternion domain [20]. **Definition 4.1 (QOD).** *A QOD* $\mathbf{Q}$*, on pure quaternion elements* $\{q_1, q_2, \ldots, q_n\}$ *of type* $\{s_1, s_2, \ldots, s_n\}$ *is an* $m \times n$ *matrix with entries from set* $\{\ 0, q_1, q_1^*, q_2, q_2^*, \ldots, q_n, q_n^*\}$ *including possible multiplications on the left and/or right by quaternion elements* $q \in Q$ *and satisfying the condition*

$$\mathbf{Q}^Q \mathbf{Q} = \sum_{h=1}^{n} (s_h(|q_h|)^2) \mathbf{I}_{n\times n} = \lambda \mathbf{I}_{n\times n}\,, \tag{4.1}$$

where $\mathbf{I}_{n\times n}$ *is the* $n \times n$ *identity matrix and* λ *is a positive real number.*

Based on the above realization, it is now natural to redefine code rates for QODs.

Definition 4.2.

The quaternion code rate r_q *of a QOD* $\mathbf{Q}$*, is the ratio of number of trans-*

mitted quaternions to the number of time slots through a combination of DP antennas.

For example, for a single DP antenna T^D, a QOD $\mathbf{Q}_1 = [q]$ where $q^Q q = 1$, has rate $r_q = 1$, transmitting one quaternion in one time slot. It is emphasized that the above construction is in sharp contrast to the standard procedure given in [20], [109], [120], [110], where $\mathbf{Q}_1$, is thought to be made of two complex numbers and it yields a quasi code $\mathbf{C}_q = [z_1\ z_2]$, which has "complex" rate 2. Such quasi-codes, $\mathbf{C}_q$, are not used in this chapter and they are indicated for a brief comparison.

In order to develop quaternion designs over an arbitrary number of transmitting DP antennas, following steps have been considered. First, it is pertinent to mention that in all previous approaches [20], [120], the QODs were obtained by employing conditions on underlying CODs **A** and **B**, namely, they must form a symmetric-pair and satisfy the amicable condition. It was shown that by permuting columns of one of the CODs, another COD can be constructed which satisfy both of these conditions. Furthermore, the matrix **B** needs not necessarily be a permuted version of **A** [121]. In Theorem 9 of [20], the authors proved that two CODs which form a symmetric pair design generate a QOD. It was then required to see whether all QODs arise from symmetric pair designs. In the following theorem, we show that it is indeed a case.

Theorem 4.1. *The necessary and sufficient condition for an STBC in the quaternion domain* $\mathbf{Q} = \mathbf{A} + \mathbf{B}j$*, to be a QOD is that both* **A** *and* **B** *are CODs and satisfy the symmetric property, i.e.,* $(\mathbf{A}^H\mathbf{B})^T = \mathbf{A}^H\mathbf{B}$*, where* $(.)^H$ *and* $(.)^T$ *denotes the Hermitian and transpose operators, respectively.*

Proof. A QOD **Q** has a unique decomposition in the complex domain, i.e.,

$\mathbf{Q} = \mathbf{A} + \mathbf{B}j$, where both $\mathbf{A}$ and $\mathbf{B}$ are two complex matrices of the same order as of $\mathbf{Q}$. We first note that $\mathbf{A}^Q = \mathbf{A}^H$ and following identity holds. Suppose $\mathbf{A} = [a_{mn}]$, such that $a_{mn} \in \mathcal{C}$, then $\mathbf{A}j = j\mathbf{A}^*$ where $*$ denotes the conjugation operation. Consequently, $\mathbf{A}^H j = j\mathbf{A}^T$ and $j\mathbf{A}^H = \mathbf{A}^T j$. Note that the multiplication with quaternion j eats up the conjugation in Hermitian. It turns out that

$$\mathbf{Q}^Q\mathbf{Q} = (\mathbf{A}^H - j\mathbf{B}^H)(\mathbf{A} + \mathbf{B}j), \tag{4.2}$$

$$= \mathbf{A}^H\mathbf{A} - j\mathbf{B}^H\mathbf{B}j + \mathbf{A}^H\mathbf{B}j - j\mathbf{B}^H\mathbf{A}, \tag{4.3}$$

$$= \mathbf{A}^H\mathbf{A} - j\mathbf{B}^H\mathbf{B}j + \left(\mathbf{A}^H\mathbf{B} - (\mathbf{A}^H\mathbf{B})^T\right)j, \tag{4.4}$$

therefore the condition in Equation (4.1) is true if and only if

$$\mathbf{A}^H\mathbf{A} = \lambda_1\mathbf{I},\ \mathbf{B}^H\mathbf{B} = \lambda_2\mathbf{I},\ \mathbf{A}^H\mathbf{B} - (\mathbf{A}^H\mathbf{B})^T = \mathbf{0}.$$

Hence, proved. □

There were four ways indicated in [20] which can be used to obtain viable QODs of order two. Theorem 4.1 indicates that the construction of QODs entirely depends on finding two CODs which form symmetric-pair design, therefore, it can be shown that all those QODs arise from symmetric-pairs. The problem of generating QODs of order two from a geometrical point of view has been considered. Supposing a quaternion matrix

$$\mathbf{Q}_1 = \begin{bmatrix} q_1 & q_2 \\ q_3 & q_4 \end{bmatrix} = \begin{bmatrix} z_1 + z_2 j & z_3 + z_4 j \\ z_5 + z_6 j & z_7 + z_8 j \end{bmatrix}, \tag{4.5}$$

which can provide a coding matrix of maximum rate $r_q = 2$, for two DP antennas provided $\mathbf{Q}_1$ satisfies orthogonality in Equation (4.1). Geometrically, the requirement in Equation (4.1) can be interpreted as follows. The quaternion vectors $\mathbf{q}_1 = [q_1\ \ q_3]^T, \mathbf{q}_2 = [q_2\ \ q_4]^T$, reside in an eight dimensional space $\mathbf{q}_1, \mathbf{q}_2 \in Q \times Q \simeq \mathbb{R}^8$, containing eight linearly independent real vectors. The problem now is to choose two such orthogonal vectors from $Q \times Q$, resulting in $\binom{8}{2} = 28$ combinations to choose from, such that the resulting pair ensures the proposal of Equation (4.1). Since the diagonal entries of $\mathbf{Q}_1^Q\mathbf{Q}_1$ are same, i.e., $|\mathbf{q}_1|^2+|\mathbf{q}_2|^2$. Therefore, to meet the orthogonality condition in Equation (4.1), the off-diagonal terms must vanish necessarily, i.e.,

$$
\begin{aligned}
&q_1^Q q_2 + q_3^Q q_4 = 0,\\
&q_2^Q q_1 + q_4^Q q_3 = 0.
\end{aligned} \tag{4.6}
$$

As there are two algebraic equations in four unknown quaternions, therefore, the quaternionic rate is bounded above by 1, i.e., $|r_q| \leq 1$ for QODs of order two. This result seems analogous to an Alamouti code which has a maximum rate 1 for two single-polarized transmit antennas. However, in QODs the quaternionic rate $r_q = 1$ corresponds to an STBC with *complex* rate 2 which implies transmission of four complex symbols in two time slots. As is known, there exists no such COD of order two which has "complex" rate 2. More concretely, for four single-polarized antennas corresponding to two DP antennas, there does not exist any COD which has maximal rate equal to 1 and it is a unique code of order two which exists only for two transmit antennas [102]. Therefore, the theory of QODs offers better ways to deal with the problem of generating codes especially for DP antennas. In addition to this, there is a class of QODs of order two with maximal rate $r_q = 1$, thereby, violating the uniqueness condition of Alamouti schemes. Another essential feature of such QODs is that they provide linear decoupled decoding solution which is briefly discussed in the subsequent section.

The conditions in Equation (4.6) can be decomposed into a systems of eight real algebraic equations, which can be solved using any computer algebra system. One such solution of a QOD of order two has been considered with maximal rate $r_q = 1$, i.e.,

$$
\mathbf{Q}_2 = \begin{bmatrix} z_1 + z_2 j & z_4 + z_3 j \\ z_2^* - z_1^* j & -z_3^* + z_4^* j \end{bmatrix}, \tag{4.7}
$$

in which two DP antennas transmit two quaternions $z_1 + z_2 j$ and $z_4 + z_3 j$, in the first time slot, respectively. Subsequently, $z_2^* - z_1^* j = -j(z_1 + z_2 j)$ and $-z_3^* + z_4^* j = j(z_4 + z_3 j)$ are sent in the second time slot. For completeness, another QOD of maximal rate is constructed. Two quaternions $q_1 = z_1 + z_2 j$ and $q_2 = z_3 + z_4 j$ are supposed, such that $q_1 q_2^Q$ does not contain $k-th$ component. Then, it is easy to generate a QOD such that $\mathbf{Q}_3 = \begin{bmatrix} i(z_1+z_2 j) & -j(z_4+z_3 j) \\ j(z_4+z_3 j) & -i(z_1+z_2 j) \end{bmatrix}$. Similarly, another QOD of rate $r_q = 1/2$, given as

$$\mathbf{Q}_4 = \begin{bmatrix} z_1 + z_2 j & j(z_1+z_2 j) \\ i(z_1+z_2 j) & -k(z_1+z_2 j) \end{bmatrix}. \tag{4.8}$$

Likewise, a QOD of order three of "complex" rate 3/4 is,

$$\mathbf{Q}_5 = \begin{bmatrix} z_1 + z_2 j & z_2 + z_1 j & z_3 + z_3 j \\ -z_2^* + z_1^* j & z_1^* - z_2^* j & 0 \\ -z_3^* & -z_3^* j & z_1^* + z_1^* j \\ -z_3^* j & -z_3^* & z_2^* + z_2^* j \end{bmatrix}. \tag{4.9}$$

All such codes have decoupled decoding as shown in the subsequent section.

4.3 Quaternionic System Model

4.3.1 Channel Realization

Suppose a system with $1 \times 1-$configuration of DP antennas. As described above, the transmission through a DP antenna T^D is modeled by a single quaternion $q = z_1 + z_2 j$, where z_1 and z_2 are simultaneously transmitted through T_H and T_V, respectively. A quaternionic channel gain is proposed between transmit and receive DP antennas, given by $h = h_1 + h_2 j$. Therefore, the received vector is a quaternion and has the form

$$r = qh + n, \tag{4.10}$$

where n is a quaternionic noise, i.e., $n = n_1 + n_2 j$. It should be pointed out that in [20], [120], the same design was implemented in a different way where a quasi-code was constructed, i.e., $\mathbf{C}_q = [z_1 \; z_2]$, using an operator $\mathbb{C}$ [122]. The necessity of such an operation is due to the channel which was based on complex numbers. In the proposed model, the channel is quaternionic where the aim is to exploit quaternion domain and its intrinsic operations in full spirit. More briefly, the quaternionic form of channel is also studied in a different context in [123]. For a fair comparison, this situation may be compared with a $2 \times 2-$single polarized antennas where the system model is

$$\mathbf{R} = \begin{bmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{bmatrix} = \begin{bmatrix} z_1 & z_2 \\ -z_2^* & z_1^* \end{bmatrix} \begin{bmatrix} h_{HH} & h_{HV} \\ h_{VH} & h_{VV} \end{bmatrix} + \begin{bmatrix} n_{11} & n_{12} \\ n_{21} & n_{22} \end{bmatrix}. \tag{4.11}$$

It is clear from above that there are four complex channel coefficients owing to the fact that there must be four complex channel gains between two DP antennas. Apparently, the proposed quaternion channel seems to contain two complex numbers. In order to elaborate our point, it is important to open a brief discussion on the above design. In this standard approach in Equation (4.11), the columns refer to horizontal and vertical polarized antennas or vice versa. Without loss of generality, it can be assumed that the first column correspond to H-polarized antenna and second column to the V-polarized antenna. Let us first delve on the working of H-polarized antenna which refers to first column of $\mathbf{R}$ in Equation (4.11). Therefore, r_{11} and r_{21} are received at H-polarized antenna with a time delay. We now look at their explicit forms

$$r_{11} = z_1 h_{HH} + z_2 H_{VH}, \tag{4.12}$$

$$r_{21} = -z_2^* h_{HH} + z_1^* H_{VH}. \tag{4.13}$$

Note that in r_{21}, the *conjugates* of transmitted symbols are employed and not the original symbols. Furthermore, both received symbols r_{11} and r_{21} contain two channel coefficients. This accounts for the time diversity which is a necessity and that is why four complex channel coefficients are obtained such that two are in received symbols r_{11}, r_{21} of H-polarized antenna. The V-polarized antenna receives r_{12} and r_{22}, which contains the other two complex channel coefficients. This costs huge time delays, although, the diversity gain due to time by sending multiple copies of the original signals is a benefit. This presents that presence of four complex channel coefficients in Alamouti design is a necessity due to the time diversity. Here, the aim is to devise a mechanism to exploit the polarization diversity independently without including any time diversity. This seems to be possible by using quaternions with DP antennas.

However, it is important to note that, in the analysis, the quaternionic product "qh" is very crucial. It is only after its decomposition, the underlying operations behind quaternions became visible. For example, $qh = z_1h_1 - z_2h_2^* + (z_1h_2 + z_2h_1^*)j$, which indicates that the first complex symbol $z_1h_1 - z_2h_2^*$, which is received at $H-$polarized antenna is a combination of two complex channel gains. Similarly, the other part $z_1h_2 + z_2h_1^*$, which is received at $V-$polarized antenna again contains two complex channel gains. Therefore, a total of four complex channel gains are present as is expected in the standard approach. Apart from their simplicity as evident from Equation (4.10), the quaternions tend to exploit polarization diversity in a transparent and unique way. For example, the CP components h_{HV} and h_{VH} in Equation (4.11) are responsible for incorporating the polarization twists by reflection, scattering and other means [124]. However, their use entirely depends on sending the same copies of signals in different time slots due to the nature of Alamouti code in Equation (4.11). Therefore, the exploitation of polarization diversity relies on time diversity. On the other hand, quaternionic model Equation (4.10) does not need time diversity at all as both signals z_1 and z_2 are instantaneously sent through orthogonal polarization planes.

There is another strong reason in support of the proposed design. For example, physicaly realization of a quaternion $q = z_1 + z_2 j$, has been considered in the following way. It describes transmission of z_1 from H-polarized antenna and z_2 through V-polarized antenna *simultaneously*, while j ensures that these symbols are orthogonal because $z_2 j$ lies in a plane perpendicular to z_1, fully respecting the horizontal and vertical polarization structure of a DP antenna. According to the main idea as the signals z_1 and $z_2 j$ propagates through space in the forms of orthogonal polarizations of an em-wave, they suffer reflections, scattering and twists etc.. Although there can be substantial flipping of original transmitted polarizations but somehow the effect of orthogonal polarizations remains after reflections, scattering etc.. This also makes sense, for example due to reflection, two orthogonally polarized waves should have their counter parts in two reflected polarized waves, which are different but at least orthogonally polarized. This can be achieved by simply taking a quaternioinic channel $h = h_1 + h_2 j$, where $h_2 j$ serves the above purpose and it also follows the same spirit as for $q = z_1 + z_2 j$. Therefore, the answer lies in the explicit form of qh and not in the channel $h = h_1 + h_2 j$, which seems to contain only two complex numbers. This characteristic of quaternion designs for DP antennas gives them edge over almost all known CODs and to the best of knowledge of the author it cannot be replicated in the complex domain. Also note that in system model in Equation (4.10), there is no coding/decoding delay while in Equation (4.11) there is a delay at both ends. Besides the performance of Alamouti code, Equation (4.11) depends on maintaining spatial distance of at least half a wavelength at the receiver side while ten wavelengths at the BS in order to achieve maximum diversity gains [116]. Thus, considering the above points in favor of the physical interpretation of this model, it is very comparable to the Alamouti design where two transmit 2 receive SP antennas are explouit both time and space diversities. Here, a new form of diversity is exploited in combination with space to show its impact on the diversity gains. The gains seem to increase when time diversity is incorporated in to this system model with space and polarization diversities.

Following above lines, it is now natural to propose a general quaternionic system model for $N_T \times 1-$configuration of DP antennas which transmits symbols in $T-$times slots

$$\mathbf{R}_{T\times 1} = \mathbf{Q}_{T\times N_T}\mathbf{H}_{N_T\times 1} + \mathbf{N}_{T\times 1}, \tag{4.14}$$

where $\mathbf{H} = [h_1, h_2, ..., h_{N_T}]$, such that each entry is a quaternion $h_a = h_{a1} + h_{a2}j$, for all $a \in \{1, 2, ..., N_T\}$. The complex channel gains, h_{a1} and h_{a2} incorporate the effects of CP scattering and the channel is assumed to be Rayleigh fading, which implies that each element of channel gain matrix is a complex Gaussian random variable (RV) with zero mean and unit variance. Moreover, the noise $\mathbf{N} = [n_1, n_2, ..., n_T]$, and $n_b = n_{b1} + n_{b2}j$, such that $n_{b1}, n_{b2}\ \forall\ b = \{1, 2, ..., T\}$, represent the entries of white noise as two dimensional independent and identically distributed (i.i.d.) complex Gaussian RVs with zero mean and identical variance per dimension. It is emphasized that the above approach is different from previous attempts due to quaternionic nature of channel. For clarity, a complex "quasi-code" is included [17],

$$\mathbf{C}_q = \begin{bmatrix} z_1 & z_2 & z_2 & z_1 \\ -z_2^* & z_1^* & z_1^* & -z_2^* \end{bmatrix}, \tag{4.15}$$

constructed from QOD in Equation (4.8) in [110], where odd columns refer to transmission from one polarization plane and even columns contain symbols that are transmitted through orthogonal polarization plane. It was shown in [110] that an OSTPBC for 2×1 DP unit performs better than an Alamouti code for 2×1 single-polarized unit. Unfortunately, this is not a fair comparison because the performance of 2×1 DP unit may be compared with the performance of 4×2 single-polarized unit as is also pointed out in [116]. Furthermore, the above quasi-code in Equation (4.15) does not provide a decoupled solution at the receiver end. On the other hand, the proposed model is fully decoupled as shown below.

4.3.2 Linear Decoupled Solution

In all previous attempts, the ML-decoding rule norm is equivalent to the minimum of either the norm $\mathbf{R} - \mathbf{C}_q\mathbf{H}$ or its square for finding the transmitted symbols where the channel $\mathbf{H}$ was assumed complex. [20] proposed that a decoupled decoding can be obtained for any QOD even if the channel gain matrix of a DP transmission system is not modeled by a single quaternion gain. Later on, they corrected their decoding rule [122] and clarified that the decoupled decoding can be achieved for certain QODs only [125]. The main reason was assumption of complex nature of the channel. The following theorem confirms a linear decoupled solution to our proposed model in Equation (4.14).

Theorem 4.2. *For a given system model in Equation (4.14), the ML-decoding rule assumes a linear decoupled form*

$$\min_z ||\mathbf{R} - \mathbf{Q}\mathbf{H}||^2 = \min_z \Big(tr(\mathbf{R}^Q\mathbf{R}) + \lambda tr\big(\mathbf{H}^Q\mathbf{H}\big) - 2\Re\big(tr\big(\mathbf{R}^Q\mathbf{Q}\mathbf{H}\big)\big)\Big). \quad (4.16)$$

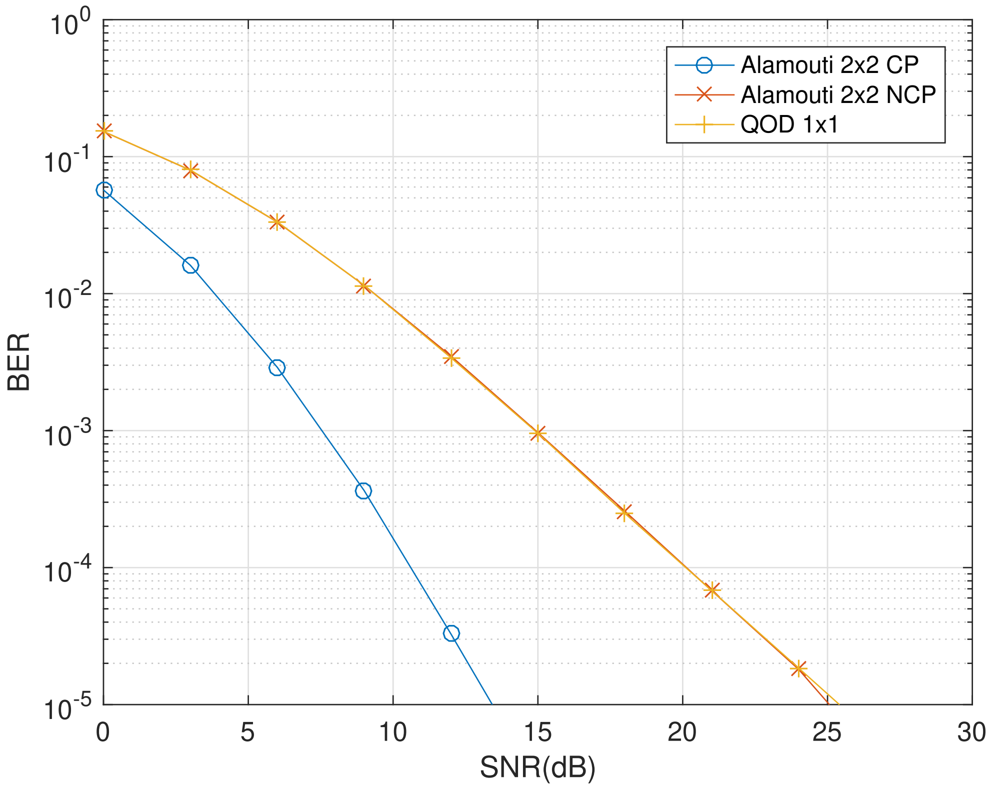

Figure 4.1: BER vs. SNR performance of (1x1) QOD.

