On the Modeling of Randomized Distributed Cooperation for Linear Multi-Hop Networks

Syed Ali Hassan
School of EECS
National University of Sciences and Technology
H-12, Islamabad, 44000
Email: ali.hassan@seecs.edu.pk

Mary Ann Ingram
School of ECE
Georgia Institute of Technology
Atlanta, GA, 30332-0250
Email: mai@gatech.edu

***Abstract*—A one-dimensional cooperative network is modeled stochastically, such that the nodes are randomly placed according to a Bernoulli process. A discrete time quasi-stationary Markov chain model is considered to characterize the multi-hop transmissions and its transition probability matrix has been derived. By the Perron-Frobenious theorem, the eigen-decomposition of the matrix gives useful information about the coverage of the network and signal-to-noise (SNR) margin that is required for obtaining a given quality of service or packet delivery ratio. An SNR penalty for the random placement of nodes, compared to regular placement, is quantified.**

I. INTRODUCTION

For large coverage areas, wireless multi-hop transmission, both in cellular as well as in ad hoc and sensor networks, has the advantage of reduced cost of deployment, compared to the networks that have a base station or access point within one hop of every user. A conventional multi-hop network employs a path or route, which is an arrangement of point-to-point links, over which the signal propagates from the source to the destination. However, in a multi-hop route through a wireless network, each link is generally subject to receiver thermal noise and multi-path effects, causing non-negligible probability of link failure. The end-to-end probability of *success* in delivering the packet, from the source to the destination, is the product of all the link probabilities of success, and therefore the end-to-end probability of success is much lower than the link probability of success when there are many hops. A multi-hop transmission or a broadcast on a line network faces similar issues. Link layer functions, such as retransmission, may attempt to save the packet, at the cost of significant extra energy and delay [1]. These challenges are present if the nodes along the link or route are equally spaced. However, if the nodes are not equally spaced, e.g., because of mobility or random placement, there is an additional probability of very weak links, or a network partition, where a gap is so large that no single-input-single-output (SISO) link can bridge the gap. Cooperative transmission (CT) has been proposed as a means to improve link reliability or provide range extension, by having multiple radios transmit the same message to a receiver through uncorrelated fading channels [2]. This paper analyzes a line network that employs a simple form of CT called the opportunistic large array (OLA) [3]. We consider a kind of quantized random deployment along a line. In particular, we study the case where the potential node locations are equally spaced, but the presence or absence of a node in each location follows a Bernoulli process.

The OLA is suitable for multi-hop networks consisting of a large number of nodes or sensors that can do diversity combination in their receivers. In an OLA transmission, all radios that decode a message, relay the message together, very shortly after reception, without coordination with other relays. This process continues until the message signal reaches the destination node or broadcasted over the entire network.

A number of papers on OLA networks have appeared, owing to the aforementioned promising advantages. Most of the theory on OLA transmission has assumed a *continuum* of nodes, where the number of nodes in any given area goes to infinity while keeping the transmit power per unit area fixed [4],[5]. This assumption is appropriate for highly dense networks. The authors in [4] and [5] derived conditions under which an infinite broadcast over an OLA network is guaranteed. Most results are supported by Monte Carlo simulation because of the analytical intractability. In [6], the authors introduced the quasi-stationary Markov chain approach for analysis of a line network of equally spaced decode-and-forward nodes, which form consecutive OLAs. In [7], the authors studied a linear cooperative network with co-located groups of nodes, where each group forms an OLA at a certain time. In this paper, we extend the above approaches of finite density networks to the case in which the nodes are randomly deployed over a line, according to a Bernoulli process. As in [6], the channel model includes path loss with an arbitrary exponent, and independent Rayleigh fading. The new formulation allows us to quantify the SNR penalty for random placement of nodes, relative to the regular placement case, for various granularities of placement possibilities. In contrast to [6], there is the new possibility of too few or no nodes in a local area.

