Coverage Analysis of a Dual Source Opportunistic Network Utilizing Cooperation

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***Abstract*—The performance of a dual source cooperative multi-hop network is investigated, where linear network coding technique is used by the intermediate nodes to combine and forward the information of two sources. A strip-shaped network is considered where each hop is characterized by a square region in which the nodes are randomly deployed that transmit the information of two sources to a common destination using cooperative decode-and-forward mechanism. The power received at a node is calculated in the presence of Rayleigh fading and path loss. The coverage of the network for a given signal-to-noise ratio (SNR) margin is found using the state distribution probabilities. Numerical simulaions are performed to validate the theoretical results.**

***Index Terms*—Opportunistic large arrays (OLAs), linear network coding (LNC), strip-shaped networks, binomial point process (BPP), state distribution.**

I. Introduction

Cooperative multi-hop systems have gained significant popularity and research efforts beacuse of the advantages they offer in both wireless sensor networks (WSNs) and the cellular networks. In cooperative communication, there is an improvement in the received signal-to-noise ratio (SNR) as multiple copies of the same signal are received from different relay nodes via uncorrelated channels, providing spatial diversity. One of the famous endowed techniques of cooperative transmission (CT) at the physical layer is opportunistic large arrays (OLAs) [1]. In basic OLA networks, a source node transmits an information signal to a group of nodes in its vicinity. The nodes that decode the information successfully participate in the next transmission and relay the information to the next level nodes. This process continues in a multi-hop fashion without any cluster head or beacon node. OLA networks can thus be used to deliver the information to far off destinations. The physical routing mechanism of OLA networks is beneficial for dense WSNs and long distance wireless communication.

The analytical modeling of OLA network was initially performed using the *continuum* assumption [1], which implies that if the nodes density approaches infinity for a constant power per unit area and the decoding threshold is below a certain critical value, then the decoding can be done successfully, no matter how far is the destination. This assumption is only applicable for dense wireless networks. A linear finite-node network is analytically modeled in [2] with a single source, whereas the stochastic modeling of random 2-dimentional $(2D)$ geometry is presented in [3]. Several other works related to these networks are presented in [11]–[14]. However, in all the above mentioned works, there is a single source, which excites the whole network. The packet insertion rate is calculated in [4] for the multiple flows, however the multiple flows are from the same single source.

In this paper, we study a cooperative multi-hop strip network in which there are two sources that are transmitting at the same time and randomly placed in a fixed boundary. Although, in OLA networks, irregular boundaries are formed at each hop, however, to reduce the complexity imposed by an irregular geometry, the fixed boundaries are considered. There are many applications of strip-shaped network such as monitoring of bridges and tunnels and hazard sensing in hallways of buildings [5], [15]. The information of two sources is transmitted using orthogonal channels to avoid interference. Opportunistic space time coding is used for the assignment of orthogonal channels [6]. The intermediate nodes, which are randomly placed within a fixed boundary will receive the two information packets from the sources. To increase the throughput of the system, linear network coding (LNC) is used at the relay nodes for combining the two sources' information. In earlier literature, network coding is proposed for wireless networks in which links are assumed to be noiseless [7]. The binary network coding was introduced first, whereas non-binary and linear network coding techniques were used later on, which increase the transmission efficiency and provide better diversity. In [8], network coding techniques, e.g., low density parity check (LDPC)-based coding, and Reed Solomon codes are proposed to acheive diversity. Random network coding within a defined field is proposed in [10], which can be useful for WSNs. The diversity and success probability is calculated for the two hop network in [9] in the presence of network coding.

In this paper, we use linear network coding technique for the cooperative multi-hop network with two sources. It is assumed that the codewords made by the relay nodes are linearly independent from one another. We consider that all the nodes transmit with the same power, while the channel impairments include Rayleigh fading and path loss. A node can decode the information if the cumulative received power is greater than a modulation-dependent threshold. The power received by the node at a level is the sum of powers transmitted by the nodes at

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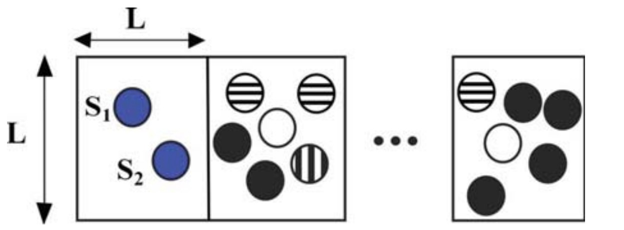

Fig. 1. Realization of a strip-shaped network topology with randomly placed nodes; $N = 6$

previous level that are transmitting the same information over orthogonal channels. In this study, as the nodes are randomly placed hence the path loss is no more deterministic. It depends on the Euclidean distance between the nodes of disjoint sets. The probability of outage calculated in [3] for single-input single-output (SISO) and multiple-input single-output (MISO) links in non-overlapping contiguous square regions, are used to find the state distribution probabilities of the system. Hence we provide coverage of the network in the presence of network coding for a given SNR margin.

