Maximum likelihood SNR Estimation for Non-Coherent FSK-based Cooperative Networks over Correlated Rayleigh Fading Channels

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***Abstract*—This paper addresses the problem of signal-to-noise ratio (SNR) estimation for a virtual multi-input single-output (MISO) communication system employing non-coherent M-ary frequency shift keying (NCMFSK) modulation scheme. The transmitted signals from L different nodes undergoing correlated Rayleigh fading with additive white Gaussian noise (AWGN) are combined at a single receiving node via equal gain combining (EGC) scheme. Maximum likelihood (ML) estimation technique is used for deriving the closed-form expressions for data aided (DA) and non-data aided (NDA) estimators. Cramer-Rao bound (CRB) has also been derived to evaluate the performance of the derived estimators. Numerical results have been shown for various parameters such as number of transmitting nodes, modulation order, and varying number of symbols**

***Index Terms*—Maximum likelihood SNR estimation, correlated Rayleigh fading, MISO, FSK.**

I. Introduction

Signal-to-noise ratio (SNR) is one of the key performance metrics in analyzing the performance of a wireless communication system. SNR estimates are used at the receiver side for the purpose of symbol decoding, power control algorithms, turbo decoding, and deploying adaptive modulation and coding (AMC) schemes. In wireless sensor networks (WSN), it serves a major role in finding candidate cooperators in the cooperative communication environment by letting power efficient routing. An SNR threshold is defined for relay recruitment by limiting the number of nodes participating in the data transfer, hence, making the routing process more energy-efficient [1-2].

In WSNs, the sensor nodes are highly energy constrained. Because of the impairments of the wireless channel and receiver noise, single link between two sensor nodes is proned to errors and imperfect decoding. Therefore, such scenarios are desirable in which power gain along with efficient data transfer is obtained. Cooperative transmission (CT) is a way to improve the decoding capability by providing array gain. In a multi-hop WSN, a group of nodes participate to transfer their data to the next group of nodes cooperatively [3]. Hence, for one receiving node, a virtual multiple-input single-output (MISO) is created as many sensors send the same message to this node. Due to simultaneous transmission of multiple nodes, power gain is achieved at the central receiving node if non-coherent combining techniques are employed. The data from each sensor node follows a separate channel but unlike [4] different channels might have correlation with each other due to insufficient spacing between the sensor nodes. Moreover, such a modulation scheme is required, which not only offers less complexity but also provides power efficiency. Non-coherent frequency shift keying (NCFSK) is well known for fulfilling both the criteria. Therefore, in this paper, we aim to estimate the SNR of a MISO communication system employing NCFSK modulation scheme considering correlated Rayleigh fading channels. The data arriving at the receiver from all sensor nodes is combined using equal gain combining (EGC) because of the non-coherent nature of the employed modulation scheme [5]. We assume perfect synchronization between the transmissions [6]-[7].

SNR estimation for the communication systems employing M-ary frequency shift keying (MFSK) and phase shift keying (MPSK) modulation schemes has been remained in practice by several authors in the recent past. For example, in [8], the authors have presented a comparison among BPSK and 8PSK modulation schemes considering real and complex additive white Gaussian noise (AWGN) channels, respectively. SNR estimation of non-coherent BFSK and MFSK receivers have been presented considering Rayleigh fading channels in [9]-[10]. Authors have considered data aided (DA) and non-data aided (NDA) scenarios and presented their comparison in the terms of performance for designed maximum likelihood (ML) and method of moments (MoM) estimators. In [13], the authors have designed SNR estimators for a slow fading environment. Carrier frequency offset effects have been taken into consideration while estimating SNR in [14]-[15]. SNR estimator design for NCMFSK receiver for Rice fading environment is presented in [16]. However, all of these works focus on single-input single-output (SISO) communication system, which is not applicable on WSN where multiple links are formed for data transfer.

In this paper, we consider a MISO system and derive ML estimators taking DA and NDA cases into account. Then, their performance is compared in terms of normalized mean squared error (NMSE) and Cramer-Rao bound (CRB).