Proof. The proof is straight forward owing to the fact that $||\mathbf{R} - \mathbf{QH}||^2 = \text{tr}\left((\mathbf{R} - \mathbf{QH})^Q(\mathbf{R} - \mathbf{QH})\right)$, which is easy to expand. The term $\text{tr}(\mathbf{H}^Q\mathbf{Q}^Q\mathbf{QH})$, which was the main source of problems in all previous attempts reduces to $\lambda\text{tr}(\mathbf{H}^Q\mathbf{H})$, using orthogonality condition in Equation (4.1) and does not contain the transmitted symbols. Consequently, there is only one term which is linear and contain transmitted symbols. Hence proved. □

Since the first two terms in Equation (4.16) are pure constants, therefore, the ML-decoding rule of minimizing the norm is equivalent to minimize $\Re\left(tr\left(\mathbf{R}^Q\mathbf{QH}\right)\right)$, where $\Re$ denotes the real part of a complex number.

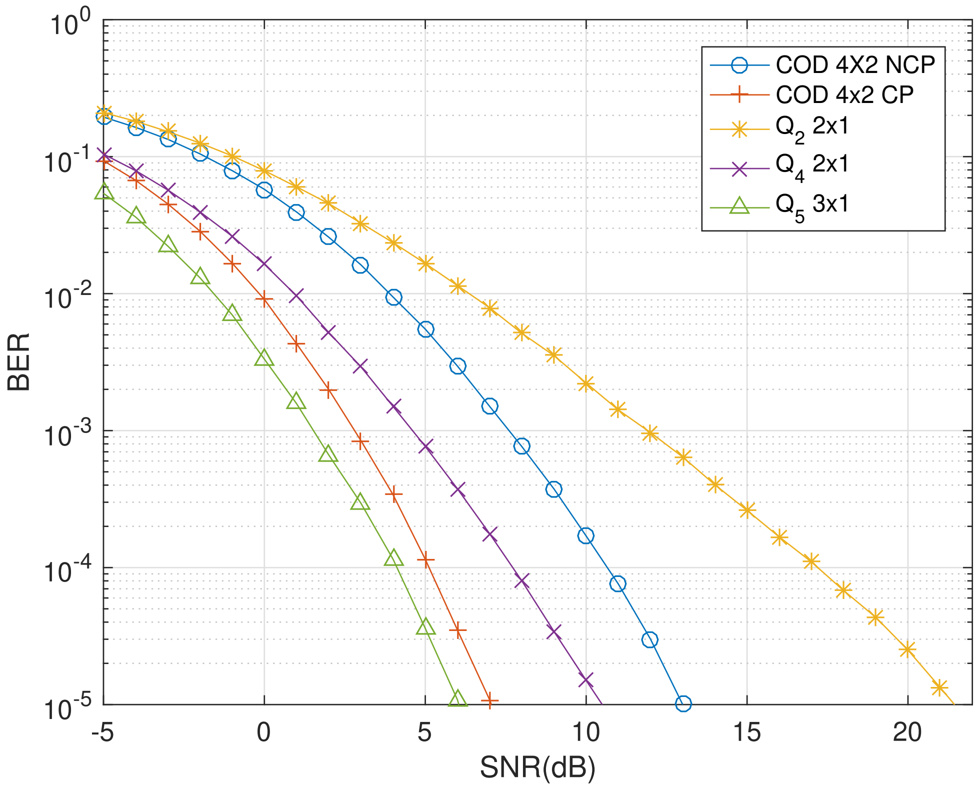

Figure 4.2: BER vs. SNR performance of $\mathbf{Q}_2, \mathbf{Q}_4, \mathbf{Q}_5$ & CODs

4.4 Simulation Results

To evaluate the performance and diversity gains, QODs given in Equations (4.7) – (4.10) are evaluated, corresponding to configurations 2×1, 3×1 and 1×1 of DP antennas, respectively. For simulations, quadrature phase shift keying is used and equal power distribution is ensured per antenna per polarization. The receivers are aware of the channel coefficients and uniform white noise is added in each polarization.

From Figure 4.1, it is clear that the performance of Equation (4.10) for a 1×1 DP system matches with that of an Alamouti design in Equation (4.11) for 2×2 single-polarized with non cross-polar (NCP) components.

However, it is worth pointing out that the Alamouti design exploits time diversity to attain it whereas Equation (4.10) achieves the same performance using polarization diversity. Contrary to what one would suspect that the performance match between codes in Equation (4.10) and Equation (4.11) with NCP components will occur in case of higher order configurations. The QOD in Equation (4.8) for 2×1 DP antennas that has complex rate 1, performs significantly better in Figure 4.2, than its counter part which is an Alamouti design for 4×2 single-polarized antennas with NCP. An improvement in the performance also comes from the fact that Equation (**??**) exploit time diversity besides polarization gain. In Figure 4.2, we also include the performance of QOD in Equation (4.7) which has its unique characteristic of sending four complex symbols in two time slots. Lastly, polarization diversity gain is attained by increasing the transmit antenna dimensions only by one and it is clear from BER performance of Q_5 that the gain will become more pronounced in higher dimensions.

It can be seen that the QODs already present and constructed using different techniques are limited in terms of exploiting the polarization diversity independent of the space and time diversities. The effect of space and time diversities in the works [20, 109, 110] could not explain the transmission of a pure QOD using a single DP antenna. They have represented the analogy between a single DP antennas and two single-polarized antennas. This leaves a constraint environment towards freedom of the possible dimensions that can be exploited to generate QODs. The quaternionic channel model presented in this chapter is a completely novel concept that relates to the use of DP antennas for the transmission of pure quaternions. It has been made evident that this representation of exploiting the higher dimensions using pure quaternions provides not only higher diversity gains, but this also results in higher code rates. The remarkable contribution of these pure QODs using DP antennas is the existence of linear and decoupled solution that remained a major area of concern for researchers in past. Thus, the QODs present in literature are developed on existing code designs in complex domain. However, this chapter defines the generation of QODs completely on its own over a quaternionic channel that supports higher diversity gains due to fully exploiting the polarization diversity.

4.5 Conclusions

The main purpose behind proposal of a new systems model based on quaternion algebra was to bring quaternions at the same level to the complex numbers in a natural way and it seems that one such instance to realize them is the point-to-point communication between two DP antennas. Just like complex numbers $z = x + iy$, we believe that both quaternions $q = z_1 + z_2 j$ and $h = h_1 + h_2 j$, seem to offer us a natural and physically acceptable solution which can lead to major breakthroughs in space-time coding. There is large amount of work that needs to be done on each of the above points, however, this was an attempt to provide one possible unified way in the form of proposed system model based on quaternions and demonstrated the potential it has to achieve them. The literature presents works that have developed QODs from existing quasi or COD designs. DP antennas have also been used [109], but this does not help in exploiting the polarization diversity independent of the space and time diversities. Thus, the subsequent benefits have not been achieved to full. The proposals in this chapter addresses this deficiency by presenting a channel model with pure QODS that has the capacity to independently exploit polarization diversity and the results support this claim [57].

Chapter 5

Linear and Decoupled Decoders for Dual-Polarized Antenna-Based MIMO Systems

STBCs has long been studied using CODs and many varieties of codes, both orthogonal and quasi-orthogonal, have been designed for different antenna configurations and code rates. Recently, QODs have been used to design STBCs that provide improved performance in terms of various design parameters. In this dissertation, we show that all QODs obtained from generic iterative construction techniques based on Adams-Lax-Phillips approach have linear and decoupled decoders which significantly reduce the computational complexity at the receiver. The analysis is based on the quaternionic description of communication channels among DP antennas. The proposed solution promises diversity gain with the quaternionic channel model and provides a decoupled decoding solution with independence of the number of receive DP antennas. A brief comparison is presented at the end to demonstrate the effectiveness of quaternion designs in two DP antennas over available STBCs for four single-polarized antennas.

5.1 Introduction

Wireless communication through multiple antennas has been used extensively in today's telecommunication standards owing to the multiple benefits they offer [1]. More specifically, in addition to providing high data rates through spatial multiplexing, multiple antennas can be used effectively to combat multi-path fading. There are numerous ways in which multiple antennas can be used to provide diversity in wireless signals such as using time, frequency, space and polarization, etc., and the underlying codes carry certain desirable properties such as orthogonality [20]. These properties can be exploited effectively at the receiver side to obtain decoupled solutions with least complexity. However, because of environmental scattering and imprecise antenna spacing, the diversity gains start to diminish and also pose the problem of coupled solutions, which are computationally expensive at the receiver [16].

With the advent of new generations of wireless communication systems that demand very high data rates and ultra reliability, the paradigm is shifting from simple MIMO to massive MIMO systems where the base station in anticipated to be equipped with hundreds of antennas. Therefore, there is a dire need to study other forms of diversity techniques with new antenna designs that contain, for instance, both polarizations for transmissions and require no extra bandwidth. Polarization diversity enables the simultaneous transmission and reception of information signals using the orthogonally polarized antennas [20]. It has been shown that using a special mathematical tool of quaternion algebra, the transmission/reception/decoding of these MIMO systems can be described effectively [117].

The QODs are under rigorous research, for instance, see [20, 109, 110, 118, 122, 123, 126, 127]. However, an important construction technique in these previous designs is that they were either extensions of CODs with some necessary properties to be applicable as QODs, or they were constructed from quasi-complex STBCs. However, despite the main motivation of exploring polarization diversity in DP antennas using quaternions, these QODs focused on construction of codes and their decoding.

The decoding of the codes presented previously seem to necessitate that only square orthogonal designs can produce decoupled decoding. In [128], the decoupled decoding solution for quasi-orthogonal codes has been presented based on a wireless communication channel model which is derived from the quaternionic channel representation. This model is restricted in terms of its application to only the zero cross polar scattering environments. Also, it restricts the number of DP antennas at the receiver. Both the above are conservative conditions on a generic wireless communication arrangement.

The quaternionic channel proposed in [57] provides codes with optimal rates exploiting polarization diversity along with space and time diversities resulting in higher diversity gains. This research work contributes in providing iterative construction techniques for designing QODs. It is remarkable to note that all generic iterative constructions of QODs result in decoupled and linear decoders with enhanced throughput using the system model proposed in [57], that forms the main result of this dissertation. Secondly, it is identified that there are non-square designs which can have decoupled solutions contrary to what has been believed that these fail to attain decoupled decoding and only pair-wise decoding is possible for them. The proposed design enjoys freedom in exploiting the transmit and receive diversities with no restriction on the antenna dimensions at both the transmitter and receiver ends. It fully exploits the polarization diversity using the cross polar scattering components, making it more practical for current and future massive MIMO wireless communication systems. A brief comparison of the performance of QODs for two-input and single-output (TISO) system of DP antennas is provided with a 4×2 multiple-input two-output (MITO) system of single-polarized antennas. The former is shown to have key advantages over the latter. Also, a detailed comparison of the proposed coding and decoding design for the quasi-orthogonal STBCs is evaluated in light of the literature.

5.2 Realization of Quaternion Designs

The simultaneous transmission of symbols through a DP antenna can be modeled through a unified quaternion $q = z_1 + z_2 j$, such that symbols z_1 and z_2 are transmitted instantaneously through T_H (horizontal) and T_V (vertical) polarizations, respectively. This line of approach is resurrected in [57] for single-input single-output (SISO) system by implementing an idea that the orthogonal polarization states can be represented as quaternions [19], thereby attaining polarization diversity gain. Unfortunately, for SISO systems the gains from different form of diversities are less apparent and becomes proficient for large number of antennas in MIMO systems. For such systems, it is necessary to develop an iterative approach so that higher order quaternion designs can be generated, which forms the main topic of this paper.

It is assumed that each quaternion in a QOD comprises of two complex symbols which are obtained from standard modulation schemes, e.g., QPSK. In order to exploit diversity gains from space and time, the OSTPBCs has been defined in the quaternion domain, as in Equation 5.10.

Unlike a SISO system of single-polarized antennas, a SISO system between DP antennas has several important features to offer. For example, for one DP antenna, a QOD $\mathbf{Q} = [q]$ where $q^Q q = 1$, has quaternionic rate 1, transmitting one quaternion in one time slot. This rate corresponds to a design with rate 2 of a TITO (2×2) system of single-polarized antennas, for which we already know that there exist no such "orthogonal" design which has rate more than 1. A design with rate more than 1 is a quasi-design which fails to attain a linear and decoupled decoder. However, as we shall see in the subsequent section that the above QOD of a SISO system has optimal decoding solution.

In [57], there were three QODs considered (two of order 2×2 and one QOD with order 4×3) which were based on non-iterative construction techniques. It is demonstrated that higher order QODs can easily be generated iteratively and have fast, linear, and decoupled decoders. Indeed, [16] proposed three generic iterative construction techniques, namely Adams-Lax-Phillips, Józefiak, and Wolfe constructions. A general COD is designed for $l + 1$ symbols embedded in a square matrix of order 2^l such that

$$\mathbf{A} = \begin{bmatrix} \mathbf{G}_{2^{l-1}}(z_1, z_2, \ldots, z_l) & z_{l+1}\mathbf{I}_{2^{l-1}} \\ -z_{l+1}^*\mathbf{I}_{2^{l-1}} & \mathbf{G}_{2^{l-1}}^H(z_1, z_2, \ldots, z_l) \end{bmatrix}, \tag{5.1}$$

where $l = \{1, 2, 3 \ldots\}$ and $\mathbf{G}_{2^{l-1}}(z_1, z_2, \ldots, z_l)$ represents a COD of order $2^{l-1} \times 2^{l-1}$ defined on symbols $\{z_1, z_2, \ldots, z_l\}$. For example, for $l = 1$, $\mathbf{G}_1(z_1) = [z_1]$.

It is now easy to generate square QODs using above mechanism [121]. We briefly indicate the steps involved in generating a hierarchy of such designs along with the main proof. In particular, by systematically swapping columns $1, 2, \ldots, N_t/2$ of matrix $\mathbf{A}$ with $(N_t/2)+1, (N_t/2)+2, \ldots, N_t$ columns, respectively, an equivalent matrix $\mathbf{B}$, is generated where N_t represents the number of antennas of COD on which permutation is performed. This gives rise to the following result where the redundant argument in $\mathbf{G}$ has been omitted for the sake of simplicity.

Theorem 5.1. *For a given COD* $\mathbf{A}$ *in Equation (5.1), a complex amicable and symmetric-paired design can be constructed such that the following realization*

$\mathbf{Q}_{2^l}(z_1, z_2, .., z_{l+1}) = \mathbf{A} + \mathbf{B}j =$

$$\begin{bmatrix} \mathbf{G}_{2^{l-1}} + z_{l+1}\mathbf{I}_{2^{l-1}}j & z_{l+1}\mathbf{I}_{2^{l-1}} + \mathbf{G}_{2^{l-1}}j \\ -z_{l+1}^*\mathbf{I}_{2^{l-1}} + \mathbf{G}_{2^{l-1}}^H j & \mathbf{G}_{2^{l-1}}^H - z_{l+1}^*\mathbf{I}_{2^{l-1}}j \end{bmatrix}, \tag{5.2}$$

provides a QOD of dimension $2^l \times 2^l$ *with complex rate* $(l+1)/2^l$*.*

Proof. In order to prove the quaternion orthogonality in Equation (3.4), it was noticed that $\mathbf{Q}_{2^l}^{\mathbf{Q}} = \mathbf{A}^H - j\mathbf{B}^H$, owing to the fact that both $\mathbf{A}$ and $\mathbf{B}$ are CODs. Hence, $\mathbf{Q}_{2^l}^{\mathbf{Q}}\mathbf{Q}_{2^l} = (\mathbf{A}^H - j\mathbf{B}^H)(\mathbf{A} + \mathbf{B}j)$, in which the outer product merely yields a Frobenius norm of complex numbers $z_1, z_2, \ldots, z_l$ multiplied by an identity matrix. However, the inner product $\mathbf{A}^H\mathbf{B}j - j\mathbf{B}^H\mathbf{A}$ is identically equal to zero because $\mathbf{A}^H\mathbf{B} = \mathbf{B}^H\mathbf{A}$. This follows from the construction of $\mathbf{B}$ which is obtained from $\mathbf{A}$ upon permutation of columns. □

For completeness, we delve on another iterative construction technique as it also has decoupled decoding. It is, however, different from the above approach for there is no need of necessarily generating $\mathbf{B}$ by permutation.

Lemma 5.1. *For a given square COD* $\mathbf{G}_{2^{l-1}}(z_1, z_2, .., z_{l+1})$*, the matrix*
$\mathbf{Q}_{2^1 \times 2^{l-1}}(z_1, z_2, .., z_{l+1}) =$

$$\begin{bmatrix} \mathbf{G}_{2^{l-1}}(z_1, z_2, \ldots, z_l) + z_{l+1}\mathbf{I}_{2^{l-1}}j \\ -z_{l+1}^*\mathbf{I}_{2^{l-1}} + \mathbf{G}_{2^{l-1}}^H(z_1, z_2, \ldots, z_l)j \end{bmatrix} \tag{5.3}$$

provides a quaternion design of order $2^l \times 2^{l-1}$*, with rate* $(l+1)/2^l$*.*

Proof. The Equation (5.10) for above code is simplified into $(\mathbf{G}^H + z^*\mathbf{I})(\mathbf{G} + z\mathbf{I}) + (-z\mathbf{I} + \mathbf{G})(-z^*\mathbf{I} + \mathbf{G}^H)$. As before, the outer products of both terms result into Frobenius norm due to the orthogonality of $\mathbf{G}$. The inner product is $\mathbf{G}^H z + z^*\mathbf{G} - z\mathbf{G}^H - \mathbf{G}z^*$, which is identically equal to zero due to commutativity of the complex numbers. □

Note that due to identity matrix in the term $z_{l+1}\mathbf{I}_{2^{l-1}}j$, there will be at least one element in the first time slot which does not contain j. Hence, this construction lacks in providing non-zero QODs. Codes with non-zero entries ensure fixed average power codeword by maintaining reduced peak power transmission from every antenna. This results in favorably low peak-to-average power ratio (PAPR) and diminishes the hardware implications to switch antennas on and off while transmitting a non-zero and zero, respectively [129]. An iterative technique without such a drawback is considered below.

Lemma 5.2 *For two generalized CODs* $\mathbf{G}_{2^l}(z_1, z_2, \ldots, z_{l+1})$ *and* $\mathbf{L}_{2^l}(z_{l+2}, z_{l+3}, .., z_{2l+2})$ *with same structure, which are constructed on the COD formulation, as shwon in Equation (5.1), it follows that*

$$\mathbf{G}_{2^l}^H \mathbf{L}_{2^l} + \mathbf{L}_{2^l}^H \mathbf{G}_{2^l} = \mathbf{G}_{2^l} \mathbf{L}_{2^l}^H + \mathbf{L}_{2^l} \mathbf{G}_{2^l}^H = \gamma \mathbf{I}_{2^l}, \tag{5.4}$$

where $\gamma = 2\Re\left(\sum_{k=1}^{l+1} z_k^* z_{l+1+k}\right)$.

It is important to mention that the above lemma does not hold true for two general CODs. For example, two Alamouti codes with different structures $\begin{bmatrix} z_1 & z_2 \\ z_2^* & -z_1^* \end{bmatrix}$ and $\begin{bmatrix} z_3 & z_4 \\ -z_4^* & z_3^* \end{bmatrix}$, fail to satisfy it while they can be used effectively in generating a consistent COD of the form, as in Equation (5.1).

By using the above lemma, the following theorem is defined which can be proved in a similar way.

Theorem 5.2. *For generalized CODs* $\mathbf{G}_{2^{l-1}}(z_1, z_2, \ldots, z_l)$ *and* $\mathbf{L}_{2^{l-1}}(z_{1+2}, z_2, \ldots, z_{2l+2})$*, a symmetric-paired design,*

$$\mathbf{Q}_{2^{l+1}\times 2^l}(z_1, \ldots, z_{2(l+1)}) = \begin{bmatrix} \mathbf{G}_{2^l} + \mathbf{L}_{2^l} j\,, \\ \mathbf{L}_{2^l} + \mathbf{G}_{2^l} j \end{bmatrix}, \tag{5.5}$$

is a QOD of dimension $2^{l+1} \times 2^l$ *with complex rate* $(l+1)/2^l$.