The rest of the paper is organized as follows. In the next section, we define the network parameters. Section III proposes a model of the network while Section IV derives the transition probability matrix for the proposed model. The results are given in Section V and the paper then concludes in Section VI.

978-1-4577-2053-6/12/$31.00 ©2012 IEEE

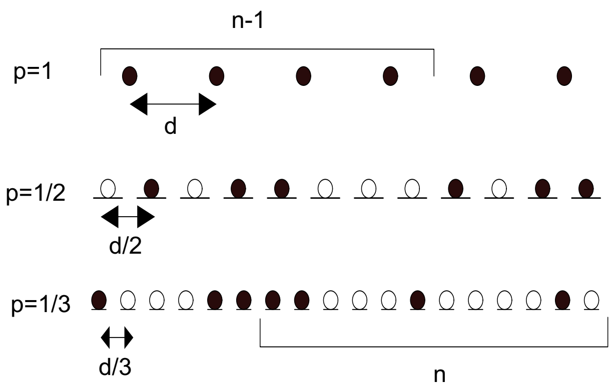

Fig. 1. Deterministic and random placement of nodes

II. System Model

As shown in Fig. 1, our deployment model is to place nodes according to a Bernoulli process on equally spaced candidate locations, such that at most one node can be placed at a location. In other words, for every candidate location, a Bernoulli random variable $\mathcal{B}$ has the outcome $\mathcal{B} = 1$ with probability p if a node is present, and $\mathcal{B} = 0$ with probability $1 - p$, if the node is not present. We wish to compare line networks with the same average density of nodes, but with different degrees of randomness and spatial granularity. If p is a very small number, this Bernoulli deployment can be considered to be an approximation to a Poisson point process (PPP). In Fig. 1, the $p = 1$ case shows a deterministic deployment of nodes with a fixed density. We assume that the node locations are integer multiples of d, where d is the inter-node distance on the one-dimensional grid. The subsequent plots in Fig. 1 show examples of possible Bernoulli deployments with $p = 1/2$ and $p = 1/3$, respectively. The filled-in circles indicate the existence of a node while the hollow circles show the absence of a node. Thus, p can be regarded as the *granularity parameter* where smaller values of p corresponds to *higher* granularity. As $p \to 0$, the resulting deployment follows a PPP.

For all cases, we assume that cooperating nodes transmit synchronously in OLAs or levels, and that a *hop* occurs when nodes in one level transmit a message and at least one node, in the forward direction, is able to decode the message for the first time. Correct decoding is assumed when a node's received SNR at the output of the diversity-combiner, from the previous level only, is greater than or equal to a modulation-dependent threshold, τ. Exactly one time slot later, all the nodes that just decoded the message relay the message. Once a node has relayed a message, it will not relay that message again. Let $p_n(m)$ be the *membership* probability that the mth node transmits in the nth level, given that at least one node transmitted in the $(n-1)$th level. We assume that $p_n(m)$ has a finite region of support of width M. We assume that there exists some M_0 and M such that $p_n(m) \geq 0$ for $M_0 \leq m \leq M_0 + M - 1$ and $p_n(m) = 0$ otherwise. As we will show later, the quasi-stationary property implies that there exists a hop distance, h_d, such that $p_{n-1}(m-h_d) = p_n(m)$. Hence h_d can be considered as a shift to the window of size M, on the reference deterministic case. The horizontal square brackets in Fig. 1 show an example of adjacent window positions, corresponding to $M = 4$ and hop distance $h_d = 2$. We assume that all the nodes transmit with the same transmit power P_t. A node receives superimposed copies of the message signal from the nodes that decoded the message correctly in the previous level, over orthogonal fading channels using maximum ratio combining. Let us define $\mathbb{N}_n = \{1, 2, ..., k_n\}$, where k_n is the cardinality of the set $\mathbb{N}_n$ such that $\sup_n k_n \leq M$, to be the set of slot indices of those nodes that decoded the signal perfectly at the time instant (or hop) n. The received power at the jth node at the next time instant $n + 1$ is given by