The rest of the paper is organized as follows. System model is discussed in the next section while derivations of DA and NDA (ML) estimators are presented in the Section III. Section IV contains the derivation of CRB. At the end, simulation

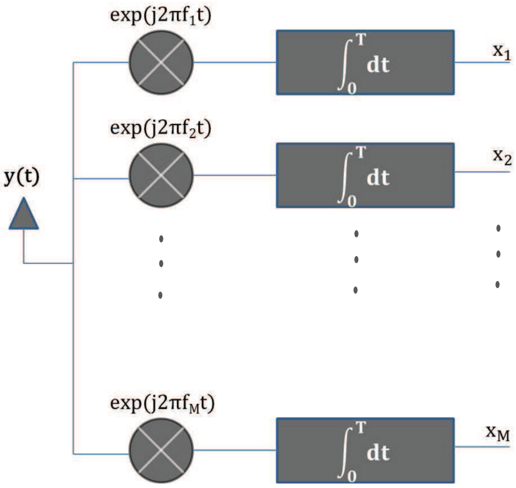

Fig. 1: Non-coherent MFSK receiver

results for the derived estimators and conclusion are discussed in Section V and Section VI, respectively.

II. System Model

Consider a communication system having L transmitting sensor nodes and a single sink or a central receiving node. Non-coherent MFSK modulation scheme is employed where each transmitted symbol undergoes correlated Rayleigh fading and is corrupted by additive white Gaussian noise (AWGN) independently. It can be noticed that transmission system creates a virtual MISO, i.e., all of the L nodes transmit the same signal $s(t)$. The received signals from L nodes are combined at the receiver using non-coherent EGC can be written as

$$y(t) = s(t)\sum_{\ell=1}^{L}\alpha^{(\ell)}(t) + n(t), \qquad 0 \le t \le T, m \in \{0,1,...,M\} \quad (1)$$

where T is the symbol period, $s(t)$ is the message signal modulated at one of the FSK frequency, $\alpha^{(\ell)}(t)$ is the Rayleigh fading envelope and $n(t)$ is the AWGN. The received data symbol x_m from M branches is obtained after quadrature demodulation of the above signal $y(t)$. The block diagram of the non-coherent MFSK receiver is shown in Fig. 7. Each x_m $\forall m = \{0,1,...,M\}$ is a complex number and can be written as

$$x_m = s_m \sum_{\ell=1}^{L}\left(\alpha_c^{(\ell)} + j\alpha_s^{(\ell)}\right) + \left(n_{c_m} + jn_{s_m}\right), \quad (2)$$

where the subscript m=$\{0,1,...,M\}$ represents the respective branch of the non-coherent MFSK receiver. Conventionally, a complex number η is written as $\eta = \eta_c + j\eta_s$, with j=$\sqrt{-1}$. Thus

$$x_m = x_{c_m} + jx_{s_m}, \quad (3)$$

where x_{c_m} and x_{s_m} are inphase and quadrature components of the symbol received at m^{th} branch of the receiver. Since Rayleigh fading is considered, the elements of the channel gain, $\alpha^{(\ell)}$, and the additive white Gaussian noise, n_m, are drawn from zero mean complex Gaussian distribution with variances of $S/2$ and $N/2$ per real dimensions, respectively. Additionally, we assume that all the channels are not independent from each other, therefore, correlation between any two different channels, i.e., $\alpha^{(i)}$ and $\alpha^{(j)}$ is represented by ρ_{ij}, where $i,j \in \{1,2,...,L\}$. However, x_m, s_m and n_m are the elements of the vector $\mathbf{x_{M\times 1}}$, $\mathbf{s_{M\times 1}}$ and $\mathbf{n_{M\times 1}}$ respectively. Thus at one time instant the received data is represented as $\mathbf{x}$=$[x_1, x_2, ..., x_M]^T$, where T is the transpose operator. In this paper, we are interested in estimating the average SNR of the received complex data vector, that is

$$\mathbf{x} = \mathbf{x_c} + \mathbf{j}\mathbf{x_s}, \quad (4)$$

Hence, $\mathbf{x}_{\mathbf{c}(M\times 1)}$ and $\mathbf{x}_{\mathbf{s}(M\times 1)}$ can be viewed as two zero mean real Gaussian random vectors

$$\begin{bmatrix} x_1 \\ \vdots \\ x_M \end{bmatrix} = \begin{bmatrix} x_{c_1} \\ \vdots \\ x_{c_M} \end{bmatrix} + j\begin{bmatrix} x_{s_1} \\ \vdots \\ x_{s_M} \end{bmatrix}, \quad (5)$$

with covariance matrices $\mathbf{K}_{\mathbf{cc}(M\times M)}$ and $\mathbf{K}_{\mathbf{ss}(M\times M)}$ and cross-covariance matrices $\mathbf{K}_{\mathbf{cs}(M\times M)}$ and $\mathbf{K}_{\mathbf{sc}(M\times M)}$. Thus

$$\mathbb{E}\left[\mathbf{x_c}\mathbf{x_c}^T\right] = \mathbf{K_{cc}}, \mathbb{E}\left[\mathbf{x_s}\mathbf{x_s}^T\right] = \mathbf{K_{ss}}, \mathbb{E}\left[\mathbf{x_c}\mathbf{x_s}^T\right] = \mathbf{K_{cs}}, \quad (6)$$