QODs are evaluated for the proposed construction technique for (2×1), (4×1) and (8×1) DP antenna arrangements in subsequent sections.

5.3 Higher Order Designs For DP Antennas

5.3.1 Designs for $(2 \times 1)-$DP Antennas

To elaborate the generalized construction technique, a QOD of rate 1 is presented, where the COD **A** contains symbols z_1 and z_2, while the COD **B** contains independent symbols z_3 and z_4, respectively. Using Equation (5.5), the following symmetric-paired design of order 4×2 is obtained with a complex code rate of 1,

$$\mathbf{Q}_1 = \begin{bmatrix} z_1 + z_3 j & z_2 + z_4 j \\ z_2^* + z_4^* j & -z_1^* - z_3^* j \\ z_3 + z_1 j & z_4 + z_2 j \\ z_4^* + z_2^* j & -z_3^* - z_1^* j \end{bmatrix}, \tag{5.6}$$

where $l = 1$, $\mathbf{G}_1 = [z_1]$, and $\mathbf{L}_1 = [z_3]$ from Equation (5.5). Note that in the above code, both ends of a DP antenna will be used at each time slot.

Therefore, the QODs obtained through this procedure will contain non-zero complex symbols in each time slot.

Following example is considered for illustration. Unlike [121], this design is used in the proposed system model and show that it has decoupled decoding without the need of applying projection operator.

Distinctiveness of QODs

An interesting property of QODs which distinguishes them from CODs is that there exists QODs of complex rate greater than one, which have decoupled decoders. For instance, the following QOD has code rate 2 and is shown to posses decoupled decoder

$$\mathbf{Q}_2 = \begin{bmatrix} z_1 + z_2 j & z_4 + z_3 j \\ z_2^* - z_1^* j & -z_3^* + z_4^* j \end{bmatrix}. \tag{5.7}$$

The main reason behind is that the Alamouti code is proposed for single-polarized antennas while QODs are developed for DP antennas. Similarly, there is a QOD of rate 1 which is given as

$$\mathbf{Q}_3 = \begin{bmatrix} z_1 + z_2 j & j(z_1 + z_2 j) \\ i(z_1 + z_2 j) & -k(z_1 + z_2 j), \end{bmatrix}. \tag{5.8}$$

which provides decoupled decoding.

5.3.2 Design for $(4 \times 1)-$DP Antennas

An Alamouti code $\mathbf{G}_2 = \begin{bmatrix} z_1 & z_2 \\ -z_2^* & z_1^* \end{bmatrix}$ is selected. Then using Equation (5.1), a COD of order 4 is obtained. Through permutations, the matrix $\mathbf{B}$ is generated using Equation (5.5) and finally the following QOD for 4 DP antennas

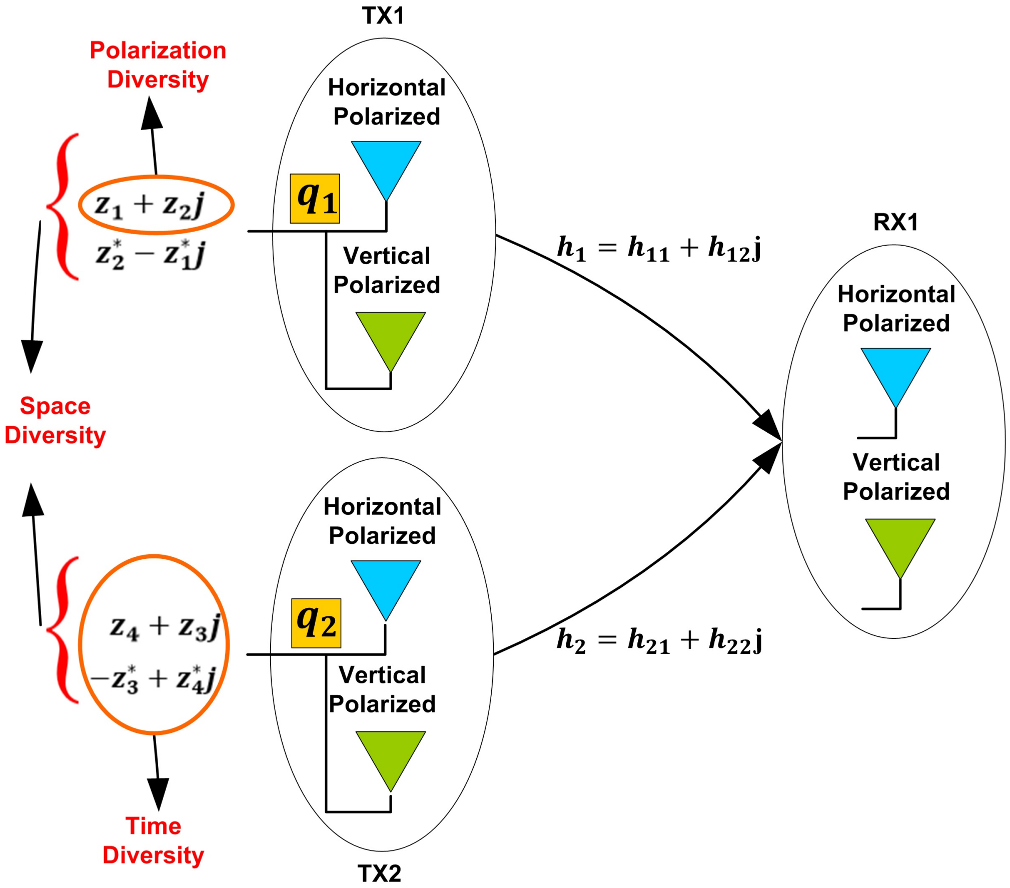

Figure 5.1: TISO DP antenna configuration exploiting space, time and polarization diversities.

with rate 3/4 is obtained.

$$\mathbf{Q}_4 = \begin{bmatrix} z_1 + z_3 j & z_2 & z_3 + z_1 j & z_2 j \\ -z_2^* & z_1^* + z_3 j & -z_2^* j & z_3 + z_1^* j \\ -z_3^* + z_1^* j & -z_2 j & z_1^* - z_3^* j & -z_2 \\ z_2^* j & -z_3^* + z_1 j & z_2^* & z_1 - z_3^* j \end{bmatrix}.$$

In comparison to the last three QODs, the above QOD suffers one drawback that in each time slot two polarizations (H or V) of at least two DP antennas needs to be switched, which results in a high peak-to-average power ratio and

$$\mathbf{Q}_5 = \begin{bmatrix} z_1 + z_4 j & z_2 & z_3 & 0 & z_4 + z_1 j & z_2 j & z_3 j & 0 \\ z_2^* & -z_1^* + z_4 j & 0 & z_3 & z_2^* j & z_4 - z_1^* j & 0 & z_3 j \\ z_3^* & 0 & z_1^* + z_4 j & z_2 & z_3^* j & 0 & z_4 + z_1 j & z_2 j \\ 0 & z_3^* & z_2^* & -z_1 + z_4 j & 0 & z_3^* j & z_2^* j & z_4 - z_1 j \\ -z_4^* + z_1^* j & z_2 j & z_3 j & 0 & z_1^* - z_4^* j & z_2 & z_3 & 0 \\ z_2^* j & -z_4^* - z_1 j & 0 & z_3 j & z_2^* & -z_1 - z_4^* j & 0 & z_3 \\ z_3^* j & 0 & -z_4^* + z_1 j & z_2 j & z_3^* & 0 & z_1 - z_4^* j & z_2 \\ 0 & z_3^* j & z_2^* j & -z_4^* - z_1^* j & 0 & z_3^* & z_2^* & -z_1^* - z_4^* j \end{bmatrix}. \tag{5.10}$$

is not practically desirable.

5.3.3 Design for $(8 \times 1)-$DP Antennas

A maximal rate square COD of order $2^2 = 4$ is given by

$$\mathbf{G}_4 = \begin{bmatrix} z_1 & z_2 & z_3 & 0 \\ z_2^* & -z_1^* & 0 & z_3 \\ z_3^* & 0 & z_1^* & z_2 \\ 0 & z_3^* & z_2^* & -z_1 \end{bmatrix}, \tag{5.9}$$

as a seed matrix to generate the required QOD using Equation (5.5). Thus for a massive MIMO comprising of an 8×1 system, a QOD which has code rate 1/2 and is given in Equation (5.10).

5.4 System Model and Decoding

A TISO of DP antennas has been considered where it is necessary to emphasize the role of quaternions which is more recognizable in this case, therefore, it is given as

$$\mathbf{R} = \begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} q_1 & q_2 \\ q_3 & q_4 \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}, \tag{5.11}$$

where each element in the above construction is a quaternion. This configuration is represented diagrammatically in 5.1. It is clear that each DP antenna consists of both horizontal and vertical polarizations that are completely orthogonal to each other. Considering the transmission of a (2x2) code, the first column entries are representing the quaternion symbol being transmitted through the horizontal and vertical polarization of the first DP antenna at the transmitter, i.e. TX1, while the second column represent the quaternion symbol transmitted through the horizontal and vertical polarizations of the second DP antenna, TX2. The spacing between these two DP antennas at the transmitter introduces the space diversity. The second row of the code block represents the second time slot that incorporates the time diversity. The beneficial aspect of this design is its ability to exploit the polarization diversity independent of the space and time diversities. The separation defined as orthogonality between the horizontal and vertical polarizations in a single DP antenna exploits the polarization diversity by ensuring that the signals transmitted remain uncorrelated and remain in orthogonal polarizations even if they encounter scattering, reflections, etc. during transmission. Thus, this supports the idea presented in Chapter 4 that QODs designed using pure quaternion algebra are able to exploit all the polarization diversity fully independent of the space and time diversities when used with DP antennas.

Through first antenna in the above TISO system, the transmission of a pair of two complex symbols is encoded in q_1 and another pair in q_3. This indicates that the above QOD exploits time and space diversities along with polarization diversity, as shown in Figure 5.1. It is worth pointing out that each quaternionic product, e.g., $q_a h_b$ contains a crucial information about the nature of quaternion domain. If it is decomposed for a general quaternionic product then $q_{a1}h_{b1} - q_{a2}h_{b2} + j(q_{a1}h_{b2} + q_{a2}h_{b1})$ is obtained, where $q_a = q_{a1} + jq_{a2}$ and $h_b = h_{b1} + jh_{b2}$. Therefore, four complex channel gains for each antenna in a 2×1 system are obtained. Subsequently, a system model for a MISO system of $N_T \times 1$ DP antennas is given as,

Subsequently, a system model for a MIMO system of dual-polarized antennas can be constructed in the same way for such a system with $N_t \times N_r$ dual-polarized antennas

$$\mathbf{R}_{T\times N_r} = \mathbf{Q}_{T\times N_t}\mathbf{H}_{N_t\times N_r} + \mathbf{N}_{T\times N_r}, \tag{5.12}$$

In Equation (5.12), symbols are transmitted in $T-$times slots where $\mathbf{H} = [h_1, h_2, \ldots, h_{N_T}]$, such that each entry is a quaternion $h_a = h_{a1} + h_{a2}j$, for all $a \in \{1, 2, \ldots, N_T\}$. The complex channel gains, h_{a1} and h_{a2} incorporate the effects of cross polar scattering and the channel is assumed to be Rayleigh fading, which implies that each element of channel gain matrix is a complex Gaussian RV with zero mean and unit variance. Moreover, the noise $\mathbf{N} = [n_1, n_2, \ldots, n_T]$, and $n_b = n_{b1} + n_{b2}j$, such that $n_{b1}, n_{b2}\ \forall\ b = \{1, 2, \ldots, T\}$, represent the entries of white noise as two dimensional i.i.d. complex Gaussian RVs with zero mean and identical variance per dimension.

Based on the system model given in Equation (5.12), the following theorem confirms a linear decoupled solution at the receiver for all QODs constructed in this chapter. It was previously proved for non-iterative QODs in [57] but its validity is now confirmed for all QODs obtained in previous sections in this chapter.

Theorem 5.3. *For a given system model in Equation (5.12), the ML-*

decoding rule assumes a linear decoupled form

$$\min_{z} ||\mathbf{R}-\mathbf{QH}||^2 = \min_{z} \Big(tr(\mathbf{R}^Q\mathbf{R}) + \lambda tr\big(\mathbf{H}^Q\mathbf{H}\big) - 2\Re\big(tr\big(\mathbf{R}^Q\mathbf{QH}\big)\big)\Big). \tag{5.13}$$

There are two main advantages of the above ML-decoding rule. The presence of QOD, $\mathbf{Q}$, in the term $\Re\big(tr\big(\mathbf{R}^Q\mathbf{QH}\big)\big)$, contributes only linear terms of complex symbols. Secondly, it significantly reduces the computational load at the receiver for the reason that the term $\Re\big(tr\big(\mathbf{R}^Q\mathbf{QH}\big)\big)$, can easily be expressed without involving matrices at all which may be cumbersome for large MIMO systems.

It is emphasized here that for all QODs obtained in the previous section, the decoupled decoding rule, similar to Corollary 5.1, can be derived explicitly. As an illustration of the above result, we choose among them the QODs given in Equations (5.6) and (5.7) and demonstrate that the above ML-decoding rule is both linear and decoupled.

Corollary 5.1. *The ML-decoding rule (5.13) for QOD given in Equation* (5.6)*, reduces to*

$$\begin{aligned}
&-2\min_{z_1} \Re(r_1^q z_1 h_1 - r_2^q z_1^* h_2 + r_3^q z_1 h_1 j - r_4^q z_1^* h_2^* j)\,,\\
&-2\min_{z_2} \Re(r_1^q z_2 h_2 + r_2^q z_2^* h_1 + r_3^q z_2 h_2^* j + r_4^q z_2^* h_1^* j)\,,\\
&-2\min_{z_3} \Re(r_1^q z_3 h_1^* j - r_2^q z_3^* h_2^* j + r_3^q z_3 h_1 - r_4^q z_3^* h_2)\,,\\
&-2\min_{z_4} \Re(r_1^q z_4 h_2^* j + r_2^q z_4^* h_1^* j + r_3^q z_4 h_2 + r_4^q z_4^* h_1)\,,
\end{aligned} \tag{5.14}$$

where $\mathbf{R} = [r_1\ r_2\ r_3\ r_4]^T$, *is the received vector with each element is a quaternion and* $h_1 = h_{11} + h_{12}j$ *and* $h_2 = h_{21} + h_{22}j$.

The ML-decoding rule (5.13) for QOD given in Equation (5.10), reduces to

$$
\begin{aligned}
&-2\min_{z_1}\Re(r_1^q z_1 h_1 + r_1^q z_1 h_5^* j - r_2^q z_1^* h_2 - r_2^q z_1^* h_6^* j + r_3^q z_1^* h_3 - r_3^q z_1^* h_7^* j - r_4^q z_1 h_4 \\
&- r_4^q z_1 h_3^* j + r_5^q z_1^* h_1^* j + r_5^q z_1^* h_5 - r_6^q z_1 h_2^* j - r_6^q z_1 h_6 + r_7^q z_1 h_3^* j + r_7^q z_1 h_7 - r_8^q z_1^* h_4^* j \\
&- r_8^q z_1^* h_8)\,, \\
&-2\min_{z_2}\Re(r_1^q z_2 h_2 + r_1^q z_2 h_6^* j + r_2^q z_2^* h_1 + r_2^q z_2^* h_5^* j + r_3^q z_2 h_4 + r_3^q z_2 h_8^* j + r_4^q z_2^* h_3 \\
&+ r_4^q z_2^* h_7^* j + r_5^q z_2 h_2^* j + r_5^q z_2 h_6 + r_6^q z_2^* h_1^* j + r_6^q z_2^* h_5 + r_7^q z_2 h_4^* j + r_7^q z_2 h_8 + r_8^q z_2^* h_3^* j \\
&+ r_8^q z_2^* h_7)\,, \\
&-2\min_{z_3}\Re(r_1^q z_3 h_3 + r_1^q z_3 h_7^* j + r_2^q z_3 h_4 + r_2^q z_3 h_8^* j + r_3^q z_3^* h_1 + r_3^q z_3^* h_5^* j + r_4^q z_3^* h_2 \\
&+ r_4^q z_3^* h_6^* j + r_5^q z_3 h_3^* j + r_5^q z_3 h_7 + r_6^q z_3 h_4^* j + r_6^q z_3 h_8 + r_7^q z_3^* h_1^* j + r_7^q z_3^* h_5 + r_8^q z_3^* h_2^* j \\
&+ r_8^q z_3^* h_6)\,, \\
&-2\min_{z_4}\Re(r_1^q z_4 h_1^* j + r_1^q z_4 h_5 + r_2^q z_4 h_2^* j + r_2^q z_4 h_6 + r_3^q z_4 h_3^* j + r_3^q z_4 h_7 + r_4^q z_4 h_4^* j \\
&+ r_4^q z_4 h_8 - r_5^q z_4^* h_1 - r_5^q z_4^* h_5^* j - r_6^q z_4^* h_2 - r_6^q z_4^* h_6^* j - r_7^q z_4^* h_3 - r_7^q z_4^* h_7^* j - r_8^q z_4^* h_4 \\
&- r_8^q z_4^* h_8^* j)\,,
\end{aligned}
\tag{5.15}
$$

Note that there are four complex channel gains between a TISO system of DP antennas, as shown in Figure 5.1. As this system is equivalent to a MIMO 4×2 system of single-polarized antennas, therefore, it may appear that it should have eight channel gains in total with two for each link. However, in the proposed model each quaternionic product results in the same number of channel gains. The receiver now computes the decision metric $\min_z ||\mathbf{R} - \mathbf{QH}||^2$, which involves matrices. On the other hand, an optimal decoder, as shown in Equation (5.13), is also used to receive signal that significantly reduces total time consumed. In the end, the decoder for QOD in Equation (5.10) is given.

Corollary 5.2. *The ML-decoding rule (5.13) for QOD in Equation* (5.10) *is given in Equation* (5.15) *where* $\mathbf{R} = [r_1\ r_2\ r_3\ r_4\ r_5\ r_6\ r_7\ r_8]^T$, *is the received vector with each element is a quaternion and* $h_i = h_{i1} + h_{i2}j, i = 1, 2, ...8.$

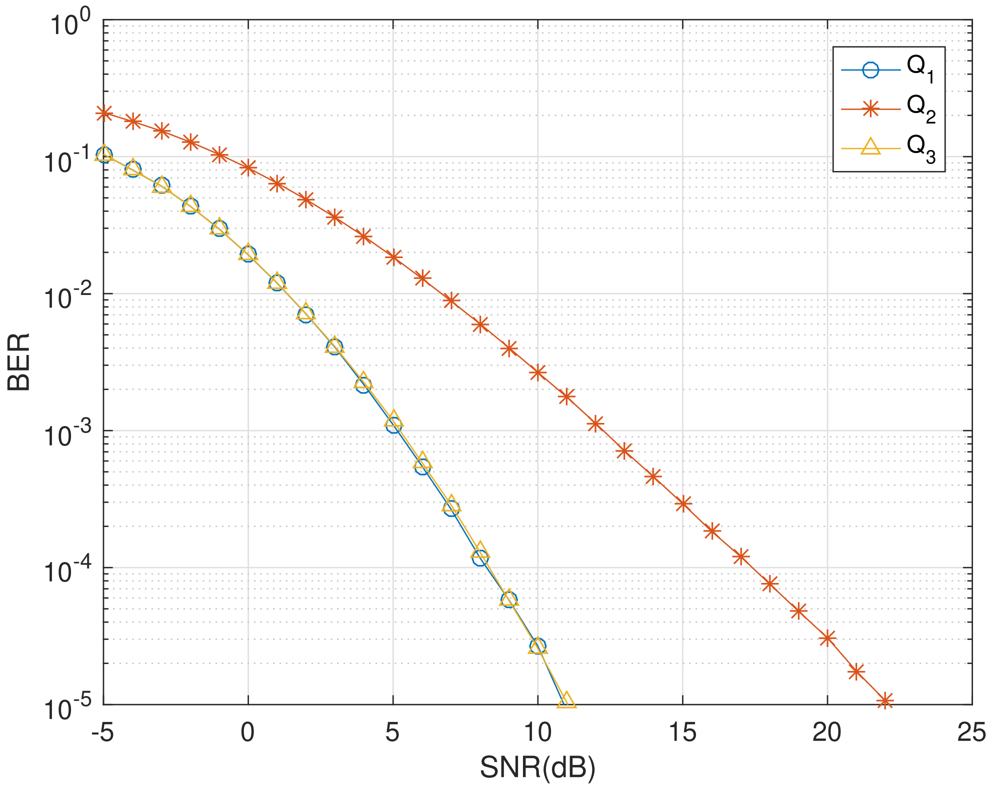

Figure 5.2: BER vs. SNR performance of QODs $\mathbf{Q}_1, \mathbf{Q}_2$ and $\mathbf{Q}_3$.