$$P_{r_j}(n+1) = \frac{P_t}{d^\beta} \sum_{m \in \mathbb{N}_n} \frac{\mu_{mj}}{|h_d - m + j|^\beta}, \tag{1}$$

where the summation is over the nodes that decoded correctly in the previous level. Here d denotes the inter-node distance for $p = 1$. For other values of granularity, we use corresponding values of distance as shown in Fig. 1. The flat fading Rayleigh channel gain from node m in the previous level to node j in the current level is denoted by μ_{mj}, where each μ_{mj} is independently and identically exponentially distributed with σ_μ^2=1; β is the path loss exponent. Consequently, the received SNR at the jth node is given as $\gamma_j = P_{r_j}/\sigma^2$, where σ^2 is the variance of the noise in the receiver. We assume perfect timing and frequency recovery at each receiver, and we also assume that there is sufficient transmit synchronization between the nodes of a level, such that all the nodes in a level transmit to the next level at the same time [8]. In other words, the transmissions only occur at discrete instants of time, $n, n+1, ...$, such that the hop number and the time instants can be defined by just one index n.

III. The Markov Model

At a certain hop number n, a node, if present at a slot, will take part in the next transmission, if it has decoded the data perfectly for the first time, or it will not take part, if it did not decode correctly or it has already decoded the data in one of the previous levels. The states of all the slots in the nth level can be represented as $\mathcal{X}(n) = [\mathbb{I}_1(n), \mathbb{I}_2(n), ..., \mathbb{I}_M(n)]$, where $\mathbb{I}_j(n)$ is the ternary indicator random variable for the jth slot at the nth time instant given and is 0 if slot j has a node, which as not decoded, 1 if slot j has a node, which has decoded, and 2 if slot j has no node or it has a node that has decoded at some earlier time. Thus, each slot in a level is represented by either 0, 1 or 2 depending upon node presence and successful decoding of the received data. We observe that the outcomes of $\mathcal{X}(n)$ are ternary M-tuples, each outcome constituting a state,and we may write

$$\begin{aligned}\mathbb{P}\{\mathcal{X}(n) = i_n | \mathcal{X}(n-1) = i_{n-1}, ..., \mathcal{X}(1) = i_1\} = \\ \mathbb{P}\{\mathcal{X}(n) = i_n | \mathcal{X}(n-1) = i_{n-1}\},\end{aligned} \tag{2}$$

where $\mathbb{P}$ indicates the probability measure. Equation (2) implies that $\mathcal{X}$ is a discrete-time finite-state Markov Process.

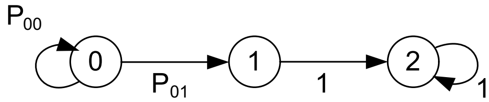

Fig. 2. The state diagram of a single slot

Absorption in the Markov chain occurs when a state comprises any combination of 0s and 2s. Hence we consider the Markov chain, $\mathcal{X}$, on a state space $\mathcal{A} \cup \mathcal{S}$, where $\mathcal{S}$ is a set of transient states and $\mathcal{A}$ is the set of absorbing states. To make the irreducible transition matrix, we strike the rows and columns corresponding to transitions to and from the absorbing states. Note that the transition matrix is square, irreducible, non-negative and right sub-stochastic. We therefore, invoke the Perron Frobenious theorem [9] which says that, there exists a unique maximum eigenvalue, ρ, such that the eigenvector associated with ρ is unique and has strictly positive entries, where $0 < \rho < 1$. Since $\forall n$, $\mathbb{P}\{\mathcal{X}(n) \in \mathcal{A}\} > 0$, eventual killing is certain. We let $T = \inf\{n \geq 0 : \mathcal{X}(n) \in \mathcal{A}\}$ denote the end of the survival time, i.e., the time at which killing occurs. It follows then,

$$\mathbb{P}\{T > n + m | T > n\} = \rho^m, \tag{3}$$

while the quasi-stationary distribution of the Markov chain is given as [10]

$$\lim_{n \to \infty} \mathbb{P}\{\mathcal{X}(n) = j | T > n\} = u_j, \quad j \in S. \tag{4}$$