Furthermore, $\mathbf{x}$ is assumed to be proper random vector with vanishing pseudocovariance matrix [11]. That is

$$\mathbb{E}\left[(\mathbf{x_c} + j\mathbf{x_s})(\mathbf{x_c} + j\mathbf{x_s})^T\right] = 0, \quad (7)$$

which implies

$$\mathbf{K}_{cc} = \mathbf{K}_{ss}, \mathbf{K}_{cs}^T = -\mathbf{K}_{cs}. \quad (8)$$

We also assume that there is no correlation between the inphase and quadrature parts of any two different channel gains, i.e., $\mathbb{E}\left\{\alpha_c^{(i)}\alpha_s^{(j)}\right\}$=0. Thus $\mathbf{K_{cs}}$=$\mathbf{K_{sc}}$=0. However, correlation among inphase-inphase and quadrature-quadrature is assumed to be present, and the correlation coefficient between two channels is given as

$$\frac{\mathbb{E}\left\{\alpha_c^{(i)}\alpha_c^{(j)}\right\}}{\sigma_{\alpha_c^{(i)}}\sigma_{\alpha_c^{(j)}}} = \frac{\mathbb{E}\left\{\alpha_s^{(i)}\alpha_s^{(j)}\right\}}{\sigma_{\alpha_s^{(i)}}\sigma_{\alpha_s^{(j)}}} = \rho_{ij}, \quad (9)$$

where $\sigma_{\alpha_c{(i)}}$ and $\sigma_{\alpha_s^{(i)}}$ represent the standard deviations of the inphase and the quadrature components respectively, related to channel i. Hence, the covariance matrix $\mathbf{K}$ of the complex vector $\mathbf{x}$=$\mathbf{x_c} + j\mathbf{x_s}$ is only a function of $\mathbf{K}_{cc}$=$\mathbf{K}_{ss}$, i.e.,

$$\mathbf{K} = \begin{bmatrix} \mathbf{K_{cc}} & 0 \\ 0 & \mathbf{K_{cc}} \end{bmatrix}, \quad (10)$$

In this paper, we are interested in estimating the average SNR of received data using different estimation schemes. These schemes are discussed in the forthcoming section.

III. Estimation Techniques

A. Data Aided MLE

In pilot-assisted or data aided estimation, the objective is to estimate the average SNR of the received data $\mathbf{x}$ by transmitting the known data symbols from the transmitter. The L sensor nodes transmit the same data vector, i.e., $\mathbf{s}$=$[s_1 = 1, s_2 = 0, \cdots, s_M = 0]$. Here, s_1=1 and $s_m = 0$ for m=$\{2, 3, \cdots, M\}$ implies that f_1 is transmitted from all nodes and that the data part will be received in the first receiver branch. However, the remaining $(M-1)$ branches contain noise. The data to be estimated follows complex a Gaussian distribution, therefore, the joint probability density function (PDF) of $\mathbf{x_c}$ and $\mathbf{x_s}$ can be written as

$$p_{\mathbf{x_c},\mathbf{x_s}}(\mathbf{x_c},\mathbf{x_s}) = \frac{1}{(2\pi)^M\sqrt{\det(\mathbf{K})}} \exp\left\{-\frac{1}{2}\begin{bmatrix}\mathbf{x_c}^T & \mathbf{x_s}^T\end{bmatrix}\mathbf{K^{-1}}\begin{bmatrix}\mathbf{x_c}\\ \mathbf{x_s}\end{bmatrix}\right\}. \quad (11)$$