5.5 Key Aspects of QODs under Quaternion Channel

5.5.1 Comparison with Benchmark Codes

DP antennas can exploit the space, time and polarization diversities suitably and the designed codes based on QODs are used to serve this purpose. The BER performance of the codes $\mathbf{Q}_1, \mathbf{Q}_2$ and $\mathbf{Q}_3$ is given in Figure 5.2. Notice that the designs $\mathbf{Q}_1$ and $\mathbf{Q}_3$ have overlapping BER curves, attaining same diversity gains, however, $\mathbf{Q}_3$ has a relatively better throughput than $\mathbf{Q}_1$. A consolidated comparison of QODs is performed, developed for two

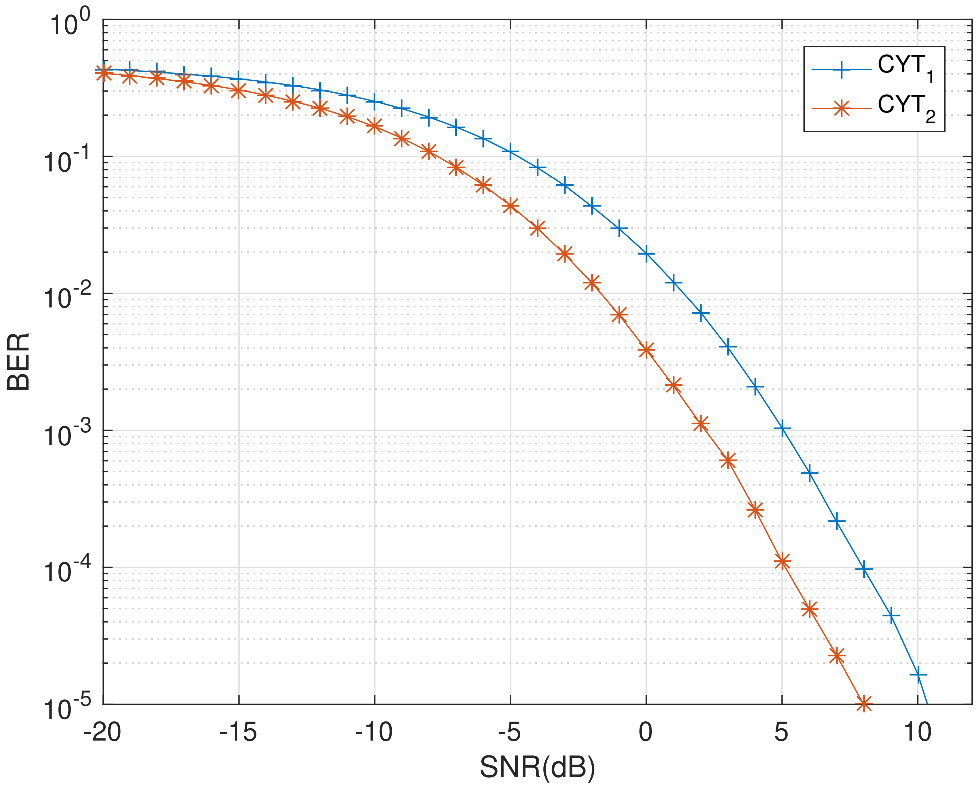

Figure 5.3: BER vs. SNR performance of QODs $\mathbf{CYT}_1$ and $\mathbf{CYT}_2$.

DP antennas against conventional STBCs designed for four single-polarized antennas to indicate the major differences. For four transmit single-polarized antennas, the authors in [130] and [131] used *amicable designs* to construct minimum decoding quasi STBCs that essentially require the products $\mathbf{A}^H\mathbf{B}$ and $\mathbf{B}^H\mathbf{A}$ to be equal where $\mathbf{A}$ and $\mathbf{B}$ are amicable STBCs. This drastically reduces the code rate which in our designs remain stable as they require only the property of being symmetric property. In particular, for four and eight transmit antennas, there are two "square" STBCs constructed (Equation (20) in [130]), which have rates 1 and 3/4, respectively. Their STBCs are denoted with **CYT** given as

$$\mathbf{CYT}_1 = \begin{bmatrix} z_1 & z_2 & -z_3 & -z_4 \\ -z_2^* & z_1^* & -z_4^* & z_3^* \\ z_3 & z_4 & z_1 & z_2 \\ z_4^* & -z_3^* & -z_2^* & z_1^* \end{bmatrix}, \tag{5.16}$$

where $z_1 = c_2^R + jc_3^I$, $z_2 = c_2^R + jc_4^I$, $z_3 = c_3^R + jc_1^I$ and $z_4 = c_4^R + jc_1^I$, where j refers to imaginary unit. Previously, the authors in [131] obtained the following square OSTBC (Equation (11) in [131])

$$\mathbf{CYT}_2 = \begin{bmatrix} x_1^* - x_2 & x_1^* + x_2 & x_3^* & -x_3^* \\ jx_1 + jx_2^* & -jx_1 + jx_2^* & jx_3^* & jx_3^* \\ -x_3 & x_3 & x_1^* - x_2^* & x_1^* + x_2^* \\ -jx_3 & -jx_3 & jx_1 + jx_2 & -jx_1 + jx_2 \end{bmatrix}, \tag{5.17}$$

which was shown to have significant performance edges over previously known codes proposed in [102] and [132]. Because of the orthogonality condition, the above code $\mathbf{CYT}_2$ has less code rate than quasi code $\mathbf{CYT}_1$. Figure 5.3 provides a comparison of the analog of $\mathbf{CYT}_1$ and $\mathbf{CYT}_2$ in quaternion domain where the later results in improved performance, i.e., a 5dB gain in SNR at 10^{-5} BER. These codes do not have decoupled decoders in complex domain while a decoder has been presented in quaternion domain which ensures decoupling. On the other hand, the state-of-art linear dispersion STBCs are proposed for four transmit antennas in [133] of maximal rate 1 when the distance between transmit antennas satisfies a physical constraint.

For a brief fair comparison of QODs with benchmark codes, we construct the complex analogues of QODs by applying operator $\mathcal{C}$ such that $\mathcal{C}(z_1 + z_2 j) = [z_1 \ z_2]$. In this way, we obtain four equivalent quasi-codes (yet quaternion orthogonal) for four transmit single-polarized antennas given as

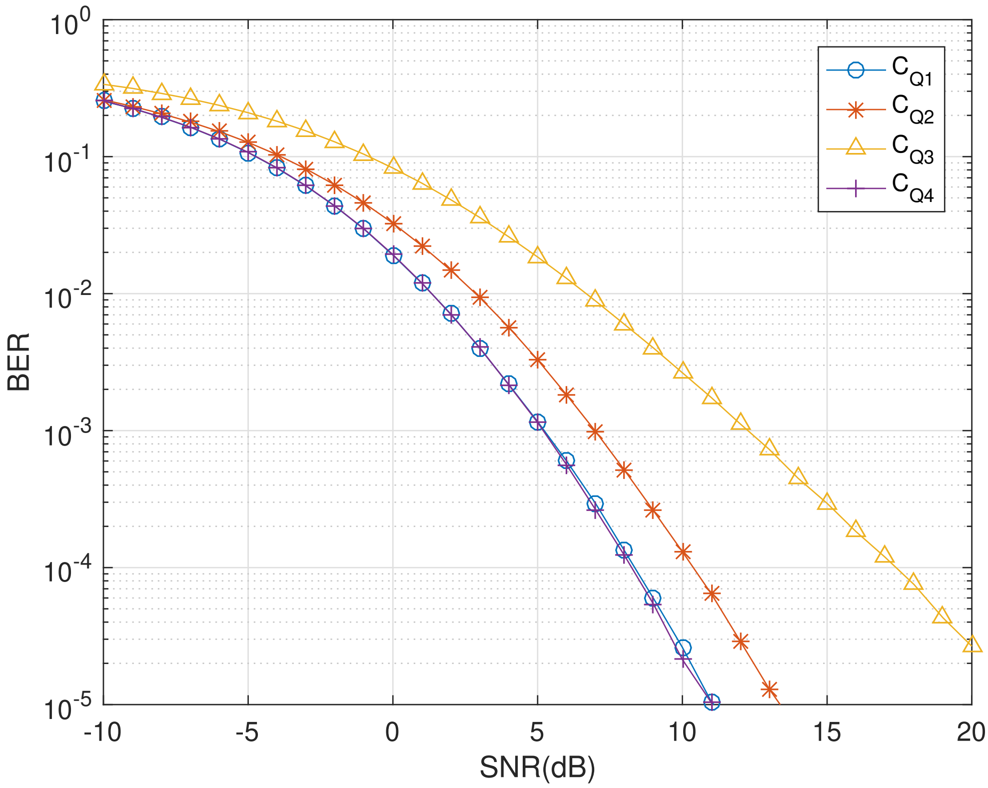

Figure 5.4: BER vs. SNR performance of QODs $\mathbf{C}_{Q_1}, \mathbf{C}_{Q_2}, \mathbf{C}_{Q_3}$ and $\mathbf{C}_{Q_4}$.

$$\mathbf{C}_{\mathbf{Q}_1} = \begin{bmatrix} z_1 & z_3 & z_2 & z_4 \\ z_2^* & z_4^* & -z_1^* & -z_3^* \\ z_3 & z_1 & z_4 & z_2 \\ z_4^* & z_2^* & -z_3^* & -z_1^* \end{bmatrix}, \tag{5.18}$$

$$\mathbf{C}_{\mathbf{Q}_2} = \begin{bmatrix} x_3 + x_4 i & x_0 + x_2 i & x_3 + x_4 i & x_1 + x_2 i \\ x_3 + x_4 i & -x_1 + x_2 i & -x_3 - x_4 i & x_0 - x_2 i \end{bmatrix}, \tag{5.19}$$

$$\mathbf{C}_{\mathbf{Q}_3} = \begin{bmatrix} z_1 & z_2 & z_3 & z_4 \\ z_2^* & -z_1^* & -z_4^* & z_3^* \end{bmatrix}, \tag{5.20}$$

$$\mathbf{C}_{\mathbf{Q}_4} = \begin{bmatrix} z_1 & z_2 & z_1^* & -z_2^* \\ iz_1 & iz_2 & iz_2^* & -iz_1^* \end{bmatrix}, \tag{5.21}$$

Figure 5.4 shows the simulation results of these codes which can be compared with $\mathbf{CYT}_1$ or $\mathbf{CYT}_2$. Subsequently, a detailed comparative analysis has been carried out, which proves that the designs developed in the quaternion domain have performance edge at many fronts such as computational complexity, improved throughput, exploitation of polarization diversity, decoding delays and linear decoupled decoding, etc.

5.5.2 Computational Complexity

The proposed design eliminates the dependence of the decoder on ζ, i.e., the number of unique transmitted symbols. This has been the case with coupled decoding which arises in the case of $\mathbf{CYT}_1$. In terms of the floating point calculations, the computational complexity for N number of DP transmit antennas reduces to $O(4(N)(T)(2))$ from $O(4^{\zeta}(N)(T)(2))$ of the coupled decoder, where T represents the time slots required to transmit a single block of code. For the code in Equation (5.8), the proposed decoupled decoder executes 256 number of floating point operations (FLOPs), resulting in an increase of 98.4% computational efficiency. This efficiency is greatly enhanced as the antenna dimensions increase. Thus, the decoupled decoding of the quasi-orthogonal STBCs corresponding to $\mathbf{Q}_1, \mathbf{Q}_2, \mathbf{Q}_3$ and $\mathbf{Q}_4$ with the proposed decoder using the quaternionic channel can be easily achieved while the coupled ML decoder would have failed completely.

Here it is very important to emphasize the impact of increased number of antenna elements on the computational complexity of this proposal. This is also critical as the future generation of wireless communication aims to integrate hundreds of antennas and this demands the evaluation of the designs to practical considerations. It is already presented that the computational complexity is entirely dependent on the number of transmit antennas and the number of timeslots used to transmit a single code block. Thus, the more the number of antenna elements are added to the system, the computational complexity seem to rise. For example, the computational complexity of the proposed design for T timeslots will result drastically demanding extra processing time and incurring more delays.

In Table 5.1, the main points has been summarized which play a dominant role in the quaternion domain when employed for DP antennas. A comparison has been presented between the quaternion designs and the complex designs. These designs are evaluated in terms of different characteristics of the wireless communication system. A valuable feature that is visible is the increased code rates with the use of quaternionic channel model for the orthogonal codes. This is traded with the coding and decoding delays in the quaternionic channel-based system model incurred at both the transmitter and receiver ends to generate quaternion codes and then decode them to the complex symbols, respectively. Quaternion channel inherently embeds orthogonality due to the use of pure quaternions and DP antennas. This provides linear and decoupled decoding at the receiver end. This has been seen possible due to the quaternion channel exploiting the polarization diversity independently in addition to the space and time diversities. This remained hidden for long unless dual-polarized antennas were used for wireless communication systems using space time coding [20, 109, 110].

5.5.3 Number of Receive Antennas

The physical implementation of the design in [128] is limited with the use of even number of DP antennas at the receiver. For massive MIMO systems, this is space and cost inefficient with restrictions on the freedom of diversity at the receiver end. The proposed model works for any number of receive

Table 5.1: Significant features of the quaternion domain and their comparison with their counter codes in complex domain.

	Complex Designs		Quaternion Designs			
	CYT_1	**CYT_2**	**Q_1**	**Q_2**	**Q_3**	**Q_4**
Type	Quasi	Orthogonal	Orthogonal	Orthogonal	Orthogonal	Orthogonal
Code Rates	1	3/4	1	2	1	3/4
Coding/Decoding Delay	✓	✓	×	×	×	×
Decoupled Decoder	×	×	✓	✓	✓	✓
Space & Time Diversities	✓	✓	✓	✓	✓	✓
Polarization Diversity	×	×	✓	✓	✓	✓

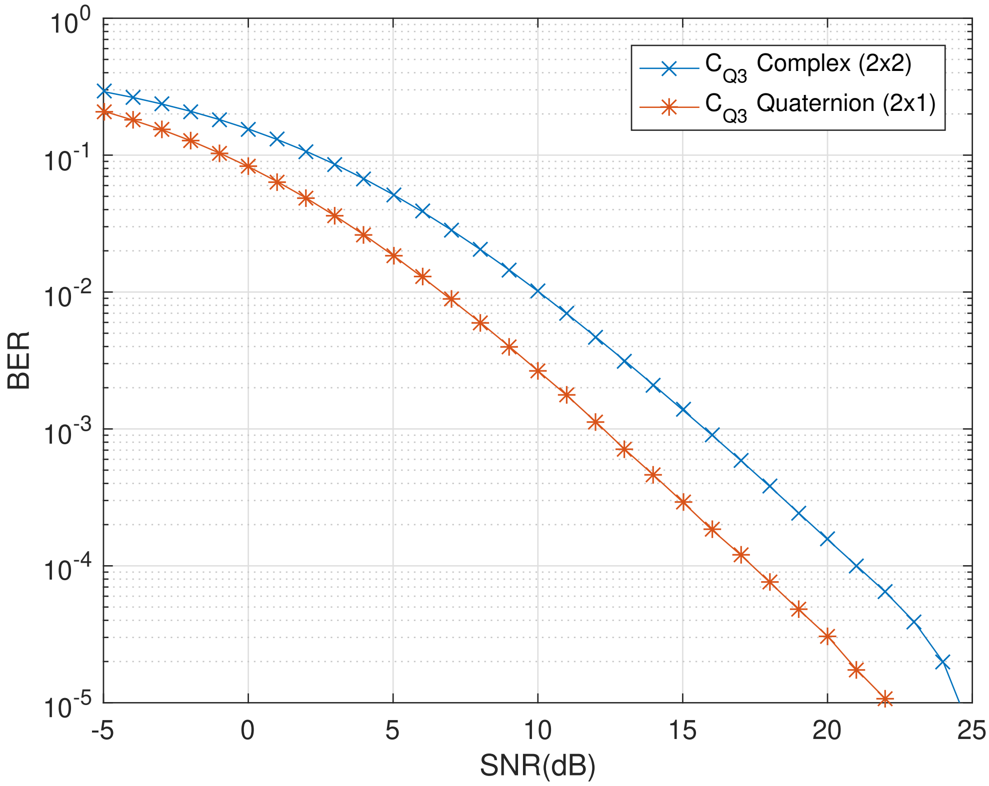

Figure 5.5: BER vs. SNR performance comparison of $\mathbf{C}_{Q_3}$ for decoupled decoder in [128] and the quaternionic channel based decoder.

DP antennas, N_R, i.e., $(N_T \times 1), \ldots, (N_T \times N_R)$; $N_R \geq 1$.

5.5.4 Diversity Gain

The diversity gain of $\mathbf{C}_{Q_3}$ is 3dB at a BER of 10^{-5} in comparison to the diversity gain achieved in [128] using the complex representation of the quater-

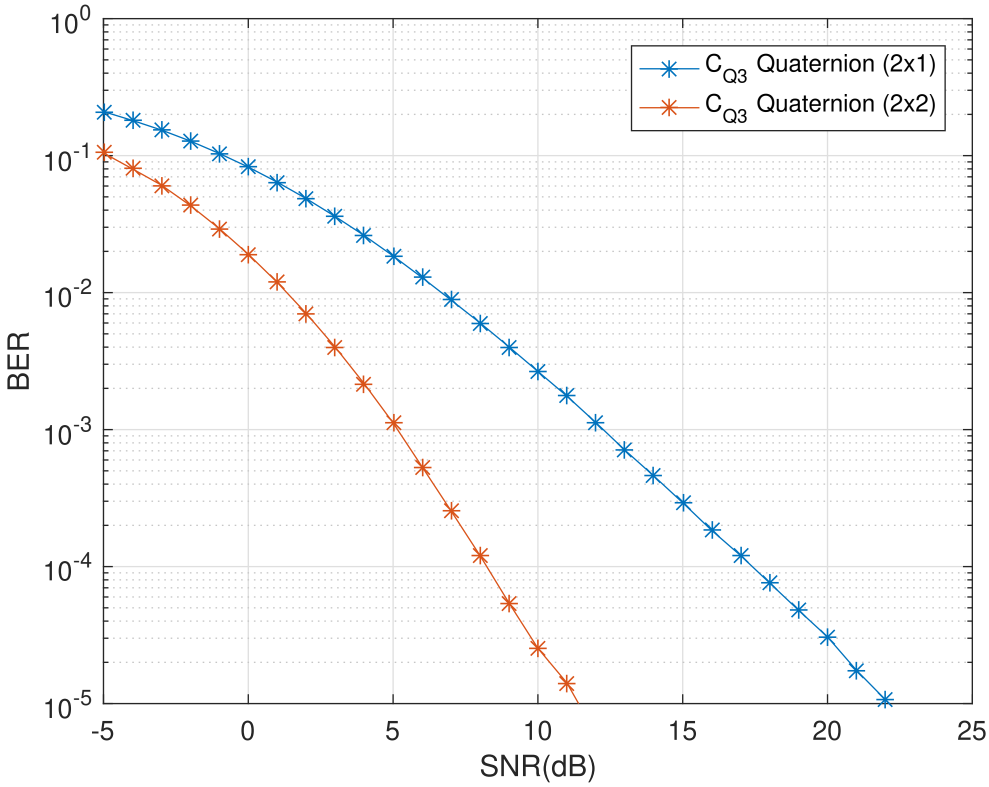

Figure 5.6: BER vs. SNR performance of $\mathbf{C}_{Q_3}$ for one and two receive DP antennas

nionic channel with two transmit and two receive DP antennas, as shown in Figure 5.5. This gain approaches 13dB at the same bit error rate if the antenna dimensions are matched at both the transmitter and receiver ends, as evident in Figure 5.6. Thus, the best exploitation of polarization diversity is executed using the cross polar as well as the polar components without any dependence on the number of receive antennas.

5.5.5 Cross-Polar Scattering

Scattering and reflections result in polarization variations where cross polar scattering is natural. The orthogonal quaternion codes are decomposed into quasi-orthogonal STBCS in [128] to provide a decoupled decoding solution. Yet, this model has constraints to have zero cross polar scattering environment, a limiting scenario in real communication systems. Such an exercise appears redundant as decoupled decoding solution for DP antennas based on a generalized quaternionic channel model has already been detailed in [57], which considers both the polar as well as non-cross polar scattering and provides linear decoupled decoding solution for quasi-orthogonal STBCs.

This work presents a generalized decoupled decoding solution for the quasi-orthogonal STBCs using the quaternionic channel model irrespective of any constraints regarding the cross polar scattering, the number of received DP antennas and coding/decoding delays. This design provides a decoupled decoding solution for any number of transmit and receive DP antennas.