IV. THE TRANSITION PROBABILITY MATRIX

Let i and j denote a pair of states of the system such that $i, j \in \mathcal{S}$, where each i and j are the decimal equivalents of the ternary words formed by the set of indicator random variables. To determine the possible destination states in a transition from level $n-1$ to level n, it is helpful to distinguish between two mutually exclusive sets of nodes in the nth level: 1) the nodes that were also in the M-slot window of the $(n-1)$th level, i.e., nodes that are in the $M - h_d$ overlap region of the two consecutive windows, and 2) the remaining h_d nodes that are not in the overlap region. We denote these two sets of nodes as $\mathbb{N}_{OL}^{(n)}$ and $\overline{\mathbb{N}}_{OL}^{(n)}$, respectively, where *OL* stands for *overlap*.

Suppose node k in $\mathbb{N}_{OL}^{(n)}$ decoded in the previous $(n-1)$th level; this would be indicated by $\mathbb{I}_{h_d+k}(n-1) = 1$. This node will not decode again, and therefore $\mathbb{I}_k(n) = 2$. Similarly, if that node decoded prior to the $(n-1)$th level, or if there were no node in the kth slot of $(n-1)$th level, then $\mathbb{I}_{h_d+k}(n-1) = 2$. In this case also, we must have $\mathbb{I}_k(n) = 2$. Alternatively, if the node is present and has not previously decoded, then $\mathbb{I}_{h_d+k}(n-1) = 0$, and $\mathbb{I}_k(n)$ can equal 0 or 1, depending on the previous state and the channel outcomes; $\mathbb{I}_k(n) = 2$ is not possible. If the location k is in the $\overline{\mathbb{N}}_{OL}^{(n)}$, then there is no previous level index for this node, and, we can have $\mathbb{I}_k(n) \in \{0, 1, 2\}$ depending on the node presence, previous state and channel outcomes. Hence from this discussion, we note that a slot can have three possible states. Hence each individual slot is a state machine, and $\mathbb{I}_k(n)$ is generally a non-homogeneous Markov chain itself; the probabilities of transition for a single node are non-zero only at certain times. This slot Markov chain is depicted in Fig 2.

Let the superscript on an indicator function show the state associated with that indicator function. For example, if $i = \{22110\}$, then $\mathbb{I}_5^{(i)}(n) = 0$. Therefore, considering the above discussion, the one-step transition probability going from the State i in Level $n-1$ to State j in Level n is always 0, $\forall k = \{0, 1, 2, ..., M\}$, when either is true:

$$\textit{Condition I}: \quad \mathbb{I}_k^{(j)}(n) \in \{0, 1\} \text{ and } \mathbb{I}_{h_d+k}^{(i)}(n-1) \in \{1, 2\}$$

$$\textit{Condition II}: \quad \mathbb{I}_k^{(j)}(n) = 2 \text{ and } \mathbb{I}_{h_d+k}^{(i)}(n-1) = 0.$$

In the following, we assume that the previous state is a transient state, i.e., $\mathcal{X}(n-1) \in \mathcal{S}$. For each node $k \in \mathbb{N}_{OL}^{(n)}$, the probability of being able to decode at time n given that the node exists but failed to decode in the previous level is given as

$$\begin{aligned}\mathbb{P}\left\{\mathbb{I}_k^{(j)}(n) = 1 \,|\, \mathbb{I}_{h_d+k}^{(j)}(n-1) = 0, \mathcal{X}(n-1)\right\} = \\ \mathbb{P}\left\{\gamma_k(n) > \tau \,|\, \mathbb{I}_{h_d+k}^{(j)}(n-1) = 0, \mathcal{X}(n-1)\right\}.\end{aligned} \tag{5}$$