Assuming the data in the first branch, $\mathbf{K}_{cc}$=$\mathbf{K}_{ss}$ can be easily computed using (6) as

$$\mathbf{K}_{cc} = \begin{bmatrix} L\frac{S}{2}+\frac{N}{2}+\frac{S}{2}\Psi & 0\cdots & 0\\ 0 & \frac{N}{2}\cdots & 0\\ \vdots & \vdots\ddots & \vdots\\ 0 & 0\cdots & \frac{N}{2}\end{bmatrix}, \quad (12)$$

where $\Psi = \sum_{a=1}^{L}\sum_{b=1}^{L}\rho_{ab_{(a\neq b)}}$. Hence, using (10), we can have

$$\sqrt{\det(\mathbf{K})} = \frac{N}{2}^{M-1}\left[L\frac{S}{2}+\frac{N}{2}+\frac{S}{2}\Psi\right], \quad (13)$$

as both $\mathbf{K}$ and $\mathbf{K}_{cc}^{-1}$ are non-singular matrices, so their inverse can easily be found

$$\mathbf{K^{-1}} = \begin{bmatrix}\mathbf{K_{cc}^{-1}} & 0\\ 0 & \mathbf{K_{cc}^{-1}}\end{bmatrix}. \quad (14)$$

Therefore, we can re-write (11) as follows

$$p_{\mathbf{x_c},\mathbf{x_s}}(\mathbf{x_c},\mathbf{x_s}) = \frac{\exp\left\{-\frac{1}{2}(\mathbf{x_c}^T\mathbf{K}_{cc}^{-1}\mathbf{x_c}+\mathbf{x_s}^T\mathbf{K}_{cc}^{-1}\mathbf{x_s})\right\}}{(2\pi)^M\sqrt{\det(\mathbf{K})}}. \quad (15)$$

Similarly, for a data packet containing g independent data symbols, the likelihood function is the product of their marginal PDFs and is given as

$$L(\mathbf{x_1},\mathbf{x_2},\cdots,\mathbf{x_g};S,N) = (2\pi)^{-Mg}(\sqrt{\det(\mathbf{K})})^{-g} \exp\left\{-\frac{1}{2}\left[\frac{\sum_{i=1}^{g}|x_{1,i}|^2}{[L\frac{S}{2}+\frac{N}{2}+\frac{S}{2}\Psi]}+\frac{\sum_{i=1}^{g}\sum_{m=2}^{M}|x_{m,i}|^2}{\frac{N}{2}}\right]\right\}. \quad (16)$$

Note that $|x_{m,i}|$=$\sqrt{x_{c_{m,i}}^2+x_{s_{m,i}}^2}$. Here, the sub-indices m and i denote the receiver branch and time respectively, where m=$\{1,2,\cdots,M\}$ and i=$\{1,2,\cdots,g\}$. The log-likelihood function is given as

$$\mathbf{\Lambda}_{\mathbf{x_{m,i}}}(x_{m,i};S,N) = -Mg\log(2\pi) - g\log\sqrt{\det(\mathbf{K})} - \frac{1}{2}\left[\frac{\sum_{i=1}^{g}|x_{1,i}|^2}{[L\frac{S}{2}+\frac{N}{2}+\frac{S}{2}\Psi]}+\frac{\sum_{i=1}^{g}\sum_{m=2}^{M}|x_{m,i}|^2}{\frac{N}{2}}\right]. \quad (17)$$

We are interested in finding the individual maximum likelihood estimates of the signal power, $\hat{S}_{ML}$ and the noise power, $\hat{N}_{ML}$, because SNR expression is the ratio of these individual estimates

$$\hat{\gamma}_{DA} = \frac{\hat{S}_{ML}}{\hat{N}_{ML}}. \quad (18)$$

By differentiating (17) with respect to S and N individually and setting their derivatives equal to zero, we get the estimate of the signal power

$$\hat{S}_{ML} = \frac{(M-1)\sum_{i=1}^{g}|x_{1,i}|^2 - \sum_{i=1}^{g}\sum_{m=2}^{M}|x_{m,i}|^2}{g(M-1)(L+\Psi)}, \quad (19)$$

and the noise power

$$\hat{N}_{ML} = \frac{\sum_{i=1}^{g}\sum_{m=2}^{M}|x_{m,i}|^2}{g(M-1)}. \quad (20)$$

Now put the value of (19) and (20) in (18) to get the final expression of SNR given as

$$\hat{\gamma}_{DA} = \frac{(M-1)\sum_{i=1}^{g}|x_{1,i}|^2 - \sum_{i=1}^{g}\sum_{m=2}^{M}|x_{m,i}|^2}{(L+\Psi)\sum_{i=1}^{g}\sum_{m=2}^{M}|x_{m,i}|^2}. \quad (21)$$