5.6 Conclusion

This chapter presented an evaluation of the conditions employed on the construction of QODs that achieve better diversity gains by exploiting space, time and polarization diversities using quaternion algebra. The main contributions of this work are:

- In the presence of fully quaternion-valued channel model, design of linear and decoupled decoder for QODs based on Adam-Lax-Phillips approach.
- Iterative construction techniques for QODs.
- Extension to more robust MIMO systems considering freedom in transmit as well as receive diversities.

Quaternionic channel provides decoupled decoding solutions using quaternion algebra and presents iterative construction techniques for QODs. A remarkable contribution of this work [58] is linear decoupled solution of codes including square as well as non-square designs. This was not present before. Additionally, the solution presented here is generalized for both polar as well as cross polar scattering environments and is independent of the number of receive DP antennas.

Chapter 6

Quaternionic Channel-based Modulation For DP Antennas

STBCs have been studied to exploit the spatial and temporal diversities in wireless systems. OSTPBCs designed using the quaternion algebra promise gains in terms of higher data rates, diversity and spectral efficiency. In this context, quaternion modulation has been proposed using the DP antennas to generate efficient selection of the polarization and optimal decoding at the receiver end. In this thesis, the quaternion modulation technique has been evaluated considering the quaternionic channel using the DP antennas. The results show promising diversity gains with benefits in terms of spectral efficiency and data rates. An extension of this scheme for higher number of symbols and higher DP antenna dimensions has also been presented. The proposal includes linear decoupled decoding of the QODs at the receiver end where the complexity stays independent of the number of transmitted symbols. The design of the quaternion modulation using the quaternionic channel fully exploits the polarization diversity in addition to unfolding its applicability for future massive MIMO wireless systems.

6.1 Introduction

Design of orthogonal codes in higher dimensions has been investigated and proven to have beneficial results on the diversity as well as spectral efficiency and code rates [1, 16, 20]. The multi-dimensional orthogonal codes have been successful in addressing the limitations in capacity by exploiting different forms of diversity, i.e., space, time and polarization using the DP antennas [109, 110, 116]. QODs have been successful in targeting higher data rates and reduced receiver complexity [120]. In [57], pure quaternion codes have been designed where a unique construction for quaternionic channel has been exploited. This supports the importance of utilizing higher dimensions like quaternions and integrating them with the theory of DP antennas to achieve higher data rate codes with greater diversity gain, demanded for the massive MIMO systems.

Modulation in the quaternion domain has been used to achieve better performance, yet the design has hardware complexities and remains inefficient when combined with the DP antennas, [134]. In [118], a quaternion modulation scheme, QMod, uses the notion of quaternions and DP antennas to gain the benefits of spectral efficiency, diversity and data rate. The authors claimed that the diversity gain is maximized with the presented channel model for the DP antennas using quaternions due to the simultaneous use of both the polarizations at a single DP antenna. The four states of the polarizations of the DP antenna, i.e., VV, VH, HV and HH, have been used to generate two extra bits of data to be transmitted, where $V\big(0\big)$ represents the vertical polarization and $H\big(1\big)$ represents the horizontal polarization of the DP antenna. However, this diversity gain obtained using the proposed modulation scheme is limited as it does not fully exploit the benefits of combining the polarization diversity with time and space diversities using the quaternions due to its dependence on the cross polar scattering effects. In [57], a quaternionic channel model is presented using the DP antennas which exploits time, space and polarization diversities with the QODs. This model represents the quaternion structure fully embedded into the codes as well as the wireless channel, where (1×1) QOD performs similar to the (2×2) Alamouti code with zero cross-polar components. This is possible due to the coupling with 'j'(where j is the orthogonality in quaternion domain between the two complex symbols), which inherently brings the effect of cross-polar scattering. Furthermore, in [57], it has been explored that in higher dimensions, e.g., (2×1) or higher, the diversity gain with the quaternionic channel model is significantly high in comparison to the unipolarized schemes of the same order with efficiency in terms of reduced number of timeslots required to transmit the same number of complex symbols. Using this quaternionic channel model and the QODs with DP antennas, the modulation scheme presented in [118] provides promising results.

This chapter presents the design of a quaternion modulation using the quaternionic channel model developed in [57]. In [118], the authors have exploited the use of cross-polar scattering to achieve maximum diversity gains when the isolation across the two polarizations of the DP antenna is maximum. Thus, for the design of OSTPBCs, the presented quaternion modulation design using the quaternionic channel is independent of the cross-polar scattering effects which fully exploits polarization using the DP antennas in addition to space and time diversities to achieve maximum performance. A further extension of this design to higher dimension has been presented which makes it a valuable application for next generation massive multiple-input multiple-output (MIMO) networks.

6.2 System Model

Consider a MIMO transmission system with N DP antennas at the transmitter. The receiver is equipped with a single DP antenna. Both the transmitter and receiver perform data transmission using the QODs. For a $\left(1 \times 1\right)$ DP antenna configuration, the discrete-time input-output relation over a block of data bits for Rician fading channel is given by the received signal $r = qh + n$, where $n = n_1 + n_2 j$ is the AWGN, which is two dimensional i.i.d. complex Gaussian random variables with zero mean and identical variance per dimension and h is the channel given as

$$h = \sqrt{\frac{1}{K+1}}(h_1 + h_2 j) + \sqrt{\frac{K}{K+1}}(1 + 1j), \tag{6.1}$$

which is a combination of pure Rayleigh $\left(K = 0\right)$ and AWGN $\left(K = \infty\right)$ channels.

For $\left(N \times 1\right)$ antenna dimension, the received signal $\mathbf{R}$ can be written as

$$\mathbf{R} = \mathbf{QH} + \mathbf{N} = \mathbf{Q} \begin{bmatrix} h^{(1)} \\ \vdots \\ h^{(N)} \end{bmatrix} + \begin{bmatrix} n^{(1)} \\ \vdots \\ n^{(N)} \end{bmatrix}, \tag{6.2}$$

where $h^{(m)}, n^{(m)} \in \mathbb{Q},\ m = \left[1, 2, \ldots N\right]$. In Equation (6.2), $\mathbf{H}$ is the quaternion channel matrix, with coefficients representing gains between each pair of transmit and receive DP antenna for a Rician fading channel, with infinite cross-polar isolation $\left(\chi^{-1}\right)$ and cross-polar discrimination $\left(\alpha^{-1}\right)$ [116]. $\mathbf{N}$ is the quaternion noise matrix and $\mathbf{Q}$ is a quaternion code matrix containing transmitted symbols.

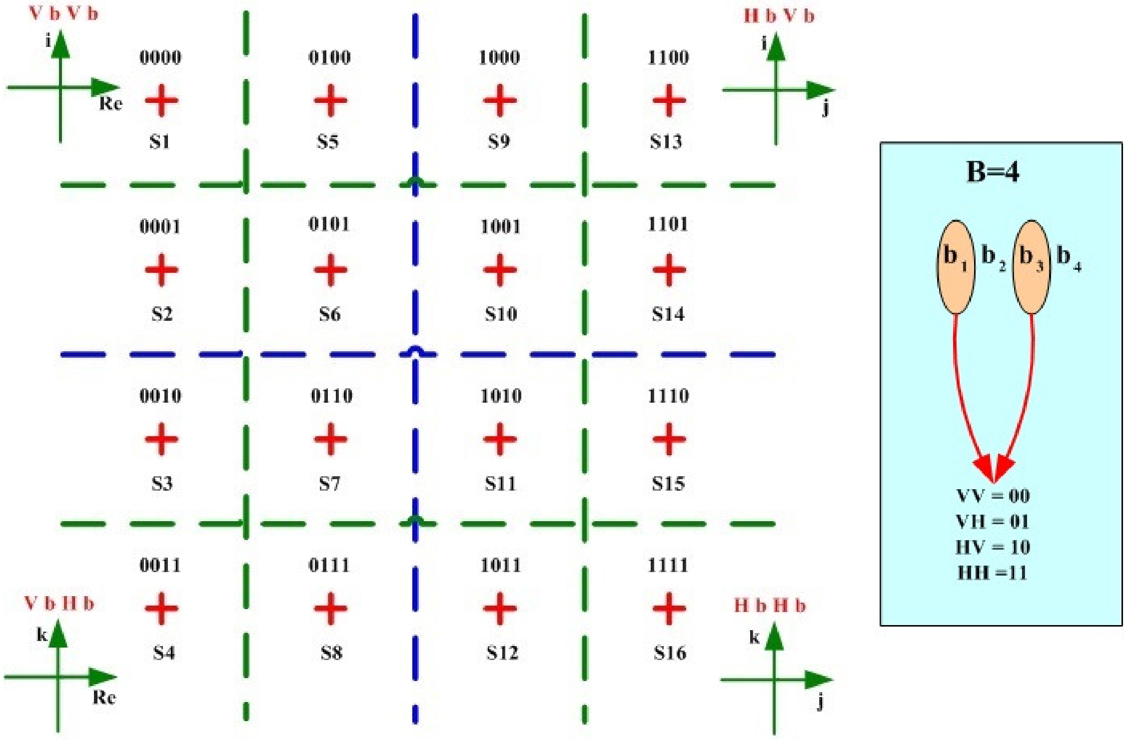

Figure 6.1: Quaternion modulation bit patterns and constellation, for $B = 4$

6.3 Quaternion Modulation using Quaternionic Channel

In [118], the quaternion modulation scheme, QMod, uses the notion of quaternions and DP antennas to gain the benefits of diversity and data rate. Among the two parts of the transmit data block, one is the information block which is transmitted using the standard modulation schemes while the other represents one of the two polarizations, H or V, that is to be used for transmitting the data symbols.

The total number of data bits per symbol are represented as B, where B is even, which is divided into four parts, as in [118]. The first and the $(\frac{B}{2}+1)^{th}$ bit are used to select the polarization state V/H of the DP antenna while the rest of the bits are considered as information bits. If the first and the $(\frac{B}{2}+1)^{th}$ bits turn out to be same $\big(e.g.\ 00$ or $11\big)$, the modulation scheme similar to the one presented in [134] is followed, where one of the two polarizations is selected to transmit all of the information bits. For the first and $(\frac{B}{2}+1)^{th}$ bits being different, the first half of the information bits are modulated using the amplitude-shift keying (ASK) and transmitted through the real part of the complex symbol sent through one of the polarizations of the DP antenna while the second half of the information bits is transmitted as an ASK symbol through the imaginary part of the complex symbol sent through the other polarization of the DP antenna.

As an example, for $B = 4$, four bits are available to be transmitted per quaternion symbol, i.e., $\big[b_1b_2b_3b_4\big]$, as shown in Fig. 1. Bits b_1 and b_3 selects the polarization state whereas the remaining bits, i.e. b_2 and b_4, are the information bits. For one DP antenna at the transmitter and receiver, i.e., (1x1) DP antenna configuration, the transmit data is represented as a quaternion, q_1, which comprises of two complex symbols $z_1 = b_1 + b_2j$ and $z_2 = b_3 + b_4j$ and is given as

$$q_1 = z_1 + z_2j. \tag{6.3}$$

The first bit, b_1, determines that the BPSK modulated symbol using the second information bit, b_1, is transmitted through the real or j^{th} dimension of the quaternion while the third bit, b_3, determines that the BPSK modulated symbol using the fourth information bit, b_4, is transmitted through i^{th} or k^{th} dimension of the quaternion. Thus, in comparison to [134], two additional bits of data are obtained which increases the modulated transmission rate.

The proposed model is a generalized solution for quaternion modulation for any number of transmit DP antennas, where (2×1) DP antenna configuration is explained. For (2×1) DP antenna configuration, the received signal $\mathbf{R}$ can be written as,

$$\mathbf{R} = \mathbf{Q} \begin{bmatrix} h^{(1)} \\ h^{(2)} \end{bmatrix} + \begin{bmatrix} n^{(1)} \\ n^{(2)} \end{bmatrix}. \tag{6.4}$$

Two QODs, $\mathbf{Q}_2$ and $\mathbf{Q}_3$, are considered from [57] and their performance is evaluated for the quaternion modulation using the quaternionic channel with the results presented in [118]. These codes have been already shown in [57] to promise high diversity gains as well as quaternion code rates for (2×1) DP antenna configuration specifically $r_q^{(Q_2)} = 1$ and $r_q^{(Q_3)} = 0.5$, respectively.

$$\mathbf{Q}_2 = \begin{bmatrix} z_1 + z_2 j & z_4 + z_3 j \\ z_2^* - z_1^* j & -z_3^* + z_4^* j \end{bmatrix}. \tag{6.5}$$

$$\mathbf{Q}_3 = \begin{bmatrix} z_1 + z_2 j & j(z_1 + z_2 j) \\ i(z_1 + z_2 j) & -k(z_1 + z_2 j) \end{bmatrix}. \tag{6.6}$$

6.3.1 Demodulation and Decoupled Decoding

Considering all the previous studies including [134] and [118], the decoding has stayed as an issue. [118] has utilized the Maximum-Likelihood (ML) decoding with a consequence to design a method to estimate the states and transmission information simultaneously. [57] has presented linear decoupled decoding solution for the quaternionic channel model. This provides a decoupled decoding solution of the proposed quaternion demodulation. The ML-decoding rule assumes a linear decoupled form as Equation (5.13).The same decoding method can be applied to the proposed modulation scheme using the quaternionic channel to maximize the diversity gains. As an illustration of the above result, the QODs given in Equation (6.6) has been chosen to demonstrate that the above ML-decoding rule is both linear and decoupled.

Corollary 5.1. *The ML-decoding rule (5.13) for QOD given in Equation* (6.5)*, reduces to*

$$
\begin{aligned}
&\min_{z_1} -2\Re(r_1^q z_1 h_1)\,, \\
&\min_{z_2} -2\Re(r_2^q z_2^* h_1)\,, \\
&\min_{z_3} -2\Re(-r_2^q z_3^* h_2)\,, \\
&\min_{z_4} -2\Re(r_1^q z_4 h_2)\,,
\end{aligned}
\tag{6.7}
$$

where $z_l \in \mathbb{C}$ *and* $r_l, h_l \in \mathbb{Q},\ l = 1, 2.$

6.3.2 Computational Complexity

The proposed quaternion modulation using the quaternionic channel model promises less computational complexity and independence from the number of transmitted symbols, ζ. Thus, For BPSK modulation, the complexity remains $\Big(2\big(2N\big)\big(t\big)2\Big)$ complex floating point calculations, instead of $\Big(2^{\zeta}\big(2N\big)\big(t\big)2\Big)$ for the coupled ML decoder. Although, [118] has presented a modulation scheme with real computations only, yet this has been done at the expense of the diversity gains, spectral efficiency and the overhead of designing the joint detection for every different antenna arrangement and modulation scheme. Also, the authors have not included the influence of increased number transmitted symbols, mapping directly to the requirement of greater number of DP antennas and thus need for higher number of timeslots for transmission, on the complexity of the decoder. The proposed scheme is generalized and can easily be mapped to any number of antenna configurations. This decoupled decoding solution reduces the receiver's computational complexity by refraining the use of matrices and thus, becomes a valuable candidate for large MIMO systems.

6.4 Simulation and Results

To analyze the transmit diversity gains for the quaternion modulation using quaternionic channel against the QMod scheme, presented in [118], different QODs in Equations (6.3), (6.5) and (6.6), have been considered for 1 and 2

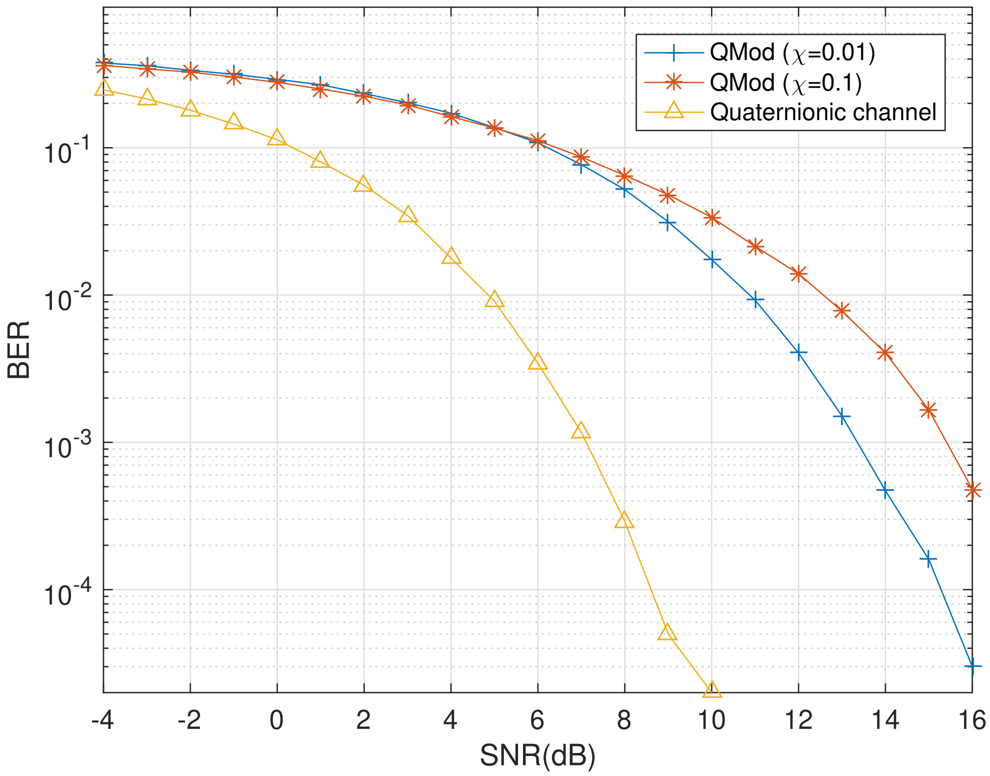

Figure 6.2: Performance of the quaternion modulation using quaternionic channel for pure AWGN channel.

DP transmit antennas with code rates, $r_q^{\left(q_1\right)} = 1$, $r_q^{\left(Q_2\right)} = 1$ and $r_q^{\left(Q_3\right)} = 0.5$. Monte Carlo simulations are performed for the transmission of 10^5 symbol blocks. Each symbol comprises of four bits where BPSK has been considered as the data modulation scheme. Results have been presented for both pure Rayleigh and AWGN channels where the channel coefficients are assumed to be perfectly known at the receiver, whereas, the white Gaussian noise is added to the horizontal and vertical polarizations uniformly.

Figure 6.2 highlights the performance of the proposed quaternion modulation using the quaternionic channel model for pure AWGN environment. It is clear that QMod shows better diversity gains as the cross-polar isolation $\left(\chi^{-1}\right)$ increases. Comparatively, the performance of the proposed quaternion modulation model outperforms in terms of the diversity gains showing the independence of the channel model over the cross-polar scattering effects due to the embedded coupling in the quaternion space, $\mathbb{Q}$.

Figure 6.3 presents the simulation results for a single transmit and single receive DP antenna configuration. The system model presented in (1) in [118] has been considered for comparison where the cross polar discrimination factor, α, has been considered to incorporate the cross-polar scattering effects. The increase in this factor results in greater isolation between the horizontal and vertical polarizations of the DP antenna which is directly related to the increase in diversity gain. The comparison has been simulated considering the proposed system design presented in Equation (6.3) for $\left(1 \times 1\right)$ DP antenna configuration. It is clear that the proposed model promises higher diversity gains with the cross-polar scattering effects already incorporated into the quaternion code and quaternionic channel design. The quaternion modulation presented in [118] is dependent on the cross-polar scattering, which is an unavoidable phenomenon when the DP antennas are used with complex rep-

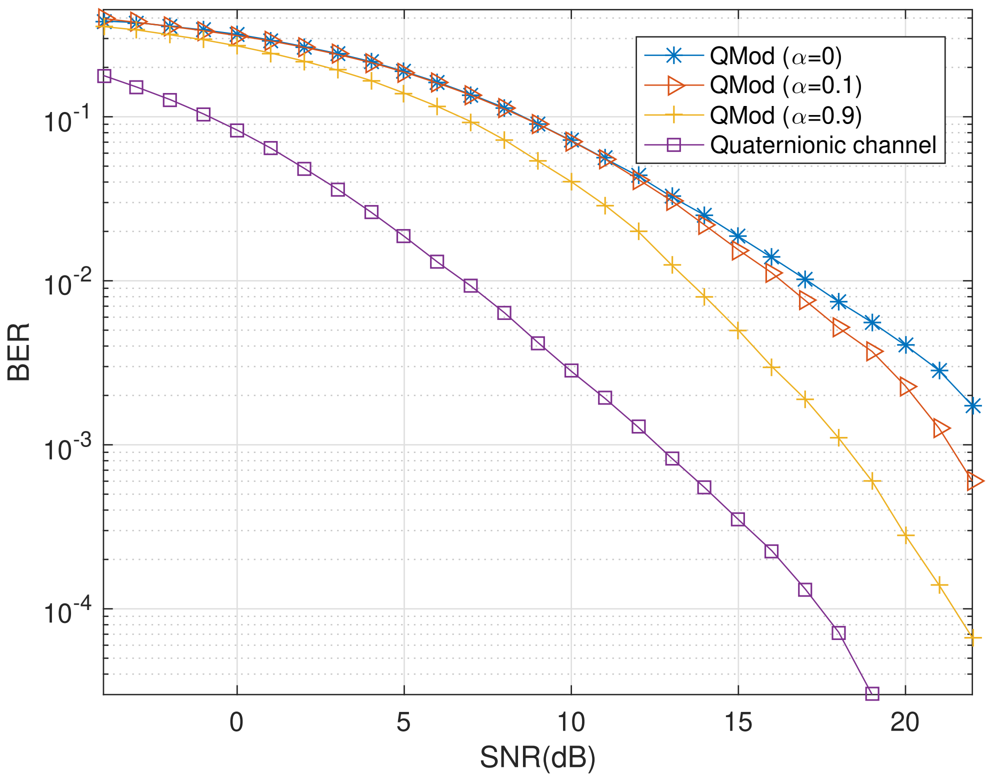

Figure 6.3: Performance of the quaternion modulation using quaternionic channel for pure Rayleigh channel for (1×1) DP antenna configuration.

resentation of the codes and channel. Whereas, with the quaternionic channel model, the quaternion modulation promises maximum diversity gains.