The remaining part of the paper is ordered as follows. In Section II, the network topology is described along with network coding scheme and state distribution probabilities. Section III validates our numerical results and provides some useful insights regarding the system performance. Section IV concludes the paper along with future recommendations.

II. System Model

Consider a network topology in which there are two source nodes that have independent information to be transmitted to a far off common destination. In this topology, the source nodes are uniformily distributed in a region of an area $L \times L$. Similarly, at each hop there are N relay nodes, which are randomly placed in a region of area $L \times L$ each. The network is stretched in horizontal direction with each $L \times L$ region that are contiguous as shown in Fig. 1. The randomly placed nodes make a binomial point process (BPP) at each hop. Let Ψ is the binomial point process on compact set $\mathbb{A} \in \mathbb{R}^2$ such that $\mathbb{A}$ is the bounded square region $L \times L$. A total of N points are uniformaly distributed in the bounded set $\mathbb{A}$, where $\Psi(\mathbb{A}) = N$ with probability 1.

The source nodes broadcast their information using the orthogonal channels to the nodes at the first hop. Hence there is no interference in the reception of both sources' information for the relay nodes at first hop. The sources S_1 and S_2 transmit information I_1 and I_2, respectively, with the same power P_t. Due to wireless channel impairments, all nodes at the first hop do not decode both sources' information. Some of the nodes decode only I_1 and/or I_2 and some nodes decode nothing. The nodes that decode either I_1 & I_2 or any of the sources' information, i.e., I_1 or I_2, will participate in the next level transmission. The technique used by relay nodes at the each hop is decode-and-forward (DF) to transmit the decoded information to the next hop. The relay nodes that decode both sources' information will make a network codeword and transmit them to the next level. The network coding technique is explained in next subsection.

It can be observed that the receiving nodes at each hop can thus be categorized in three different groups of nodes; one having the information of source 1, second with the information of source 2 and third group contains the nodes that receive both sources' information. For example, in Fig. 1, the nodes represented by hollow circles decode nothing, filled circle nodes decode both informations, horizontally-hashed nodes decode I_1 and vertically-hashed nodes decode I_2, respectively. Therefore, there are four possible states for a node at a receiving level as follows:

- State 0= if node decodes nothing,
- State 1= if node decodes only I_1,
- State 2= if node decodes I_1 and I_2,
- State 3= if node decodes only I_2.

If this network is modeled with a Markov chain then there are a total of 4^N states in the system e.g., if $N = 6$ then the number of total states will be 4096. As the number of nodes in a hop increases, the problem of state-space explosion occurs, which implies that the states of the system become enormously large. Therefore, we calculate the probability of state distribution at each hop to analyze this network. In other words, we are interested in finding the percentage of the nodes in a level that are in 0, 1, 2,or 3 state, respectively.

A. Network Coding Technique

The linear network coding technique is used to make the network codewords. The nodes which receive the information of both sources i.e., I_1 & I_2 will make network codeword e.g., $C_j^{(n)} = 2I_1 + 3I_2$, where j is the transmitting node at level n. It is assumed that the coefficients selected for making the network codewords are random and independent. Hence if only two codewords or one codeword with any of the sources' information i.e., I_1 or I_2 are receive, the information can be extracted sucessfully. For instance, if $C_j^{(n)} = 2I_1 + 3I_2$ and $C_i^{(n)} = I_1 + 2I_2$ are the two codewords from two nodes at a specific level n, the information of two sources can be recovered by a simple Gaussian elimination method. And also if one codeword with any of the source information I_1 or I_2 are received, both informations can also be recovered. Therefore, the linearly independent codewords are a way to increase the diversity in the system.