If $k \in \overline{\mathbb{N}}_{OL}^{(n)}$, and $\mathcal{V}_{\overline{OL}}$ is the cardinality of set $\overline{\mathbb{N}}_{OL}^{(n)}$, then we define a sequence of Bernoulli random variables $\mathcal{B}(n)$ such that $\mathcal{B}(n) = \left\{\mathcal{B}_1(n), \mathcal{B}_2(n), ..., \mathcal{B}_{\mathcal{V}_{\overline{OL}}}(n)\right\}$, and also denote the event $\left\{\mathbb{I}_{h_d+k}^{(j)}(n-1) = 0, \mathcal{X}(n-1), \mathcal{B}_k(n) = 1\right\}$ as ξ, then

$$\begin{aligned}\mathbb{P}\left\{\mathbb{I}_k^{(j)}(n) = 1 \,|\, \mathbb{I}_{h_d+k}^{(j)}(n-1) = 0, \mathcal{X}(n-1)\right\} = \\ \mathbb{P}\{\gamma_k(n) > \tau \,|\, \xi\} \, \mathbb{P}\{\mathcal{B}_k(n) = 1\},\end{aligned} \tag{6}$$

$$\mathbb{P}\left\{\mathbb{I}_k^{(j)}(n) = 2, \mathcal{B}_k(n) = 0\right\} = \mathbb{P}\{\mathcal{B}_k(n) = 0\}, \tag{7}$$

and

$$\mathbb{P}\{\gamma_k(n) > \tau | \xi\} = \int_\tau^\infty p_{\gamma_k|\xi}(y) dy. \tag{8}$$

$p_{\gamma_k|\xi}(y)$ is the conditional probability density function (PDF) of the received SNR at the kth node, conditioned on state $\mathcal{X}(n-1)$ and the node existing but not having decoded yet, and is given by the hypoexponential distribution [6]. For the formulation of one step transition probability from State i to State j, define

$$\mathbb{N}_{\theta_{OL}}^{(j)} = \left\{k : k \in \mathbb{N}_{OL}^{(n)}, \mathbb{I}_k^{(j)}(n) = \theta\right\}, \quad \theta \in \{0, 1\},$$
$$\mathbb{N}_{\theta_{\overline{OL}}}^{(j)} = \left\{k : k \in \overline{\mathbb{N}}_{OL}^{(n)}, \mathbb{I}_k^{(j)}(n) = \theta\right\}, \quad \theta \in \{0, 1, 2\},$$

and let $\mathcal{V}_\theta$ be the cardinality of $\mathbb{N}_{\theta_{\overline{OL}}}^{(j)}$, $\theta \in \{0, 1, 2\}$. Then the one step transition probability for going from state i to state j is 0 if Conditions I or II holds and otherwise it is given as

$$\begin{aligned}\mathbb{P}_{ij} = \prod_{k \in \mathbb{N}_{1_{OL}}^{(j)}} \left(\psi_m^{(k)}\right) \prod_{k \in \mathbb{N}_{1_{\overline{OL}}}^{(j)}} \left(\psi_m^{(k)}\right) (p)^{\mathcal{V}_1} (1-p)^{\mathcal{V}_2} \times \\ \prod_{k \in \mathbb{N}_{0_{OL}}^{(j)}} \left(1 - \psi_m^{(k)}\right) \prod_{k \in \mathbb{N}_{0_{\overline{OL}}}^{(j)}} \left(1 - \psi_m^{(k)}\right) (p)^{\mathcal{V}_0}\end{aligned} \tag{9}$$