Figure 6.4 provides the diversity gains achieved when the QODs $\mathbf{Q}_2$ and $\mathbf{Q}_3$ are transmitted for the quaternion modulation in the two transmit and single receive DP antenna configuration. It is evident that the codes designed using the scheme in [57] uses lesser number of timeslots to transmit a code block when compared to a complex STBC with similar antenna configuration, which directly maps to better utilization of the spectrum. [118] did not discuss how the codes will be extended for higher dimensions considering the issues of orthogonality. The proposed quaternion modulation using the quaternionic

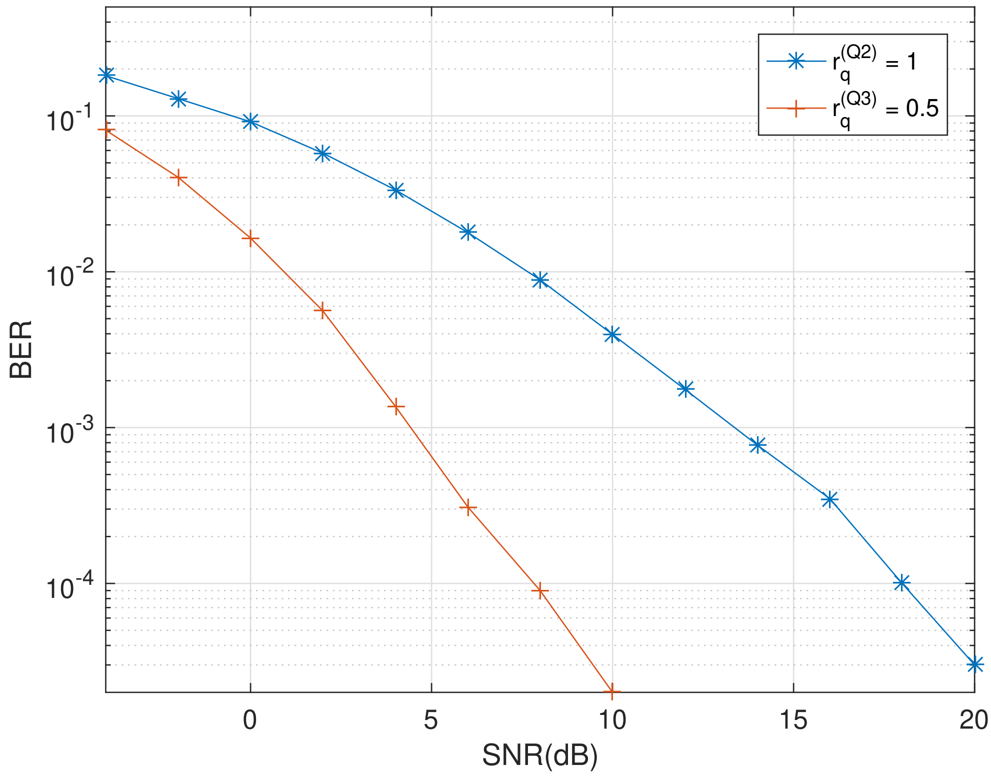

Figure 6.4: Performance of the quaternion modulation using quaternionic channel for pure Rayleigh channel for (2×1) DP antennas configuration.

channel model from [57] provides a solution for the same modulation in higher antenna dimensions with gains in terms of maximized diversity which makes this model a promising candidate for the next generation massive MIMO wireless communication networks.

In literature, the quaternion modulation has been done through optimal manipulation of the two polarizations in the DP antenna to encode the data for efficient transmission and detection at the receiving end. This design demanded frequent switching of the polarizations on a single DP antenna that adds considerbale hardware complexity in terms of electrical specifications. It is deemed far more optimal to transmit a null symbol than to switch of the relevant polarization. It is expensive and might also result in delays due to extra time required to switch off one polarization and turn the other one on and vice versa. The presented modulation scheme, [59], prevents such problems by fully exploiting the available polariztaion states and utilizing both polarizations of the DP antenna. The increase in the diversity gains is a similar reflection of exploiting polarization diversity independent of the space and time diversity. This was never the case when the quaternions constructed using the existing QODs were used with DP antennas. The quaternion modulation using the quaternionic channel model provides the maximum diversity gains incorporating the cross-polar scattering effects embedded into the code and channel designs which are based on pure quaternion space, $\mathbb{Q}$. This is a generalized model which can be extended to any DP antenna dimensions. The number of DP antennas at the receiver can be increased with the same construction mechanisms, providing an increase in the diversity gains at the expense of increased space and cost requirement at the receiver end. It is beneficial in terms of diversity gains, spectral utilization, data rates and its application to future communication systems.

6.5 Conclusion

This chapter presented the quaternion modulation in [118] using the quaternionic channel model [57]. The coupling of the channel coefficients and code variables embeds the cross-polar scattering effects. The quaternion modulation achieves better diversity gains by exploiting space, time and polarization diversities using the quaternionic channel model and independence from the number of transmit and receive antennas. The literature consists of quaternion modulation developed using the notion of switching the polarizations of the DP antennas. However, they fail to fully exploit the benefits of polarization diversity and leads to adding hardware complexities in terms of the overhead of switching the polarizations of the DP antennas. The proposed model for quaternion modulation, [59], using the quaternionic channel can be mapped to future communication systems due to its advantages in terms of diversity gains, data rates, reduced receiver complexity and spectral efficiency.

Chapter 7

Quaternion Codes in MIMO System of Dual-Polarized Antennas

7.1 Introduction

Surge of high speed communication services has accelerated the demand for efficient communication techniques that have the potential to make reliable data transmissions without compromising on data rates. In this regard, STBCs, based on orthogonal designs, are considered one of the key techniques that have moved the capacity of wireless communication close to theoretical limits. STBCs have been used extensively such as in third generation (3G) standard and wireless local area networks (LANs) based on IEEE 802.11n. Initially, they were proposed by Tarokh *et. al* [102] as a generalization of the famous Alamouti code which is a COD [1]. The most attractive feature of these orthogonal designs is the provision of full diversity along with low complexity ML decoder. However, they achieve this attribute at the expense of code rate, i.e., the ratio of the number of independent complex transmitted symbols and the number of total time slots taken to transmit a coding matrix. On the other hand, a COD with full rate and maximum diversity exists only for two single-polarized transmit antennas and maximum code rate approaches half with as the number of transmit antennas increase [16]. To meet higher data rate demands, other designs such as complex quasi-orthogonal STBCs have also been explored that provide comparatively higher code rates but compromise on optimal decoders due to nonlinear and coupled decoding issues [17]. To further enhance the capacity of communication systems, other combinations of diversity providing techniques are being investigated.

In [19], orthogonally polarized transmissions through both ends of a DP antenna were modeled through quaternions and later [20] laid the foundation of OSTPBCs that utilize polarization diversity together with space and time diversity. Polarization diversity can provide nearly similar performance to spatial diversity without any measurable increase in antenna dimensions [21]. This is achieved with the use of DP antennas, which have two antennas of orthogonal polarizations co-existing on a single antenna platform and there has been a growing interest recently [135, 136].

Based on the combination of polarization diversity with space and time diversities, various QOD construction techniques have been proposed by Seberry *et. al* [20]. The primary motivation of these designs has been their ability to provide higher code rates along with a low complexity quaternion norm-based ML decoder. To illustrate the benefit of these designs, [20] presented an example of 2×2 order QOD and derived linear equation based decoding solution for this configuration. They argued that quaternion decoding statistics can provide decoupled decoding for any QOD. Their subsequent studies [109, 110] used the same QOD and emphasized the similar postulate that quaternion ML norm can provide optimal decoupled decoding for any QOD construction. However, the authors corrected their decoding rule in [122] and highlighted that the proposed decoding decoding rule does not yield optimal decoding for all QODs, and therefore, the design of semi-optimal or optimal low complexity decoders remained an open research problem [125]. In this regard, [120] explored the designs for which quaternion norm-based ML decoder resulted in optimal decoding solutions. However, it is important to note that their proposed ML decoder works for a special class of STBCs.

In this chapter, three famous generalized QOD construction techniques are investigated [120] and two main short falls are identified which restricts their use for large MIMO systems. Firstly, this iterative approach works only in the case when the number of transmit antennas are in powers of 2, which clearly restricts their use to other antenna configurations. Secondly, the code rate decreases very sharply for higher order designs based on these iterative techniques. Therefore, it was deemed necessary to develop codes that work for any number of antenna systems besides having the main advantage of attaining decoupled decoders in the presence of quaternionic channel as was the case with iteratively generated designs [120]. This has been done following the line of approach indicated in [16] which gives us a class of QODs that are non-square and the code rate is bounded below by $1/2$. The idea has been to exploit the impact of a DP transmission channel at the receiver side in such a way that ML quaternion norm criterion simplifies to a decoupled decoding solution which reduces the decoder complexity significantly.

After obtaining a generalized ML decoder, quasi-QODs are explored. The proposed quasi-QOD provides a code rate of two for four transmit DP antennas. However, the quasi nature of these codes leads to a slight compromise on decoding complexity. The solutions obtained with this compromised complexity-based decoder are still better than the coupled traditional ML decoder based solutions.

The main contributions made in this section are:

- Proposal of a new class of QODs based on Liang mechanism [16] with stable code rate as the number of transmit antennas increases.
- The class is shown as best suitable in describing point-to-point communication among DP antennas.
- The proposed decoder is shown to provide linear decoding solution for all STBCs obtained from QODs.
- A brief performance analysis is carried out for all obtained QODs.

7.2 Theory behind the QODs

As mentioned before that a quaternion is a combination of two complex numbers $q = z_1 + z_2 j$, therefore, it is natural to think of QODs satisfying $\mathbf{Q} = \mathbf{A} + \mathbf{B}j$, where $\mathbf{A}$ and $\mathbf{B}$ are two complex matrices. It turned out that any two arbitrary complex matrices do not necessarily give rise to a QOD which satisfies Equation (**??**). Essentially, the authors in [20] found the key requirements on $\mathbf{A}$ and $\mathbf{B}$ to ensure Equation (**??**), for the resulting QOD. Interestingly, the *amicable* and *symmetry* conditions were found to play main role for which an extensive theory was already in place and they used it to generate class of QODs. Later it was found that all proposed quaternion-based designs employed symmetric-paired complex matrices. Since the symmetry property is crucial in our study therefore we state it for a brief and self-contained exposition.

Definition 7.1 (QOD). *Two CODs $\boldsymbol{A}$ and $\boldsymbol{B}$ based on complex variables $\{z_1, z_2, \ldots, z_u\}$ form a symmetric-paired design $(\boldsymbol{A} + \boldsymbol{B}j)$ provided $\boldsymbol{A}^H\boldsymbol{B}$ or $\boldsymbol{B}^H\boldsymbol{A}$ is symmetric.*

A relatively simple way to find such symmetric-paired designs arise from the observation that swapping of certain columns of a COD generates an equivalent COD. The resulting COD along with the original COD form a symmetric-paired design. This technique was used in [20], for the search of viable QODs. However, it is important to note that as the dimension of COD matrix gets larger, not every permutation of columns of a COD yields a valid QOD. Therefore, only one permutation per column is allowed to generate valid QODs under this construction [121]. Following subsections describe other possible ways to generate QODs. For square designs, three recursive construction methods to find CODs were presented in [16], namely Adams-Lax-Phillips, Józefiak and Wolfe constructions. It is easy to realize that all three constructions recursively generate the same class of square QODs.

According to these constructions, a recursive COD $\mathbf{A}$ is designed for $l+1$ symbols embedded in a square matrix of order 2^l such that

$$\mathbf{A} = \begin{bmatrix} \mathbf{G}_{2^{l-1}}(z_1, z_2, \ldots, z_l) & z_{l+1}\mathbf{I}_{2^{l-1}} \\ -z_{l+1}^*\mathbf{I}_{2^{l-1}} & \mathbf{G}_{2^{l-1}}^H(z_1, z_2, \ldots, z_l) \end{bmatrix}, \tag{7.1}$$

where $\mathbf{G}_{2^{l-1}}(z_1, z_2, \ldots, z_l)$ represents a COD of order $2^{l-1} \times 2^{l-1}$ defined on symbols $z_1, z_2, \ldots, z_l$ and $l = \{1, 2, 3, \ldots\}$. For example, for $l = 1$, $\mathbf{G}_1(z_1) = [z_1]$. Taking this as a seed element, higher order CODs such as $\mathbf{G}_2(z_1, z_2)$ and $\mathbf{G}_4(z_1, z_2, z_3, z_4)$ can be constructed [16], recursively. In the subsections, this way of COD generation is used to form different generalized QOD constructions. In Figure 7.1, we briefly explains the basic nomenclature which describes the main working in quaternion domain. We start

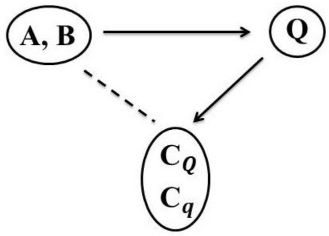

Figure 7.1: Quaternionic Nomenclature: Two symmetric-paired CODs **A** and **B** generate a QOD **Q**, which gives rise to different quasi-codes $\mathbf{C_Q}$ and $\mathbf{C_q}$ with linear and decoupled decoders.

with two CODs that form a symmetric-pair such that they give rise to a QOD. In earlier works, this was not directly used to describe communication among DP antennas. Rather, an STBC was constructed from a QOD and its even columns represent signals being sent through one polarization while entries in odd columns are signals transmitted through an orthogonal polarization plane. However, the simultaneous transmission through a DP antenna is being modeled by quaternions here. Although, an enriched theory of quasi-orthogonal designs can be discussed yet the main focus is to work on an approach which algorithmically identify among them, those designs that have decoders with two main characteristics, i.e., linear and decoupled. It is noticed that the departure from the complex to quaternion domain serve this purpose. The dotted line connecting CODs $\mathbf{A}, \mathbf{B}$ with quasi-STBCs $\mathbf{C}_Q$ or $\mathbf{C}_q$ indicates a vivid difference between their working which distinguishes them in terms of code rates, decoding delays at both ends, zero vs non-zero entries, linear and decoupled decoders.

The generalized construction techniques employ Equation (7.1), which provides the symmetric-paired square CODs that act as seeds to generate three classes of square and non-square QODs.

7.2.1 Symmetric-Paired Design 1: (Square QODs)

This construction technique constructs QODs $\mathbf{A}+\mathbf{B}j$ in which COD $\mathbf{B}$ is obtained from $\mathbf{A}$ through permutation of columns, where permutation operation on two columns m and n results in swapping the positions of these two columns with each other. For all CODs based on the permutation of CODs for a specific antenna dimension, the diversity order remains the same, therefore, without loss of generality we employ Equation (7.1) to prove following theorem for which the details are added for self-contained exposition. We will briefly present the proofs of remaining theorems.

Definition 7.2 (QOD). Theorem 7.1. *For a given COD* $\mathbf{A}$ *in Equation (7.1) and its permuted version* $\mathbf{B}$*, a complex amicable and symmertic-paired design can be constructed such that the following realization*

$$\mathbf{Q}_{2^l}(z_1, z_2, \ldots, z_{l+1}) = \mathbf{A}+\mathbf{B}j = \begin{bmatrix} \mathbf{G}_{2^{l-1}} + z_{l+1}\mathbf{I}_{2^{l-1}}j & z_{l+1}\mathbf{I}_{2^{l-1}} + \mathbf{G}_{2^{l-1}}j \\ -z_{l+1}^*\mathbf{I}_{2^{l-1}} + \mathbf{G}_{2^{l-1}}^H j & \mathbf{G}_{2^{l-1}}^H - z_{l+1}^*\mathbf{I}_{2^{l-1}}j \end{bmatrix}, \tag{7.2}$$

provides a QOD of dimension $2^l \times 2^l$ *with rate* $(l+1)/2^l$. **Proof.** We first prove that CODs $\mathbf{A}$ and $\mathbf{B}$ given in Equation (7.2) hold the symmetric property. It can be seen that

$$\mathbf{A}_{2^l}^H \mathbf{B}_{2^l} = \begin{bmatrix} \mathbf{O}_{2^{l-1}} & 2\lambda_1\mathbf{I}_{2^{l-1}} \\ 2\lambda_1\mathbf{I}_{2^{l-1}} & \mathbf{O}_{2^{l-1}} \end{bmatrix}, \tag{7.3}$$

where $\lambda_1\mathbf{I}_{2^{l-1}} = \mathbf{G}_{2^{l-1}}^H\mathbf{G}_{2^{l-1}} + z_{l+1}z_{l+1}^*\mathbf{I}_{2^{l-1}}$ and $\mathbf{O}_{2^{l-1}}$ is a $2^{l-1} \times 2^{l-1}$ order null matrix. Equation (7.3) shows that $\mathbf{A}^H\mathbf{B}$ is symmetric as its transpose remains invariant. In order to check Equation (**??**), quaternion conjugate of

Equation (7.2) can be written as

$$\mathbf{Q}^Q_{2^l}(z_1, z_2, .., z_{l+1}) = \begin{bmatrix} \mathbf{G}^H_{2^{l-1}} - jz^*_{l+1}\mathbf{I}_{2^{l-1}} & -z_{l+1}\mathbf{I}_{2^{l-1}} - j\mathbf{G}_{2^{l-1}} \\ z^*_{l+1}\mathbf{I}_{2^{l-1}} - j\mathbf{G}^H_{2^{l-1}} & \mathbf{G}_{2^{l-1}} + jz_{l+1}\mathbf{I}_{2^{l-1}} \end{bmatrix}. \tag{7.4}$$

To prove orthogonality, multiplication of Equation (7.4) with Equation (7.2) gives

$$\mathbf{Q}^Q_{2^l}\mathbf{Q}_{2^l} = \begin{bmatrix} 2\lambda_1\mathbf{I}_{2^{l-1}} & \mathbf{O}_{2^{l-1}} \\ \mathbf{O}_{2^{l-1}} & 2\lambda_1\mathbf{I}_{2^{l-1}} \end{bmatrix} = \lambda\mathbf{I}_{2^l}, \tag{7.5}$$

where $\lambda = 2\lambda_1$. Hence, Theorem 7.1 is proved.

The following example illustrates the construction in which an Alamouti code $\mathbf{G}_2 = \begin{bmatrix} z_1 & z_2 \\ -z_2^* & z_1 \end{bmatrix}$ is taken. Using Equation (7.1), a square matrix $\mathbf{A}$ of order 4 is obtained. Consequently, the result is a QOD.

Example 7.1. Using the permutation operation on $\mathbf{A}$, as described above and representing it with $\mathbf{B}$, we obtain following QOD $\mathbf{Q}_1 = \mathbf{A} + \mathbf{B}j$, to be used in the configuration of 4 DP antennas

$$\mathbf{Q}_1 = \begin{bmatrix} z_1 + z_3j & z_2 & z_3 + z_1j & z_2j \\ -z_2^* & z_1^* + z_3j & -z_2^*j & z_3 + z_1^*j \\ -z_3^* + z_1^*j & -z_2j & z_1^* - z_3^*j & -z_2 \\ z_2^*j & -z_3^* + z_1j & z_2^* & z_1 - z_3^*j \end{bmatrix}. \tag{7.6}$$

This QOD transmits 3 complex symbols z_1, z_2 and z_3 in four time slots and provides a 3/4 code rate.