B. Non-Data Aided MLE

In NDA estimation, we do not have any prior knowledge of the transmitted message symbol. Every individual symbol has equal chances to be received at the receiver. The conditional PDF of the received symbol given f_n frequency was transmitted, i.e., 1 at the n^{th} position of vector s is given as

$$p_{\mathbf{x_c},\mathbf{x_s}}(\mathbf{x_c},\mathbf{x_s}|s_n=1) = \frac{1}{M(2\pi)^M(\det\sqrt{\mathbf{K}})} \exp\left\{-\frac{1}{2}\left[\frac{|x_n|^2}{[L\frac{S}{2}+\frac{N}{2}+\frac{S}{2}\Psi]^2}+\frac{\sum_{m=1,m\neq n}^{M}|x_m|^2}{[\frac{N}{2}]}\right]\right\}. \quad (22)$$

There are M different possibilities of receiving the data in M receiver branches. Hence by using the law of total probability, we can write the joint unconditional PDF of the received data is given as

$$p_{\mathbf{x_c},\mathbf{x_s}}(\mathbf{x_c},\mathbf{x_s}) = \frac{1}{M(2\pi)^M(\det\sqrt{\mathbf{K}})} \left[\exp\left\{-\left(\frac{|x_1|^2}{[S(L+\Psi)+N]^2}+\frac{\sum_{m=2}^{M}|x_m|^2}{N}\right)\right\}+\cdots + \exp\left\{-\left(\frac{|x_M|^2}{[S(L+\Psi)+N]^2}+\frac{\sum_{m=1}^{M-1}|x_m|^2}{N}\right)\right\}\right]. \quad (23)$$

In order to simplify the above expression, we can factor out the common term $\exp\left\{\frac{-\sum_{m=1}^{M}|x_m|^2}{N}\right\}$ such that

$$p_{\mathbf{x_c},\mathbf{x_s}}(\mathbf{x_c},\mathbf{x_s}) = \frac{1}{M(2\pi)^M(\det\sqrt{\mathbf{K}})} \exp\left\{\frac{-\sum_{m=1}^{M} \mid x_m \mid^2}{N}\right\} \sum_{m=1}^{M} \exp\left\{- \mid x_m \mid^2 \Phi\right\}, \quad (24)$$

where $\Phi=\left[\frac{1}{S(L+\Psi)+N} - \frac{1}{N}\right]$. Now generalizing the joint PDF for a packet of g independent symbols, we can write the likelihood function as

$$L(\mathbf{x_1},\mathbf{x_2},\cdots,\mathbf{x_g};S,N) = (M)^{-g}(2\pi)^{-Mg}\det(\mathbf{K})^{-g/2} \prod_{i=1}^{g}\exp\left\{\frac{-\sum_{m=2}^{M} \mid x_{m,i} \mid^2}{N}\right\}\sum_{m=1}^{M}\exp\left\{- \mid x_{m,i} \mid^2 \Phi\right\}, \quad (25)$$

and log-likelihood function becomes

$$\mathbf{\Lambda_{x_{m,i}}}(x_{m,i};S,N) = -Mg\log(2\pi) - g\log\det(\sqrt{\mathbf{K}}) - \frac{\sum_{i=1}^{g}\sum_{m=2}^{M} \mid x_{m,i} \mid^2}{N} + \sum_{i=1}^{g}\log\left[\sum_{m=1}^{M}\exp\left\{- \mid x_{m,i} \mid^2 \Phi\right\}\right]. \quad (26)$$

Taking partial derivative of (26) with respect to S and N individually, setting these derivatives equal to zero and solving them simultaneously result in the following non-linear equations

$$\hat{S}(L+\Psi) + M\hat{N} = \frac{1}{g}\sum_{i=1}^{g}\sum_{m=1}^{M} \mid x_{m,i} \mid^2, \quad (27)$$

and

$$\sum_{i=1}^{g}\sum_{m=1}^{M} \mid x_{m,i} \mid^2 - g(M-1)\hat{N} = \frac{\sum_{i=1}^{g}\sum_{m=1}^{M} \mid x_{m,i} \mid^2 \exp(- \mid x_{m,i} \mid^2\Phi)}{\sum_{m=1}^{M}\exp(- \mid x_{m,i} \mid^2\Phi)}, \quad (28)$$

Finding a closed-form solution for the estimates of S and N is quite complex due to the presence of non-linear term, $\frac{\sum_{i=1}^{g}\sum_{m=1}^{M}|x_{m,i}|^2\exp(-|x_{m,i}|^2\Phi)}{\sum_{m=1}^{M}\exp(-|x_{m,i}|^2\Phi)}$ in (28). In order to make the derivation simpler and to get a closed-form solution, we approximate this term for higher SNR values. Consider the expression for the case of $M=2$, i.e., let

$$A = \sum_{i=1}^{g}\frac{\mid x_{1,i} \mid^2 \exp(- \mid x_{1,i} \mid^2\Phi) + \mid x_{2,i} \mid^2 \exp(- \mid x_{2,i} \mid^2\Phi)}{\exp(- \mid x_{1,i} \mid^2\Phi) + \exp(- \mid x_{2,i} \mid^2\Phi)}, \quad (29)$$