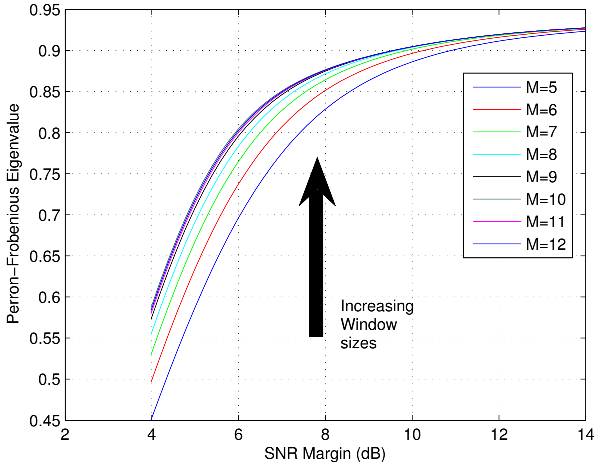

Fig. 3. Behavior of success probabilities with the increase in window size for a mean hop distance of 2

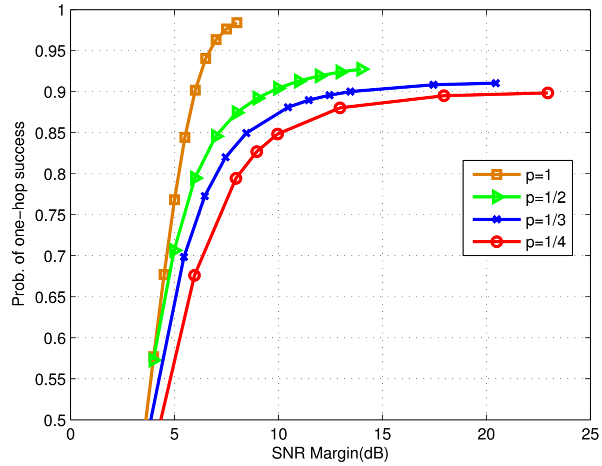

Fig. 4. Success probabilities for a mean hop distance of 2 and various granularity levels

where $\psi_m^{(k)}$ is the probability of success of Node k in Level n and is given as

$$\psi_m^{(k)} = \sum_{m \in \mathbb{N}_{n-1}} C_m^{(k)} \exp\left(-\lambda_m^{(k)} \tau\right), \tag{10}$$

where $\mathbb{N}_{n-1} = \left\{m : \mathbb{I}_m^{(i)}(n-1) = 1\right\}$ was previously defined as the set of the slot indices of all those nodes that decoded the data perfectly in the previous level, $\lambda_m^{(k)}$ is given as

$$\lambda_m^{(k)} = \frac{d^\beta \left|h_d - m + k\right|^\beta \sigma^2}{P_t}, \tag{11}$$

and $C_m^{(k)}$ is defined as

$$C_m^{(k)} = \prod_{\zeta \neq m} \frac{\lambda_\zeta^{(k)}}{\lambda_\zeta^{(k)} - \lambda_m^{(k)}}. \tag{12}$$

V. Results and System Performance

In Section III, we showed how the transition matrix is fully characterized by its Perron eigenvalue and the corresponding left eigenvector, which gives the quasi-stationary distribution of the chain. However, the Perron eigenvalue, which is the one-OLA-hop success probability in our case, depends upon many parameters like transmit power, path loss exponent, inter-slot distance etc. Hence an infinite number of solutions can be obtained by changing the values of these parameters. To decrease the design space dimension, we observe that the transition matrix, from (9) and (10), depends on the product $\lambda_m^{(k)}\tau$, from which we can extract the normalized parameter $\Upsilon = \frac{\gamma_0}{\tau} = \frac{P_t}{d^\beta \sigma^2}\frac{1}{\tau}$, which can be interpreted as the SNR margin at a receiver from a single transmitting node, a distance d away. However, Υ is not the only independent parameter, because β and h_d also separately impact the value of $\lambda_m^{(k)}\tau$ in (11), through the factor $|h_d - m + k|^\beta$ and $\mathbb{P}_{ij}$ also depends upon the granularity level p. In the following, we will discuss the case depicted in Fig. 1, i.e., a mean hop distance of 2 and its corresponding different network topologies with random placement and path loss exponent of 2. Similar results and analysis can be done for other topologies. The mean hop distance refers to the hop distance in the deterministic deployment. Note that the window size, M, is one of the factors that affects the values of ρ and the model space dimension. M is an *artificial* constraint because there is no real physical need for it. However, it strongly impacts the size of the state space and therefore the computational complexity of finding the success probability. Therefore, we would like for M to be as small as possible without significantly impacting the system performance results. We propose to find M by increasing the window size for a given hop distance, until the one-hop success probability (i.e., the Perron-Frobenius eigenvalue, ρ) ceases to change significantly. This implies that even if we add another slot at the forward edge of the window, there is no or little effect on the success probability because the transmissions reaching that specific slot are attenuated owing to the large path loss. Fig. 3 depicts the trend of eigenvalues as we increase the SNR margin for different window sizes and a mean hop distance of 2 with $p = 1/2$. The behavior is quite obvious that increasing SNR margin increases the probability of survival of the transmissions. It can be further noticed that for a given value of SNR margin, the curves start to converge as we increase the window size, thereby indicating that after a specific window size, even if we increase M, there is no change in the transmissions outcome. Thus we select the window size where this convergence is achieved.