7.2.2 Symmetric-Paired Design 2: (Non-Square QODs)

The designs which do not necessarily require the use of permutation operations on columns are important for they do not limit code rates. This can be done through a relatively simple way to recursively generate QODs using a single COD **A** and a square diagonal matrix containing an extra symbol. Therefore, these designs perform relatively better than the designs obtained from the first technique. Their generalized formulation is described below.

Theorem 7.2.*For a given square COD $\mathbf{G}_{2^{l-1}}(z_1, z_2, \ldots, z_{l+1})$, the matrix*

$$\boldsymbol{Q}_{2^1\times 2^{l-1}}(z_1, z_2, \ldots, z_{l+1}) = \begin{bmatrix} \mathbf{G}_{2^{l-1}}(z_1, z_2, \ldots, z_l) + z_{l+1}\mathbf{I}_{2^{l-1}}j \\ -z_{l+1}^*\mathbf{I}_{2^{l-1}} + \mathbf{G}_{2^{l-1}}^H(z_1, z_2, \ldots, z_l)j \end{bmatrix} \tag{7.7}$$

provides a quaternion design of order $2^l \times 2^{l-1}$, with rate $(l+1)/2^l$.

Proof. It is straightforward to check $\mathbf{Q}^Q_{2^{l-1}\times 2^l}\mathbf{Q}_{2^l\times 2^{l-1}} = 2\lambda_1\mathbf{I}_{2^{l-1}} = \lambda\mathbf{I}_{2^{l-1}}$. We again start with an Alamouti code $\mathbf{G}_2$, to obtain a square COD $\mathbf{G}_4$ following Wolfe construction which contains three symbols because $2^{l-1} = 2^2$, implies $l = 3$. Hence, we arrive at the following example.

Example 7.2. The COD $\mathbf{G}_4$ along with a diagonal matrix containing an extra symbol z_4 in Equation (7.7) gives rise to

$$\mathbf{Q}_2 = \begin{bmatrix} z_1 + z_4 j & z_2 & z_3 & 0 \\ -z_2^* & z_1^* + z_4 j & 0 & z_3 \\ -z_3^* & 0 & z_1^* + z_4 j & -z_2 \\ 0 & -z_3^* & z_2^* & z_1 + z_4 j \\ -z_4^* + z_1^* j & -z_2 j & -z_3 j & 0 \\ z_2^* & -z_4^* + z_1 j & 0 & -z_3 j \\ z_3^* j & 0 & -z_4^* + z_1 j & z_2 j \\ 0 & z_3^* j & -z_2^* j & -z_4^* + z_1^* j \end{bmatrix}. \tag{7.8}$$

This QOD transmits 4 complex symbols $z_1, z_2,$ z_3 and z_4 in eight time slots, thus provides a relatively better code rate of 1/2.

7.2.3 Symmetric-Paired Design 3: (Non-Square QODs)

The main issues related with quasi-CODs are discussed which promise a high data rate at the expense of coding and decoding delays besides coupled and non-linear decoding. This issue can be resolved in the quaternion domain. For example, in Jafarkhani [2001] a quasi-COD of rate 1 was designed for four transmit antennas

$$\mathbf{C}_q = \begin{bmatrix} \mathbf{G}_2 & \mathbf{L}_2 \\ -\mathbf{L}_2^* & \mathbf{G}_2^* \end{bmatrix}, \tag{7.9}$$

using two CODs $\mathbf{G}_2 = \begin{bmatrix} z_1 & z_2 \\ -z_2^* & z_1 \end{bmatrix}$ and $\mathbf{L}_2 = \begin{bmatrix} z_3 & z_4 \\ -z_4^* & z_3 \end{bmatrix}$, which was shown to have pair-wise decoding. These two CODs $\mathbf{G}_2$ and $\mathbf{L}_2$ are subjected to the quaternion domain which helps reclaim decoupled and linear decoding solutions. First of all, it is noted that the concept used in design 2, can be generalized such that in place of a diagonal matrix which merely contains a single extra symbol, a COD can be used. It turns out that the following theorem provides a possibility of incorporating $\mathbf{L}_2$ with $\mathbf{G}_2$, in place of a diagonal matrix. Resultantly, another recursive construction technique is obtained which can be proved easily. **Theorem 7.3.** *For two recursively generated CODs $\mathbf{G}_{2^{l-1}}(z_1, z_2, \ldots, z_l)$ and $\mathbf{L}_{2^{l-1}}(z_{1+2}, z_2, \ldots, z_{2l+2})$, a symmetric-paired design,*

$$\mathbf{Q}_{2^{l+1}\times 2^l}(z_1, \ldots, z_{2(l+1)}) = \begin{bmatrix} \mathbf{G}_{2^l} + \mathbf{L}_{2^l} j\,, \\ \mathbf{L}_{2^l} + \mathbf{G}_{2^l} j \end{bmatrix}, \tag{7.10}$$

is a QOD of dimension $2^{l+1} \times 2^l$ with rate $(l+1)/2^l$.

It is easy to implement this technique as is shown in the subsequent example.

Example 7.3. To generate a QOD for 4 DP antenna, we use $\mathbf{G}_2$ and $\mathbf{L}_2$ to obtain two CODs of higher orders $\mathbf{G}_4$ and $\mathbf{L}_4$ using Wolfe construction. Consequently, Equation (7.10) gives rise to a QOD of rate 3/4

$$\mathbf{Q}_3 = \begin{bmatrix} z_1 + z_4 j & z_2 + z_5 j & z_3 + z_6 j & 0 \\ -z_2^* - z_5^* j & z_1^* + z_4^* j & 0 & z_3 + z_6 j \\ -z_3^* - z_6^* j & 0 & z_1^* + z_4^* j & -z_2 - z_5 j \\ 0 & -z_3^* - z_6^* j & z_2^* + z_5^* j & z_1 + z_4 j \\ z_4 + z_1 j & z_5 + z_2 j & z_6 + z_3 j & 0 \\ -z_5^* - z_2^* j & z_4^* + z_1^* j & 0 & z_6 + z_3 j \\ -z_6^* - z_3^* j & 0 & z_4^* + z_1^* j & -z_5 - z_2 j \\ 0 & -z_6^* - z_3^* j & z_5^* + z_2^* j & z_4 + z_1 j \end{bmatrix}. \tag{7.11}$$

Therefore, three recursive techniques are presented to generate square and rectangular QODs from square CODs. As all of the above QODs are obtained from the recursive techniques of CODs based on famous Adams-Lax-Phillips, Józefiak and Wolfe constructions, therefore, an upper bound can be used on these to arrive at the following result.

Theorem 7.4. *The rate $r_{\mathbf{Q}}$, of all possible QODs in (3), (8) and (11) obtained from square CODs is given by*

$$r_{\mathbf{Q}} = \frac{l+1}{2^l}. \tag{7.12}$$

This provides a class of QODs which are fully diverse [137] and earlier these were shown to have decoupled decoders based on a semi-quaternionic channel model [121]. The system model [57] based on the characterization of pure quaternionic channel is used and it is shown that the above designs all have decoupled decoders and optimal decoding delays. Besides these advantages, there is one drawback as mentioned in the remark below. All of the above construction techniques generate QODs for only configurations when the number of DP transmit antennas are in powers of 2, i.e, $2, 4, 8, 16, \ldots$ which puts a heavy toll on the code rate as $1/2^l$ sharply declines as the number of antennas increase. Following above remark, it is essential to find quaternion designs for general configuration of DP antennas like for a $(n_T \times 1)-$system such that $n_T \in N$ and these codes have maximal coding rates.

7.2.4 Maximal Rate QODs for General Configuration of DP Antennas

In Liang's paper [16], rectangular CODs of maximal rates are found algorithmically. The general procedure was given in the paper, however, such designs do not carry a compact form as was found in the case of Equation (7.1). As demonstrated below that the proposed procedure successfully works on the famous examples given in [16], to generate designs for 3 and 5 DP antennas among other configurations, respectively.

Lemma 7.1. *A maximal rate* $\mathcal{R}_{\mathbf{Q}} = 3/4$ *QOD for 3 DP antennas is given by*

$$\mathbf{Q}_4 = \begin{bmatrix} z_1 + z_2 j & z_2 + z_1 j & z_3 + z_3 j \\ -z_2^* + z_1^* j & z_1^* - z_2^* j & 0 \\ -z_3^* & -z_3^* j & z_1^* + z_1^* j \\ -z_3^* j & -z_3^* & z_2^* + z_2^* j \end{bmatrix}. \tag{7.13}$$

Proof A rectangular COD of maximal rate 3/4 is

$$\mathbf{U}_1 = \begin{bmatrix} z_1 & z_2 & z_3 \\ -z_2^* & z_1^* & 0 \\ -z_3^* & 0 & z_1^* \\ 0 & -z_3^* & z_2^* \end{bmatrix}, \tag{7.14}$$

is used to construct an equivalent COD $\mathbf{V}_1$ based on the same principle of permutation of columns such that $\mathbf{U}_1^H\mathbf{V}_1$ is symmetric which can be verified easily. Consequently, we obtain a QOD for three DP antennas of rate 3/4 given by $\mathbf{U}_1 + \mathbf{V}_1 j = \mathbf{Q}_4$.

To complete the discussion, one QOD is included which is suitable for 4 DP antennas and is obtained using Liang's approach. This can be compared with the QODs based on recursive approach for the same number of antennas.

$$\mathbf{Q}_5 = \begin{bmatrix} z_1 & z_1 j & z_2 + z_3 j & z_3 + z_2 j \\ z_1 j & z_1 & z_4 + z_5 j & z_5 + z_4 j \\ -z_2^* - z_4^* j & -z_4^* - z_2^* j & z_1^* & z_1^* j \\ -z_3^* - z_5^* j & -z_5^* - z_3^* j & z_1^* j & z_1^* \\ -z_4 + z_2 j & z_2 - z_4 j & z_6 j & z_6 \\ -z_6^* j & -z_6^* & -z_3^* + z_2^* j & z_2^* - z_3^* j \\ -z_5 + z_3 j & z_3 - z_5 j & -z_6 & -z_6 j \\ z_6^* & z_6^* j & -z_5^* + z_4^* j & z_4^* - z_5^* j \end{bmatrix}. \tag{7.15}$$

Lastly, a QOD is constructed for 5 DP antennas.

Lemma 7.2. *A maximal rate* $\mathcal{R}_{\mathbf{Q}} = 2/3$ *QOD for 5 DP antennas is given* *by*

$$\mathbf{Q}_6 = \begin{bmatrix} z_1 & z_1 j & z_2 + z_3 j & z_3 + z_2 j & z_4 + z_4 j \\ z_1 j & z_1 & z_5 + z_6 j & z_6 + z_5 j & z_7 + z_7 j \\ -z_2^* - z_5^* j & -z_5^* - z_2^* j & z_1^* & z_1^* j & 0 \\ -z_3^* - z_6^* j & -z_6^* - z_3^* j & z_1^* j & z_1^* & 0 \\ -z_4^* - z_7^* j & -z_7^* - z_4^* j & 0 & 0 & z_1^* + z_1^* j \\ -z_5 + z_2 j & z_2 - z_5 j & z_8 j & z_8 & z_9 + z_9 j \\ -z_8^* j & -z_8^* & -z_3^* + z_2^* j & z_2^* - z_3^* j & 0 \\ -z_9^* j & -z_9^* & -z_4^* & -z_4^* j & z_2^* + z_2^* j \\ -z_6 + z_3 j & z_3 - z_6 j & -z_8 & -z_8 j & z_{10} + z_{10} j \\ -z_{10}^* j & -z_{10}^* & -z_4^* j & -z_4^* & z_3^* + z_3^* j \\ -z_7 + z_4 j & z_4 - z_7 j & -z_9 - z_{10} j & -z_{10} - z_9 j & 0 \\ z_8^* & z_8^* j & -z_6^* + z_5^* j & z_5^* - z_6^* j & 0 \\ z_9^* & z_9^* j & -z_7^* & -z_7^* j & z_5^* + z_5^* j \\ z_{10}^* & z_{10}^* j & -z_7^* j & -z_7^* & z_6^* + z_6^* j \\ 0 & 0 & z_{10}^* - z_9^* j & -z_9^* + z_{10}^* j & z_8^* + z_8^* j \end{bmatrix}, . \tag{7.16}$$

Proof The proof of this lemma is similar to the Lemma 1, however, in this case the underlying COD **U** is given in equation (100) in [16].

Following the same lines, it is easy to construct QODs for higher number of transmit antennas $n_T = 6, 7, 8$ starting with the CODs given in [16] (equation (101) and appendices C and D, respectively).

7.3 Comparative Analysis of the Construction Techniques

7.3.1 Code Rates

An important result which gives us bounds on the upper limits of code rates of above QODs can be proved easily. For instance, the underlying CODs which we employ in the construction of such codes have upper bounds (Theorem 5 and Theorem 6 in [16]) therefore following result follows immediately.

Theorem 7.5. *(a) For an even number of transmit DP antennas, the highest possible rate of QODs arising from rectangular CODs is bounded above by*

$$\mathcal{R}_{\mathbf{Q}} \leq \frac{n+2}{2n}. \tag{7.17}$$

(b) For an odd number of transmit DP antennas, the highest possible rate of QODs arising from rectangular CODs is bounded above by

$$\mathcal{R}_{\mathbf{Q}} \leq \frac{n+3}{2n+2}. \tag{7.18}$$

Note that there is substantial difference between the code rates of QODs obtained from the above approach, denoted by $R_{\mathbf{Q}}$, and those which are based on recursive techniques represented with $r_{\mathbf{Q}}$. Figure 7.2 clearly describes that as the number of DP transmit antennas increases the code rate $r_{\mathbf{Q}}$ sharply declines. Quite contrary, the code rates $R_{\mathbf{Q}}$ of QODs based on Liang's approach remain stable. Regardless of the number of transmit antennas, we can always obtain a QOD with code rate higher than or equal to 0.5. So we obtain a robust approach of developing QODs based on Liang's mechanism.

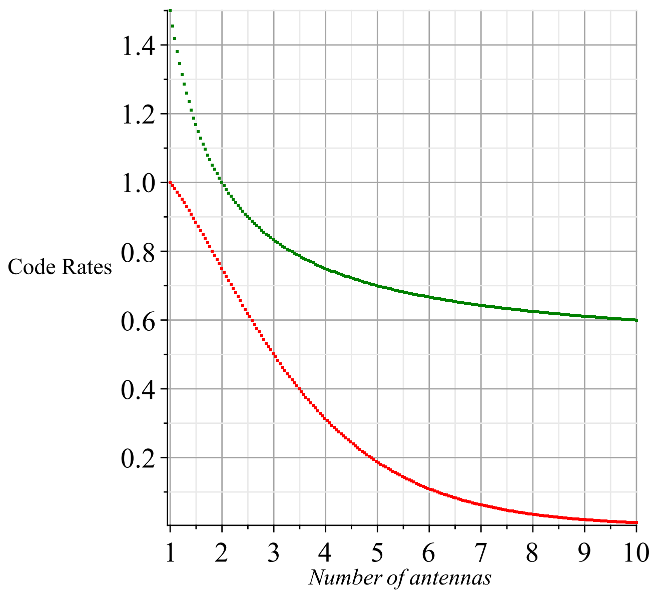

Figure 7.2: Comparison of Code Rates: Red curve represent the code rate $r_{\mathbf{Q}}$ of QODs based on recursive methods. Code rates $R_{\mathbf{Q}}$ of QODs based on Liang's approach are depicted with green curve.

7.3.2 Coding & Decoding Delays

In order to optimize throughput, it is essential to have codes with optimal coding delay and QODs have an advantage of it. The decoding delay, denoted here as ξ, is an important performance measure for STBCs. Essentially, the decoding delay signifies the total number of time slots a receiver has to wait to receive a complete block of code before starting the decoding process. This implies that higher order code matrices require larger decoding delays. To compare the performance of different STBCs schemes, i.e., symmetric-paired design 1, 2, 3 and Liang's approach respectively, we enlist values of ξ for codes in case of $3, 4$ and 5 DP antenna arrangements in Table 1. In the table, N_t denotes the number of DP transmit antennas. From the table, it is

Table 7.1: Decoding Delays

Code Designs	$N_t = 3$	$N_t = 4$	$N_t = 5$
Design 1	$*$	$\xi = 2$	$*$
Design 2	$*$	$\xi = 4$	$*$
Design 3	$*$	$\xi = 4$	$*$
Design 4	4	$\xi = 8$	15

seen that for 3 and 5 DP antenna systems there is no QOD obtainable from iterative techniques which we represent with $*$.

7.4 Quaternionic Channel Model

The simultaneous transmission from both ends of a DP antenna can be regarded as a *hyper signal* which consists of two complex numbers in two orthogonal polarizations. It propagates through space and received as a hyper signal by the DP antenna at the receiver end in a given time slot. This hyper signal can be represented as a quaternion [19] which gives us a reason to develop system model in the quaternion domain. An important component of the system model is the channel which we assume to be quaternionic following the line of approach followed in [57]. Therein, we observe that the product in the quaternion domain holds a key of consistent and viable model.

A TISO of DP antennas is considered. It is necessary to emphasize the role of quaternions which is more recognizable in this case, therefore, we have

$$\mathbf{R} = \begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} q_1 & q_2 \\ q_3 & q_4 \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}, \tag{7.19}$$

where each element in the above construction is a quaternion. Through each antenna in the above TISO system, the transmission of a pair of two complex symbols is encoded in q_1 and another pair in q_3. This indicates that the above

QOD exploits time and space diversities along with polarization diversity. It is worth pointing out that each quaternionic product, e.g., $q_a h_b$ contains a crucial information about the nature of quaternion domain. If we decompose it for a general quaternionic product then we obtain $q_{a1}h_{b1} - q_{a2}h_{b2} + j(q_{a1}h_{b2} + q_{a2}h_{b1})$, where $q_a = q_{a1} + jq_{a2}$ and $h_b = h_{b1} + jh_{b2}$. Therefore, we will obtain four complex channel gains for each antenna in a 2×1 system. Note that we have four complex channel gains between a TISO system of DP antennas. As this system is equivalent to a MIMO 4×2 system of single-polarized antennas, therefore, it may appear that it should have eight channel gains in total with two for each link. However, in our proposed model each quaternionic product results in the same number of channel gains.

Subsequently, a system model for a MIMO system of DP antennas can be constructed in the same way for such a system with $N_t \times N_r$DP antennas

$$\mathbf{R}_{T\times N_r} = \mathbf{Q}_{T\times N_t}\mathbf{H}_{N_t\times N_r} + \mathbf{N}_{T\times N_r}, \tag{7.20}$$

which transmits symbols in $T-$times slots which are assumed to be points in the QPSK constellation. The channel matrix is $\mathbf{H} = [h_{\rho\sigma}]$, where $\rho = 1, 2, \ldots, N_t$ and $\sigma = 1, 2, \ldots, N_r$. The channel is assumed to represent a flat fading channel and the path gain from ρ transmit DP antenna to receive DP antenna σ given by a quaternion $h_{\rho\sigma} = h_{\rho\sigma 1} + h_{\rho\sigma 2}j$. The complex channel gains, $h_{\rho\sigma 1}$ and $h_{\rho\sigma 2}$ incorporate the effects of cross polar scattering and each element of channel gain matrix is a complex Gaussian random variable (RV) with zero mean and unit variance. Moreover, the noise $\mathbf{N} = [n_{T\sigma}]^T$, and $n_{T\sigma} = n_{T\sigma 1} + n_{T\sigma 2}j$, such that $n_{T\sigma 1}, n_{T\sigma 2}\ \forall\ \sigma = \{1, 2, \ldots, N_r\}$, represent the entries of white noise as two dimensional independent and identically distributed (i.i.d.) complex Gaussian RVs with zero mean and identical variance per dimension.

7.4.1 Linear and Decoupled ML Decoder

Based on the system model given in Equation (7.20), the following theorem confirms a linear decoupled solution at the receiver for all QODs constructed in Section 2 of this chapter. It was previously proved for non-iterative QODs in [57] but its validity is now confirmed for all QODs obtained in Section 2. **Theorem 7.6.** *For a given system model in Equation (7.20), the ML-decoding rule assumes a linear decoupled form*

$$\min_{z} ||\mathbf{R} - \mathbf{Q}\mathbf{H}||^2 = \min_{z} \Big(tr(\mathbf{R}^Q\mathbf{R}) + \lambda tr\big(\mathbf{H}^Q\mathbf{H}\big) - 2\Re\big(tr\big(\mathbf{R}^Q\mathbf{Q}\mathbf{H}\big)\big)\Big). \tag{7.21}$$

The main contributing factor in the above rule is $\Re\big(tr\big(\mathbf{R}^Q\mathbf{Q}\mathbf{H}\big)\big)$, which needs to be minimized for any transmitted symbol encoded as a quaternion in a given time slot. The appearance of $\mathbf{Q}$ indicates the linearity of the decoder as well as the computational load at the receiver is reduced significantly.