B. State Distribution Probability

The nodes at the first hop receive two signals from both sources S_1 and S_2. As the information's are different, there is a single-input single-ouput (SISO) link between the sources and each of the nodes at the first hop. The power received at a j^{th} node in a SISO link is given as

$$P_{r_{ij}}^{(1)} = \frac{P_t \mu_{ij}}{d_{ij}^{\beta}}, \tag{1}$$

where the $i \in (S_1, S_2)$ is the transmitter, $d_{ij} \in \mathbb{R}^2$ is the Euclidean distance between nodes i and j, β denotes the path loss exponent and μ_{ij} is a random variable (RV) that denotes the channel gain between nodes i and j, which is exponentially distributed with unit mean. The sources and the nodes of the first hop are randomly deployed according to BPP so the Euclidean distance between them is also random. From [3], the distribution of the squared distance between two randomly placed node in contigious $L \times L$ regions is given by a Weibull distribution. The outage probability of a SISO link for this model in the presence of Rayleigh fading, threshold τ and arbitrary path loss exponent β is calculated as [3]

$$P_{o(SISO)} = \frac{\tau\chi}{P_t} \sum_{n=0}^{\infty} \frac{1}{(n+1)!} \Gamma\left(\frac{1+c+n}{c}\right) \left(-\frac{\tau\chi}{P_t}\right)^n, \tag{2}$$

where for Weibull distribution, the shape parameter, $k = 1.5806$ and variable scale parameter $\lambda = \frac{4L^2}{3\Gamma(1.6327)}$, $c = \frac{2k}{\beta}$ and $\chi = \lambda^{\frac{\beta}{2}}$.

As stated earlier, the nodes at the first hop can be divided in three groups ignoring the $state$ 0 nodes. These groups are G_1, G_2, and G_3 representing nodes in state 1, 2 and 3, respectively. Using Eq. (2), the success probability of a node to go in any state $state$ 1, $state$ 2 or $state$ 3 can be found. Also a node j is in outage ($state$ 0) if it does not decode any source's information. Hence the state distribution for a node j can be given by a 4×1 vector $\mathbf{V}_j$, where

$$\mathbf{V}_j = [P^2_{o(SISO)}, \quad P_{s(SISO)} \times P_{o(SISO)}, \quad P^2_{s(SISO)}, \quad P_{o(SISO)} \times P_{s(SISO)}]^T.$$

In the above definition, $P_{s(SISO)} = 1 - P_{o(SISO)}$, whereas the entries of vector $\mathbf{V}_j$ are the probabilities of node j to go in $state$ 0, 1, 2 and 3, respectively.

From second hop and onwards, the three groups of nodes (in $state$ 1, 2, and 3) transmit their respective informations. It can be observed that as time proceeds, there will be more nodes which could have decoded both the informations. This is because there is a greater possibility that the receiving node receives information by two different groups; either by the nodes which have information of a single source or by the nodes which already have information from both sources. In any case, the receiving nodes will have two different informations, i.e., from both the sources. Hence more number of nodes will have both information. As the hop count increases, the number of nodes in $state$ 2 increases until it reaches a maximum value.

The power received for node j at a level n ($n \geq 2$) from either G_1 or G_3 is given as

$$P^{(n)}_{r_{i,j}} = \sum_{\xi^{(n-1)}_k} \frac{P_t \mu_{i,j}}{d^{\beta}_{i,j}}, \tag{3}$$

where $k \in \{1, 3\}$ and $\xi^{(n-1)}_k$ denotes the set of indices of nodes which are in $state$ 1 and 3, respectively. Maximal ratio combining (MRC) is used at a receiving node, therefore, the signal-to-noise ratio (SNR) at the receiver will be the sum of the individual SNRs of signals received from the relaying nodes. MRC will be used for the node, which has either transmitted I_1 or I_2. This is a multiple-input single-ouput (MISO) link as multiple nodes are transmitting towards a single receiver. This creates a virtual MISO system. The outage probability of a MISO system is calculated as [3]

$$P_{o(MISO)} = \chi^{\sigma} \sum_{a_1=0}^{\infty} ... \sum_{a_{\sigma}=0}^{\infty} \frac{(\tau/P_t)^{a_1+a_2+...+a_{\sigma}+\sigma}}{(a_1+a_2+...+a_{\sigma}+\sigma)!} \prod_{i=1}^{\sigma} \Gamma\left(\frac{1+c+a_i}{c}\right)(-\chi)^{a_i}, \tag{4}$$

where σ is the number of transmitters in the previous level, i.e., $\sigma \in G_1$ or G_3. From second hop and onwards, we calculate the state distribution of all the states of the system using (2) and (4). We use (2) for the nodes in $state$ 2 because these nodes make linearly independent combinations. The outage probability of SISO link can be used to calculate the outage probability of $state$ 2 nodes beacause these nodes are transmitting different information in the form of network codewords. To understand, consider an example that at level $(n-1)$ two of the nodes are in $state$ 1, two in $state$ 2 and a single node in $state$ 3. Node j at level n can be in one out of four states depending upon its decoding. The probability of being in $state$ 0, i.e. it has not decoded any information and nothing to forward, is given as

$$\mathbb{P}\{node\ j\ in\ outage\} = \left(P_{o(MISO)} \times P^k_{o(SISO)}\right) + \left(P_{o(MISO)} \times P_{o(SISO)} \times \binom{2}{1} P_{o(SISO)} \times P_{s(SISO)}\right).$$