To compare the SNR margin required for a given quality of service, which can be the one-hop success probability in this case, the behavior of success probability is plotted versus SNR margin for different granularity levels in Fig. 4. In all cases, the mean hop distance is 2. Thus it can be seen that a smaller SNR margin is required to get the same success probability for deterministic deployment as compared to the random deployment. For example, for 85% success probability,

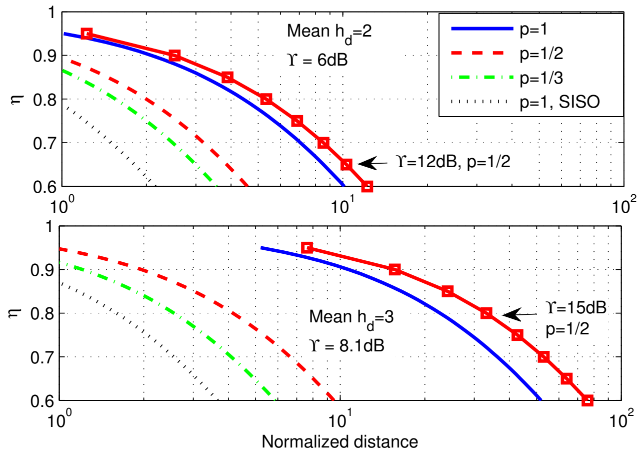

Fig. 5. Normalized distance for given quality of service with different mean hop distances. The squared-marker curves show the $p = 1/2$ case at an indicated higher SNR margin

the deterministic deployment requires 5.55dB of the SNR margin, whereas the SNR margins required are 7.1dB, 8.5dB, and 10dB for $p = 1/a$, $a = \{2, 3, 4\}$, respectively. Hence the proposed modeling gives the accurate knowledge of the success probabilities for various SNR margins and for various granularity levels.

From the deployment perspective of the network, it is sometimes desirable to determine the values of certain parameters such as transmit power of relays to obtain a certain quality of service (QoS), η. In other words, we are interested in finding the probability of delivering the message at a certain distance without having entered the absorbing state, and we desire this probability to be at least η where $\eta \sim 1$ ideally. Thus (3) gives us a nice upper bound on the value of m (the number of hops) one can go with a given η, i.e., $\rho^m \geq \eta$, which gives $m \leq \frac{\ln \eta}{\ln \rho}$. Thus if the destination is far off, we require more hops, which will require a larger value of ρ. Fig. 5 shows the relationship between η and the normalized distance for different granularity levels. The normalized distance, which is the true distance divided by d, is defined as the product of h_d and the number of hops (made to reach the destination).The upper graph corresponds to a mean hop distance, h_d, of two, while the lower graph corresponds to $h_d = 3$. In the upper graph, the four curves described in the legend have an SNR margin, Υ, of 6 dB. One of these curves corresponds to the non-CT or SISO case, given for reference. To show how a higher SNR margin can overcome losses associated with $p < 1$, we have also plotted the square-marker curve for $\Upsilon = 12$ dB and $p = 1/2$. Similarly, the lower graph has four curves with $\Upsilon = 8.1$ dB and one square-marker curve with $\Upsilon = 15$ dB and $p = 1/2$. We observe that for a fixed SNR margin (i.e., not including the square-marker curves) more random deployment or higher granularity implies a shorter distance can be covered for a given QoS. At low QoS, e.g., $\eta = 0.7$, the network is able to reach a particular distance with different granularity levels. Whereas, at high QoS, e.g., 0.9, the highest granularities are not possible. The $p = 1$ case always gives the best coverage for all values of the quality of service. The square-marker curves show that increasing the SNR margin can compensate the "random placement loss." We notice that the deterministic SISO topology has very small coverage as compared to any cooperative topology with or without random deployment. We point out that retransmissions would increase the reliability of the SISO case, as well as the CT cases. However, retransmissions are quite challenging to implement in OLA-based networks, therefore we have assumed no retransmissions in this paper.