It can be observed that, if $S \gg N$, then Φ reduces from $\Phi=\left[\frac{1}{S(L+\Psi)+N} - \frac{1}{N}\right]$ to $\Phi\approx\left[-\frac{1}{N}\right]$. Hence, using this Φ in (29), we get the expression

$$A = \sum_{i=1}^{g}\left[\frac{\mid x_{1,i} \mid^2\exp(\frac{|x_{1,i}|^2}{N}) + \mid x_{2,i} \mid^2\exp(\frac{|x_{2,i}|^2}{N})}{\exp(\frac{|x_{1,i}|^2}{N}) + \exp(\frac{|x_{2,i}|^2}{N})}\right], \quad (30)$$

Rearranging the above equation, we get

$$A = \sum_{i=1}^{g}\left[\frac{\mid x_{1,i} \mid^2}{1+\frac{\exp(\frac{|x_{2,i}|^2}{N})}{\exp(\frac{|x_{1,i}|^2}{N})}} + \frac{\mid x_{2,i} \mid^2}{1+\frac{\exp(\frac{|x_{1,i}|^2}{N})}{\exp(\frac{|x_{2,i}|^2}{N})}}\right]. \quad (31)$$

Now suppose, f_1 is transmitted, i.e., the data is transmitted for the first branch of receiver and the second branch contains noise. For higher SNR values, $S \gg N$, the expressions $1+\frac{\exp(\frac{|x_{1,i}|^2}{N})}{\exp(\frac{|x_{2,i}|^2}{N})} \to \infty$ and $1+\frac{\exp(\frac{|x_{2,i}|^2}{N})}{\exp(\frac{|x_{1,i}|^2}{N})} \to 1$. Hence, the non-linear term gets finally transformed into the following approximation

$$A \approx \sum_{i=1}^{g}\max_{m=1,\cdots,M} \mid x_{m,i} \mid^2. \quad (32)$$

Using (32) in (28), we get the estimate for noise power, $\hat{N}$, that is

$$\hat{N} = \frac{\sum_{i=1}^{g}\sum_{m=1}^{M} \mid x_{m,i} \mid^2 - \sum_{i=1}^{g}\max_{m=1,\cdots,M} \mid x_{m,i} \mid^2}{g(M-1)} \quad (33)$$

Using (33) in (27) to get the estimate for signal power, $\hat{S}$. The final expression for non-data aided SNR, $\hat{\gamma}_{NDA}$=$\frac{\hat{S}}{\hat{N}}$ is given as

$$\hat{\gamma}_{NDA} = \frac{-\sum_{i=1}^{g}\sum_{m=1}^{M} \mid x_{m,i} \mid^2 + MA}{M(L+\Psi)\left[\sum_{i=1}^{g}\sum_{m=1}^{M} \mid x_{m,i} \mid^2 - A\right]}. \quad (34)$$

IV. Cramer-Rao Lower Bound

In this section, we find the Cramer-Rao bound (CRB) for the performance evaluation of the derived estimators. We will derive the CRB for data aided case and judge its performance by comparing it with normalized mean squared error (NMSE) of the estimator. Here, we have two unknown parameters, i.e., the signal power, S and the noise power, N. We can represent them in the form of an unknown parameter vector, i.e., $\boldsymbol{\theta} = [S \quad N]^T$. Thus the CRB for vector parameter is given as [12]

$$CRB = \frac{\partial g(\boldsymbol{\theta})}{\partial\boldsymbol{\theta}}\mathbf{I}^{-1}(\boldsymbol{\theta})\frac{\partial g(\boldsymbol{\theta})}{\partial\boldsymbol{\theta}}^T, \quad (35)$$

where $g(\boldsymbol{\theta}) = \frac{S}{N}$ is the function of unknown parameter $\boldsymbol{\theta}$ and $\mathbf{I}$ is the Fisher information matrix. Taking partial derivative of the function $g(\boldsymbol{\theta})$ with respect to $\boldsymbol{\theta}$, we get

$$\frac{\partial g(\boldsymbol{\theta})}{\partial\boldsymbol{\theta}} = \left[\frac{1}{N} \quad \frac{-S}{N^2}\right]^T, \quad (36)$$