As an illustration of the above result, we choose QODs given in Equations (7.6) and (7.8) and demonstrate that the above ML-decoding rule is both linear and decoupled. For remaining QODs $\mathbf{Q}_3, \mathbf{Q}_4, \mathbf{Q}_5$ and $\mathbf{Q}_6$ in Equations (7.11), (7.13), (7.15),(7.16) respectively, a similar decoding result can be obtained easily. **Corollary 7.1.** The ML-decoding rule (7.21) for QOD given in Equation(7.13), reduces to the real part of

$$\begin{aligned}
&- 2\min_{z_1}(r_1^Q z_1(h_1 + jh_2) + r_2^Q z_1^*(jh_1 + h_2) + r_3^Q z_1^*(1 + j)h_3)\,, \\
&- 2\min_{z_2}(r_1^Q z_2(jh_1 + h_2) - r_2^Q z_2^*(h_1 + jh_2) + r_4^Q z_2^*(1 + j)h_3)\,, \\
&- 2\min_{z_3}(r_1^Q z_3(1 + j)h_3 - r_3^Q z_3^*(h_1 + jh_2) - r_4^Q z_3^*(jh_1 + h_2))\,,
\end{aligned} \tag{7.22}$$

where $\mathbf{R} = [r_1\ r_2\ r_3\ r_4]^T$, is a received quaternion vector and $h_1 = h_{11} + h_{12}j$, $h_2 = h_{21} + h_{22}j$ and $h_3 = h_{31} + h_{32}j$.

We now broaden our discussion to include designs which have significantly higher code rates than obtained from the approaches discussed in the previous sections. In order to do that we need to compromise on orthogonality in which case it is not possible to have decoupled linear decoder like Equation (7.21). It turns out that such quasi designs have other features to offer. Subsequently, we construct these quasi QODs by extending the standard approach as developed for the complex domain [18].

7.5 Quasi QODs

Unlike the complex domain, for two DP antennas there exists a QOD of rate 2 and was shown to attain a decoupled decoder [57]. We employ it

$$\mathbf{P}_1 = \begin{bmatrix} z_1 + z_2 j & z_3 + z_4 j \\ z_2^* - z_1^* j & -z_4^* + z_3^* j \end{bmatrix}, \tag{7.23}$$

to construct a higher rate quasi QOD $\mathbf{Q}_{quasi}$ as follows. By considering an identical code matrix with different symbols

$$\mathbf{P}_2 = \begin{bmatrix} z_5 + z_6 j & z_7 + z_8 j \\ z_6^* - z_5^* j & -z_8^* + z_7^* j \end{bmatrix}, \tag{7.24}$$

following the same line of approach as used for quasi CODs [18], we obtain a quasi QOD

$$\mathbf{Q}_{quasi} = \begin{bmatrix} \mathbf{P}_1 & \mathbf{P}_2 \\ -\mathbf{P}_2^Q & \mathbf{P}_1^Q \end{bmatrix}. \tag{7.25}$$

Thus, we obtain a quasi QOD to be used in the configuration of 4 DP antennas capable of sending 8 complex symbols in four time slots yielding a code rate of 2, given by

$$\mathbf{Q}_{quasi} = \begin{bmatrix} z_1 + z_2 j & z_3 + z_4 j & z_5 + z_6 j & z_7 + z_8 j \\ z_2^* - z_1^* j & -z_4^* + z_3^* j & z_6^* - z_5^* j & -z_8^* + z_7^* j \\ -z_5^* - z_6 j & -z_6 + z_5^* j & z_1^* + z_2 j & z_2 - z_1^* j \\ -z_7^* - z_8 j & z_8 - z_7^* j & z_3^* + z_4 j & -z_4 + z_3^* j \end{bmatrix}. \tag{7.26}$$

The above code $\mathbf{Q}_{quasi}$ does not satisfy the main quaternion orthogonality condition as

$$\mathbf{Q}_{quasi}^{Q}\mathbf{Q}_{quasi} \neq \lambda \mathbf{I}_{4\times 4}, \tag{7.27}$$

where $\lambda = ||z_1||^2 + ||z_2||^2 + ||z_3||^2 + ||z_4||^2 + ||z_5||^2 + ||z_6||^2 + ||z_7||^2 + ||z_8||^2$. However, the simulation curve for the above code is obtained in the next section.

7.6 Simulation and Results

To evaluate the performance and diversity gains, we employ QODs, i.e., $Q_1 - Q_6$, corresponding to single and DP receive antenna configurations. For simulations, QPSK is used. The receivers are aware of the channel coefficients and uniform white noise is added in each polarization.

The codes constructed using the Liang approach-based construction techniques provides less complex receivers. For the codes $\mathbf{Q}_1, \mathbf{Q}_6$, in Figure 7.3, it is clear that these codes have linear and decoupled decoding at the receiving end due to the use of the specific construction technique using the DP antennas and the quaternionic channel model. This has been possible due to the quaternionic channel exploiting the polarization diversity independently

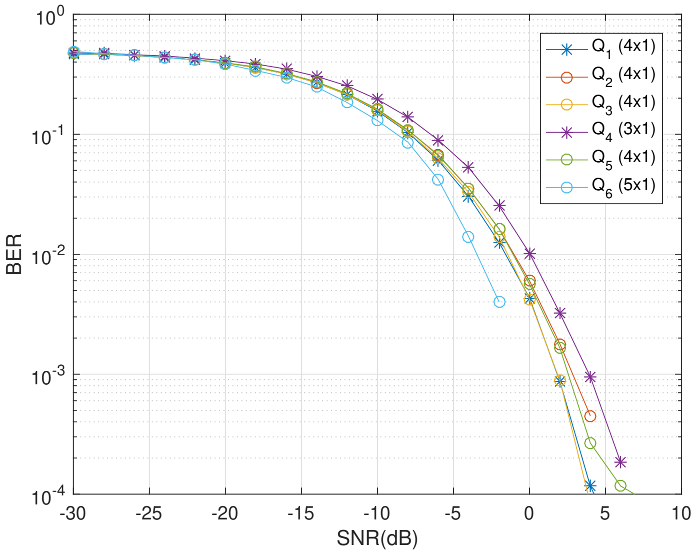

Figure 7.3: BER vs. SNR performance of $\mathbf{Q}_1, \mathbf{Q}_2, \mathbf{Q}_3, \mathbf{Q}_4, \mathbf{Q}_5$ & $\mathbf{Q}_6$ for single receive DP antenna.

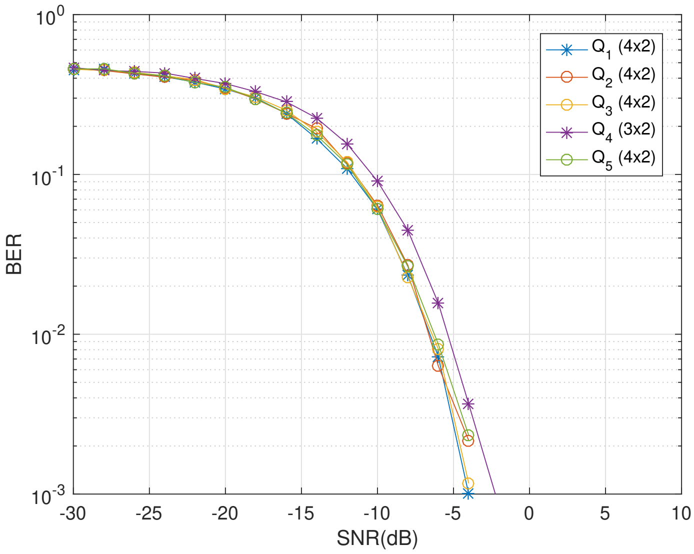

Figure 7.4: BER vs. SNR performance of $\mathbf{Q}_1, \mathbf{Q}_2, \mathbf{Q}_3, \mathbf{Q}_4, \mathbf{Q}_5$ & $\mathbf{Q}_6$ for two receive DP antenna.

using the polar as well as cross-polar scattering between the DP antennas. We can see that the codes $\mathbf{Q}_1, \mathbf{Q}_2$ and $\mathbf{Q}_3$ are transmitted using the four DP antennas at the transmitting end and have the code rates of $\frac{3}{2}$, $\frac{1}{2}$, $\frac{3}{4}$ and $\frac{3}{4}$, respectively. While, the codes $\mathbf{Q}_4$ and $\mathbf{Q}_6$ use three and four DP antennas during transmission with codes rate of $\frac{3}{4}$ and $\frac{2}{3}$, respectively. We can see that the proposed code construction technique has no restrictions on the number of transmit and receive antennas. This is demonstrated in Figure 7.3 where both even and odd number of DP antennas are used to transmit the codes.

The effects of increasing the receiver diversity are visible in Figure 7.4. Figure 7.4 shows that the receiver diversity has positive impact on the diversity gains. Use of the quaternionic channel exploits the polarization diversity and promises decoupled decoder for any number of receive DP antennas. In comparison to the work done in the past, the proposed design does not compromise the code rates when the number of transmit antennas are increased. Such an increase int he code rates with higher diversity gains are the requirements of the future MIMO systems to support greater channel utilization and efficiency. The freedom of the number of receive antennas that can be utilized by the QODs during their transmission from a quaternionic channel-based system has been further emphasized in Figure 7.5. The code $\mathbf{Q}_4$ shows increasing diversity gains as the number of receive DP antennas are increased at the receiving end.

The computational complexity of the proposed decoder used for the codes in Figure 7.3, Figure 7.4 and Figure 7.5 promises linear and decoupled decoding at the receiver. The decoder remains independent of the number of unique transmitted symbols, i.e., ζ. This has a huge impact in simplifying the complexity of the receiver in terms of the calculations to be performed. In case of the coupled decoder, the receiver complexity remains dependent on the number of unique transmitted symbols and has an exponential relationship with it. Considering N transmit antennas and T timeslots used to

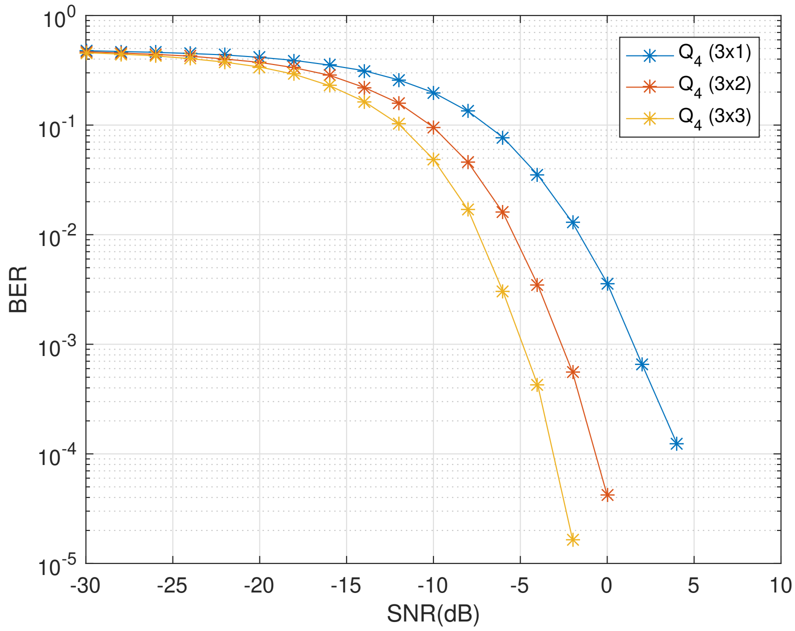

Figure 7.5: BER vs. SNR performance of $\mathbf{Q}_4$ for one, two and three receive DP antenna.

transmit a single block of code, the computational complexity of the coupled decoder is $O(4^{\varsigma}(N)(T)(2))$. However, this reduces significantly in the case of the proposed decoder design, where the computation complexity reduces to $O(4(N)(T)(2))$.

The decoupled decoding of the quasi-orthogonal codes has been a research problem as this compromises the coding rate for increased number of transmit antennas. A unique construction technique has been presented to form quasi QODs with higher code rates. For the antenna configuration of four transmit single receive DP antennas, the quasi QOD presented in Equation (7.26) has a compromised receiver complexity but promises higher gains in comparison to the coupled traditional ML-based decoder designs. The presented QOD construction mechanisms not only provides flexibility in generating new QOD designs but also supports decoupled decoding of the STBCs generated from

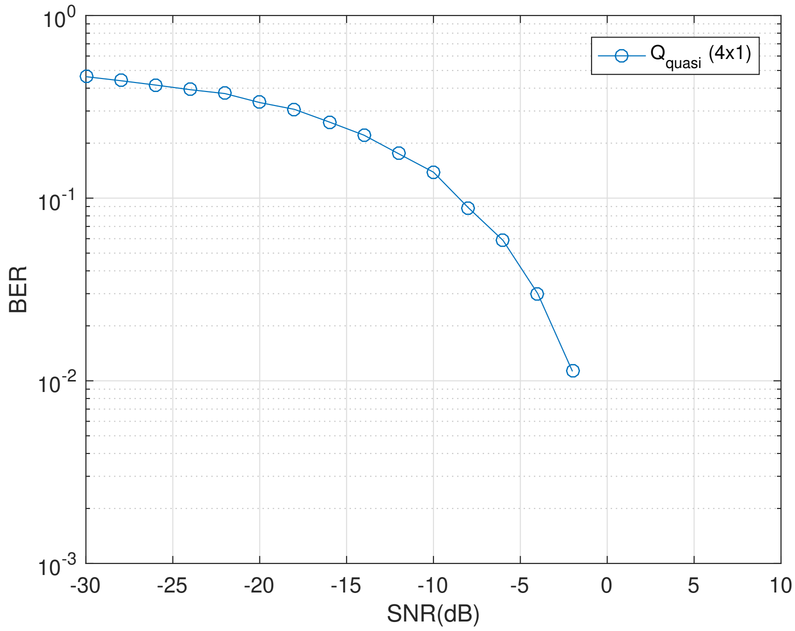

Figure 7.6: BER vs. SNR performance of $\mathbf{Q}_{quasi}$ for one receive DP antenna.

the QODs. This has remained as a problem that needed a solution since long. This research has presented a unique solution for supporting decoupled decoding of quasi-QODs with representation of maximal rates that they can achieve.

7.7 Conclusions

QODs with DP antennas have been studied to provide higher diversity gains and code rates. Research has addresseed the design of efficient QODs to present decoupled decoding but this has failed till date. This chapter presents the construction of QODs based on the Liang approach using the DP antennas and the quaternionic channel model. The unique method of constructing QODs provides linear and decoupled decoding at the receiving side where the computational complexity of the decoder remains independent of the number of unique transmitted symbols. Also, construction technique of quasi-QODs has been presented where the code rate is not compromised at higher number of transmit antennas [60]. In future, these designs can be investigated for optimizing the receiver complexity further considering their suitability for future wireless communication systems.

Chapter 8

Conclusion and Future Recommendations

8.1 Conclusion

QODs had been studied extensively [13,20,98,102,109,110,118,124,128,138]. Although, many coding and decoding techniques were proposed for STBCs, none had addressed the quaternion designs in their pure form. Most of the evaluations were carried out on system models build using the complex quasi-orthogonal STBCs. These designs supported higher diversity gains in MIMO systems, however, the compromise was made on the performance. Use of these non-orthogonal designs for the construction of higher order codes increased the complexity of the receiver. This made it difficult to obtain a decoupled decoding solution. The basic design of using the quaternion orthogonal codes was considered to exploit the benefits of polarization diversity but this failed to work. Also, the study of STBCs shows that the cross polar components contribute to the polarization diversity in combination with the time diversity. Thus, it became evident that polarization diversity, alone, can not be explored for gaining the diversity gains.

This thesis exploits the polarization diversity by proposing a new system model based on pure quaternion channel design. The space, time and polarization diversities are exploited in quaternion domain to achieve promising diversity gains. Thus, a new wireless communication channel model is presented. This channel, i.e. the quaternionic channel comprises of pure quaternion channel coefficients. Using these pure quaternions for the wireless channel as well as the quaternion orthogonal codes, polarization diversity gain is achieved independent of the other forms of diversities i.e. space and time. This system not only provides a generalized mechanism of code generation but also provides simplicity in terms of receiver design. With the use of quaternion channel and code designs, it is possible to achieve decoupled decoding solution at the receiving end. Performance evaluations have also confirmed these findings that pure quaternion designs fully exploits the polarization diversity and promise higher diversity gains.

Quaternion algebra has been explored to address construction and decoding techniques in the wireless communication channel. In this regime, DP antennas are used at both the transmitter and receiver ends, which provides the most suitable infrastructural support to implement the QODs where the quaternionic channel can be considered. Construction of QODs promising higher diversity gains by exploiting space, time and polarization diversities are restricted under conditions which have been evaluated. Through use of quaternion algebra, the proposed quaternionic channel contributes in presenting iterative as well as non-iterative construction techniques for QODs. The decoding at the receiver has been discussed at depth to reduce the receiver's computational complexity. The investigation has revealed a remarkable contribution of this dissertation where linear decoupled decoding solution exists for not only square but also non-square designs. This solution generalizes to environments experiencing polar as well as non-polar scattering. The construction techniques presented in this thesis are based on pure quaternions eliminating any restrictions on the number of DP antennas to be used at the receiver side.

An application of the proposed quaternionic channel model has been presented to contribute in better efficiency of modulated data in terms of higher diversity gain. Quaternion modulation, [59], has been evaluated using the quaternionic channel model where the cross-polar scattering effects are embedded in the coupling between the quaternion orthogonal code entries and quaternionic channel coefficients. With the quaternionic channel model and pure QODs based on quaternion algebra, space, time and polarization diversities are exploited independently and their combined impact can be seen in the diversity gains of the quaternion modulation. The proposed system model for the quaternion modulation is independent of the number of DP antennas used at transmitter or receiver end. Another contribution of implementation of quaternion modulation using quaternionic channel is the linear decoupled decoding solution that simplifying the receiver.

The quaternionic channel model and the different construction techniques (i.e. iterative and non-iterative) developed for QODs have been considered for flat-fading Rayleigh channels. This helps in protecting the signal from fluctuating across its transmission. Thus, the code is likely to perform well when it is experiencing the similar environment as it is designed for. The proposed system model using the quaternionic channel with transmit diversity will support beneficial results in frequency selective channels where the increased transmit diversity can support the signal deterioration. The presented system has been shown, in Chapter 7, to extend to higher dimensional codes using many transmit antennas. Thus, the suitability of the proposed system model based on the quaternionic channel and pure QODs is seamless for massive MIMO systems.

8.2 Future Recommendations

Massive MIMO communication systems in future will demand generalized solution exploiting the benefits of space, time and polarization diversities independently, in order to combine their composite impact for higher diversity gains. This will promote successful transition to next generation of wireless communication systems i.e. 5G, 6G and beyond. The current research has targeted the channel design considering the extensive use of DP antennas and presenting a system model based on a channel design using quaternion algebra. In this dissertation, DP antennas are explained to transmit a pure quaternion that achieves higher diversity gains as it traverses the quaternionic channel. The QODs as well as the quaternionic channel uses pure quaternions that helps in exploiting the polarization diversity by using DP antennas. The research done in this dissertation has several advantages that applies not only to the optimization of future generation of wireless communication networks but also unfolds new dimensions for research and study. A detailed mathematical model has been presented in this thesis which demonstrates that polarization diversity can be exploited independently as the quaternionic orthogonal codes are transmitted using DP antennas and provides not only diversity gains but also promises higher code rates, even full rate codes. This has been a breakthrough contribution of this dissertation, where code rates comparative to Alamouti codes are possible [1]. Some future recommendations have been listed below.

- In future, it has been observed that for QODs using more than one DP antennas, time diversity is used for achieving higher diversity gains. Thus, a possible future direction can be a detailed mathematical analysis to design the quaternion orthogonal codes such that they can provide higher order codes without such dependencies. In this domain, further extension of this work to clifford might provide us with a viable solution [139, 140]. Additionally, study can be done in designing higher order QODs and their mathematical bases.

- Generalized iterative construction techniques has been proposed providing a decoupled solution for square as well as non-square code designs. Extending the research in this dissertation to further design non-zero codes seems an interesting area for future research and analysis. Additionally, these iterative construction techniques can be further explored for designing codes for higher dimensions.

- Construction of QODs using non-iterative techniques has been presented where the decoupled decoding solution at the receiving end with promising code rates makes it a novel contribution. This can further be explored in the realm of massive MIMO systems for future generations of wireless communication systems.

- The quaternion modulation designs provides a decoupled decoding solution with higher diversity gains, increased data rate and reduced receiver complexity. This design can be extended to future wireless communication systems by evaluating its application to other modulation techniques.

- The QODs designed using the quaternionic channel model are developed using the flat fading Rayleigh channel. Practical evaluations of the performance of the proposed QOD designs in presence of frequency selective channels might reveal new insights and open further dimensions for research.

This dissertation proposes a quaternionic channel model and the design of pure quaternion orthogonal coding techniques. This has numerous advantages for future wireless communication systems, some of them have been detailed above subject to further study and evaluations.

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