VI. Conclusion

We have shown that the quasi-stationary Markov chain model can be applied to a one-dimensional ad hoc network, where the nodes can be randomly placed. We have modeled the presence or absence of a node at a location using the Bernoulli process and formulated the state space of our system using indicator random variables. The behavior of the Perron-Frobenius eigenvalue indicated the level of quality of service that can be achieved for a certain transmission. We have shown the random placement of nodes implies a disadvantage in terms of SNR as compared to deterministic placement for the same success probability. However, this loss can be compensated by increasing the SNR margin.

References

[1] Y. J. Chang, H. Jung, and M. A. Ingram, "Demonstration of an OLA-based cooperative routing protocol in an indoor environment," in *Proc. 17th European Wireless Conf.*, Vienna, Austria, April 2011.

[2] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, "Cooperative diversity in wireless networks: Efficient protocols and outage behavior," *IEEE Trans. Inf. Theory*, vol. 50, pp. 3062-3080, Dec. 2004.

[3] A. Scaglione and Y. W. Hong, "Opportunistic large arrays: cooperative transmission in wireless multi-hop ad hoc networks to reach far distances," *IEEE Trans. Signal Processing*, vol. 51, no. 8, pp. 2082-92, Aug. 2003.

[4] B. S. Mergen and A. Scaglione, "A continuum approach to dense wireless networks with cooperation," in *Proc. IEEE INFOCOM*, pp. 2755-2763, March 2005.

[5] A. Kailas and M. A. Ingram, "Analysis of a simple recruiting method for cooperative routes and strip networks," *IEEE Trans. Wireless Commun.*, vol. 9, no. 8, pp. 2415-2419, Aug. 2010.

[6] S. A. Hassan and M. A. Ingram, "A quasi-stationary Markov chain model of a cooperative multi-hop linear network," *IEEE Trans. Wireless Commun.*, vol. 10, no. 7, pp. 2306-2315, July 2011.

[7] S. A. Hassan and M. A. Ingram, "The benefit of co-locating groups of nodes in cooperative line networks," *IEEE Commun. Letters*, vol. 16, no. 2, pp. 234-237, Feb. 2012.

[8] Y. J. Chang and M. A. Ingram, "Cluster transmission time synchronization for cooperative transmission using software defined radio," in *IEEE (ICC) Workshop on Cooperative and Cognitive Mobile Networks*, May 2010.

[9] C. D. Meyer, *Matrix Analysis and Applied Linear Algebra*. SIAM publishers, 2001.

[10] E. A. van Doorn and P. K. Pollett, "Quasi-stationary distributions for reducible absorbing Markov chains in discrete time," *Journal of Markov Processes and Related Fields*, vol. 15, no. 2, pp. 191-204, 2009.

[11] A.N. Langville and W.J. Stewart, "The Kronecker product and stochastic automata networks," *Elsevier Journal of Computational and Applied Mathematics*, vol. 167, no.2, pp.429-447, 2004.