Probability of the node j being in state 1 is;

$$\mathbb{P}\{node\ j\ in\ state\ 1\} = \left(P_{s(MISO)} \times P^k_{o(SISO)}\right).$$

where k is the number of nodes that make SISO links with node j at the next level i.e., in the given example, $k = 3$. Similarly we can calculate $state$ 2 and $state$ 3 distributions.

To find the number of nodes of each state in a single hop, we multiply the probability of state distribution of the particular state with the total number of nodes at each hop. As we discussed that the number of nodes in $state$ 2 increases as the hop count increases, therefore, the coverage depends on the number of nodes that are in $state$ 2. The coverage is found as

$$Coverage\ at\ hop\, n = N.V^{(n)}(2), \tag{5}$$

where N is the total number of nodes and $V^{(n)}(2)$ is the state distribution probability for $state$ 2 at level n. If the value of (5) is greater than or equal to 2, then the n^{th} hop is in coverage or we can say that the message can be reliably delivered until the n^{th} hop.

III. Simulation and Numerical Results

In this section, the system performance is analyzed with the help of numerical and simulation results. The analytical and simulation matching of the state distribution is shown in Fig.

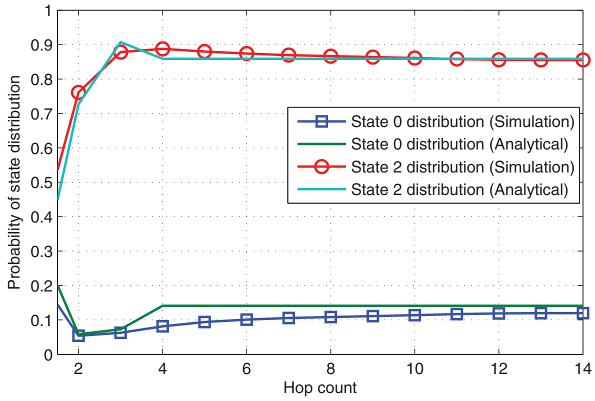

Fig. 2. States distribution for random topology; $L = 3$, $N = 8$, $P_t = 1$, $\beta = 2$ and $\gamma = 16dB$.

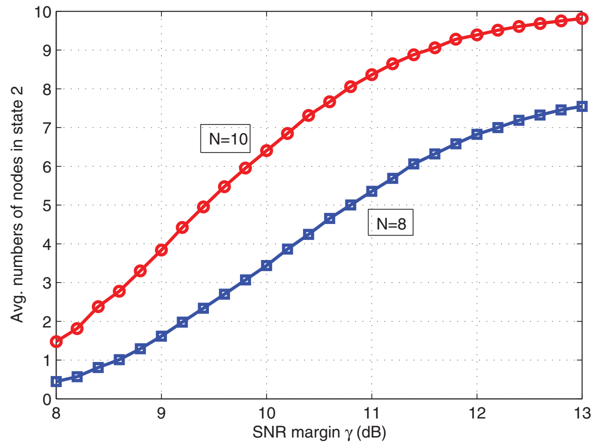

Fig. 3. Number of nodes in $state$ 2 at 5^{th} hop; $P_t = 1$, $\beta = 2$ and $L = 3$.

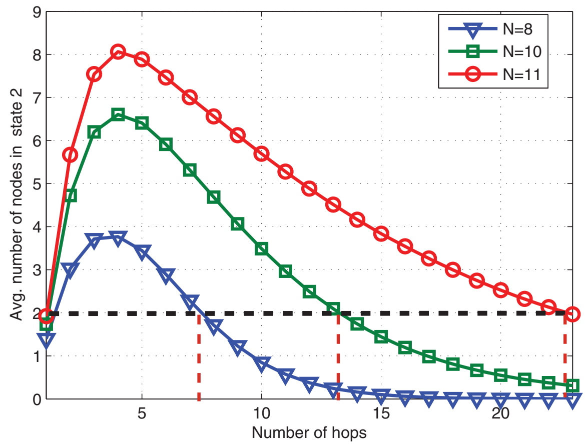

Fig. 4. Coverage of system with different N; $P_t = 1$, $\beta = 2$, $L = 3$, $\gamma = 10dB$.