The Fisher information matrix (FIM) is given as

$$\mathbf{I}(\boldsymbol{\theta}) = \begin{bmatrix} -\mathbb{E}\left(\frac{\partial^2\Lambda_{DA}}{\partial S^2}\right) & -\mathbb{E}\left(\frac{\partial^2\Lambda_{DA}}{\partial SN}\right) \\ -\mathbb{E}\left(\frac{\partial^2\Lambda_{DA}}{\partial NS}\right) & -\mathbb{E}\left(\frac{\partial^2\Lambda_{DA}}{\partial N^2}\right)\end{bmatrix}, \quad (37)$$

where Λ_{DA} is given in (17) and $\mathbb{E}$ is the expectation operator. Solving the above matrix, we have $\mathbf{I}(\boldsymbol{\theta})$, given as

$$\mathbf{I}(\theta) = \begin{bmatrix} \frac{g(L+\Psi^2)}{(S(L+\Psi)+N)^2} & \frac{g(L+\Psi)}{(S(L+\Psi)+N)^2} \\ \frac{g(L+\Psi)}{(S(L+\Psi)+N)^2} & \left(\frac{g}{(S(L+\Psi)+N)^2} + \frac{g(M-1)}{N^2}\right)\end{bmatrix}, \quad (38)$$

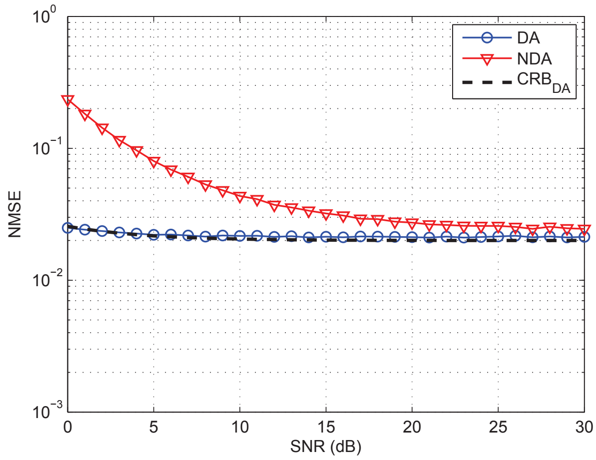

Fig. 2: NMSE for g=100 symbols, M=2, L=5 and $\rho = 0.3$

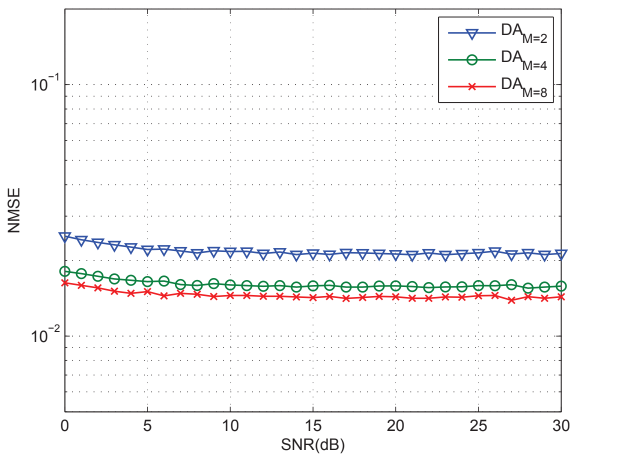

Fig. 3: Effect of increasing M on NMSE of Data aided estimator for g=100 symbols, $\rho = 0.3, L = 2$

which results in CRB from (35) as

$$CRB_{DA} = \frac{M}{g(M-1)}\left[\gamma^2 + \frac{2\gamma}{\Psi} + \frac{1}{\Psi}\right]. \qquad (39)$$

V. Simulation Results

In this section, the performance of the derived estimators has been presented in terms of normalized mean squared error (NMSE) values. Different trends for NMSE versus SNR have been analyzed on the basis of several system parameters, i.e., number of transmitting nodes, L, number of receiver branches, M and different number of symbols, g. All the results shown in this section are averaged over 25000 trials of simulations.

Fig. 2 presents the NMSE versus SNR for the derived DA and NDA estimators for the case of L=5, M=2, g=100 symbol size and for constant correlation value between every channel, i.e., ρ_{ij}=0.3 $\forall i, j$. It can be observed from the figure that for lower SNR region, data aided (DA) estimator outperforms the non-data aided (NDA) estimator. This difference in NMSE is due to the approximation made in NDA estimator for the case of higher SNR values. Additionaly, DA estimator is showing the minimum possible variance as it gives exactly the same NMSE values throughout the whole SNR region as that of CRB.

In Fig. 3, the effect of increasing the receiver branches, i.e., M on NMSE value has been analyzed for DA estimator for L=2 transmitting nodes and correlation value of ρ_{ij}=0.3 $\forall i, j$. While keeping all the other parameters constant, it can be observed that NMSE is decreased throughout the whole SNR region as M is increased. It is due to the fact that the estimated sample mean approaches the actual mean as we go on increasing the data samples. By increasing the receiver branches, M and transmitting nodes, L, we are indirectly increasing the number of data samples, which result in decreased NMSE value. Same trend is followed by the NDA estimator. It can be observed from the Fig. 4 that the NMSE for DA estimator, is decreased by increasing the number of transmitting nodes, L. This improved performance of the estimator is again due to the reason of a larger data set.

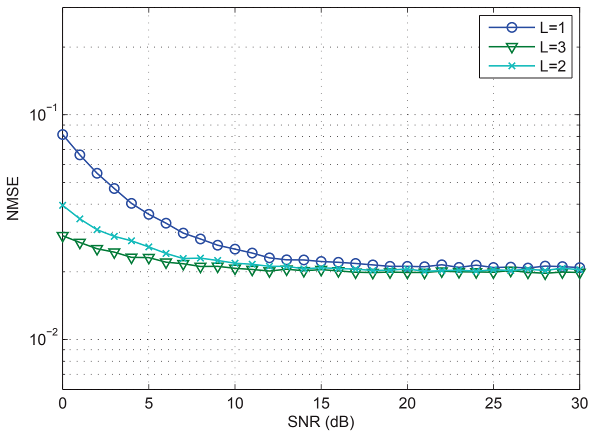

Fig. 4: Effect of increasing L on NMSE of data aided estimator for g=100 symbols, $\rho = 0.3, M = 2$

NMSE contours for DA estimator can be seen in the Fig. 5 for $\rho = 0.2$, M=4 and L=3. The effect of varying symbol sizes and SNR on the NMSE values has been analyzed in it. It can be observed that there is a decreasing trend in NMSE values as both the symbol size or SNR are increased. So, for such scenarios, where smaller NMSE value is needed, packets with larger number of symbols may be chosen and vice versa. For example, We can see from the figure that NMSE=0.003 can be achieved at the low SNR value of 5dB by choosing symbol size of 540, similarly, high SNR value of 20dB can be estimated with same NMSE by making g=470.

VI. Conclusion And Future Work

We have derived the closed form expressions for data aided and non-data aided SNR estimators using maximum

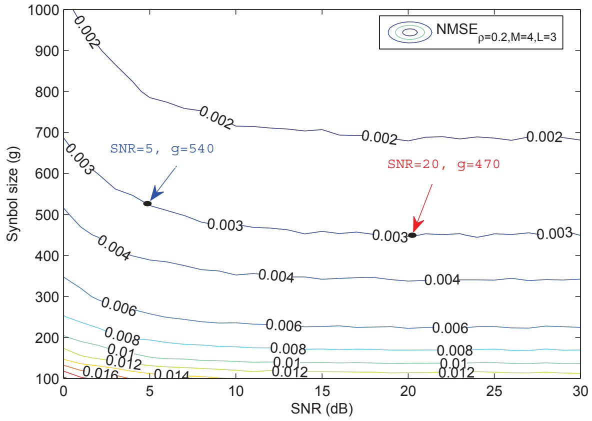

Fig. 5: NMSE contours of Data aided estimator for g=100 to 1000 symbol size, $\rho = 0.2, M = 4, L = 3$

likelihood estimation technique, which are applicable for a MISO communication system employing non-coherent M-ary frequency shift keying (NCMFSK) modulation. We have assumed correlated Rayleigh fading channels with additive white Gaussain noise (AWGN). Cramer-Rao bound (CRB) has been derived for evaluating the performance of designed estimators. On the basis of numerical analysis done in the previous section, we have found that the performance of the data aided estimator in low SNR regions is best in all the cases in comparison with that of non-data aided estimator. The performance for both the estimators can be enhanced by increasing the number of transmitting nodes, modulation order and number of symbols. However, both of the estimators perform equally well with larger number of transmitting nodes and higher correlation values at high SNR values. All the values of channel correlation are assumed to be known in this work. However, it is recommended that the channel correlation values may be estimated on the basis of received data and it is left as a future extension to this work.

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