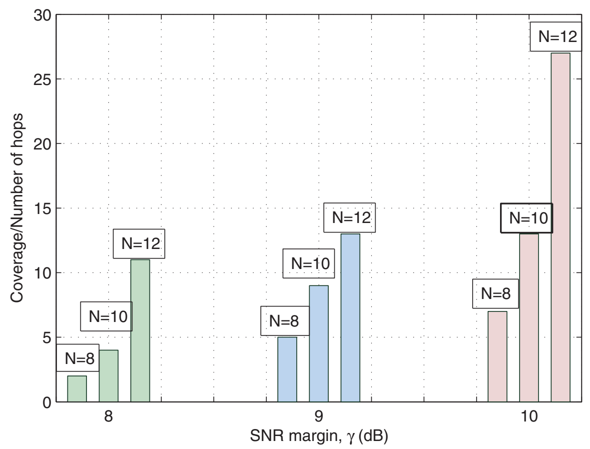

Fig. 5. Coverage for the particular values of SNR margin (γ); $P_t = 1$, $\beta = 2$, $L = 3$.

2. The distribution of $state$ 2 and $state$ 0 of both the analytical and simulation scenarios are plotted. It can be noticed that the state distribution probabilities match closely for both the cases. This plot also depicts the relationship between state distribution to the number of hops. It can be seen that as the number of hops increases, the distribution of $state$ 2 increases until a particular hop count and then it starts decreasing. Since the total number of nodes at each hop is constant, this implies that the number of nodes in $state$ 1 and 3 also decreases with time.

Fig. 3 shows the relation between the number of nodes in $state$ 2 versus the SNR margin, γ, which is the normalized SNR, i.e., $\gamma = P_t/\tau N_o$. N_o is the power spectral density of white noise. The two scenarios i.e., $N = 8$ and $N = 10$ are compared in this plot. The coverage in this network depends on the number of nodes in $state$ 2 because the number of nodes in $state$ 1 and 3 approaches to zero as we move closer to the destination. Hence we require a minimum of two nodes at each hop that are in $state$ 2 to recover both sources' informations. As we increase the SNR margin, the number of nodes in $state$ 2 increases because more nodes can decode the information of two sources. The result in Fig. 3 depicts the state of the system at the fifth hop.

In Fig. 4, we compare the coverage of the network for various values of N. In order to survive, a hop should contain two or more than two nodes, which are in $state$ 2. Therefore, the hop count with at least two nodes in $state$ 2 is the coverage of the network. From Fig. 4, it can be seen that the number of $state$ 2 nodes increases until a certain hop and then this number decreases because the probability of going in the *absorbing* state or $state$ 0 is greater than zero. Hence eventual killing of the transmissions occur. It can be seen that for $N = 8$, seven hops are in coverage, while for $N = 10$ and 12, thirteen and twenty three hops, respectively, are in coverage.

Fig. 5 decribes the relationship between the coverage and SNR margin for different values of N. From a network designer point of view, if $\gamma = 8dB$ and 4 hops are desired to be in coverage, then at least 10 nodes are required to be

deployed per hop to acheive this coverage. However, if γ is increased to $9dB$ with same $N = 10$ nodes per hop, then the coverage increases to 9 hops. A further increased γ of $10dB$ would result in 13 hops to be in coverage.
Hence the coverage depends on the N, as if destination is further away, the N should be increased for the successful delivering of message. Therefore, if we increase N for a particular value of SNR margin, the coverage is increased. The coverage also depends upon the value of L, which is used to calculate the area of square region. If the L is increased and N is fixed, the coverage is decreased as the distance between nodes of two adjacent hop increased, which eventually increases the path loss.

IV. Conclusion

In this paper, a multi-hop strip-shaped cooperative network is investigated, which uses the linear network coding to transmit information of two sources. The state distribution probabilities are calculated at each hop to find the number of nodes of each state. In this study, the nodes which decoded partial information also participate in transmission. Using the state distribution probabilities, we found the coverage and reliable delivery of the message of two sources to a far off destination. In future, we aim to extend this work with multiple sources in a random OLA network and investigate the consequences if hypothetical hop boundaries are removed.

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