Wireless One-Shot Polling of a Cluster of Sensors using Transmit Diversity

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***Abstract*—This paper considers the polling of a sensor network using a binary integration scheme. Binary integration is the combination of binary decisions from multiple sensors into a single decision at the base station. The proposed approach accomplishes binary integration in the physical layer in just two packet intervals regardless of the number of sensors, as long as the sensors are within the decoding range of the collector. We assume independent Rayleigh or Ricean links from the sensors to the collector. In the proposed scheme, the sensors simultaneously transmit their signals in each channel of a set of orthogonal channels to create diversity. The statistics of the squared envelopes of the received signals, in both line-of-sight and non line-of-sight channels, are used to perform hypothesis testing using the Neyman-Pearson criteria. It has been shown through the receiver operating characteristic (ROC) curves that the detection probability strongly depends upon the number of diversity channels available for transmission.**

***Index Terms*—Binary integration, polling of sensors, Hypothesis testing, Neyman-Pearson test, Kolmogorov–Smirnov test.**

I. Introduction

Wireless and ad hoc sensor networks have gained popularity in the past decade owing to a multitude of benefits they offer such as low cost, easy deployment, and energy-efficient operations. A basic purpose of deploying a sensor network is to gather information from a specific environment and to use this information to build a smart system. However, data fusion or polling the sensors, specifically in a wireless setting, is a challenging task. Binary integration or binary polling characterizes the process of a collector or a base station (BS) making a binary detection decision using individual decisions from a cluster of sensors [1]. Applications include an airborne collector communicating with a cluster of sensors on the ground or a BS or access point (AP) operating in uplink to receive data from a cluster of sensors, as illustrated in Fig. 1.

The conventional binary polling strategy is to communicate all sensor decisions, one at a time, to the collector or BS, which indicates a detection if at least M out of N sensors have detected the event. For instance, [2] and [3] deal with the variation of this problem in distributed detection scenarios but no channel-based solutions have been studied in these papers. The threshold for detecting an event is found optimally by considering both the individual binary sensors as well as the receiver, however, ignoring the channel conditions. Similarly, in [4] and [5], a slight variation is performed in the binary integration problem to achieve time and energy efficiency. In [4], sensors are ranked according to their detecting values depending on the local thresholds for the individual sensors and a subset of sensors are allowed to transmit to the collector. In [5], only the sensor that has the highest detection value transmits its decision to the receiver.

The previous works in the literature do not consider the wireless channel between the sensors and the BS and/or they pay a price in terms of energy and delay by requiring medium access control (MAC) layer strategies that include channel sensing, listening, and duty cycling of transmissions [6]-[7]. Similar works can be found in [8]-[10], however, the authors either revert to MAC layers, or they don't exploit the feature of diversity in wireless systems. In this paper, we study the problem of binary polling of sensors using a cooperative approach where all sensors transmit their decision to the central BS in one-shot, i.e., at the same time, using diversity channels as in orthogonal frequency division multiple access (OFDMA). We assume both scenarios where the sensors form independent line-of-sight (LOS) or non line-of-sight (NLOS) links with the BS. We employ the Neyman-Pearson (NP) test for simple hypothesis testing to decide in favor of event having happened or not, conditioned on the sensor decisions. The paper then analyzes the receiver operating characteristics (ROCs) to assess the detection performance for both LOS and NLOS channels.

II. System Model

This section presents the system model considered in this paper for binary polling where we assume N sensors report their decisions on whether an event happened. We assume the sensors are deployed in an area as shown in Fig 1. In Fig 1(a), an airborne collector receives the information from the sensors on the ground. Similarly, in Fig 1(b), the BS receives the information of a co-located cluster of sensors on the ground. In the BS scenario, the channel model consists of either the NLOS or LOS propagation medium. The number of sensors that decide in the favor of the occurrence of event is S_1 and the number of sensors that decide that an event has not occurred is S_2.

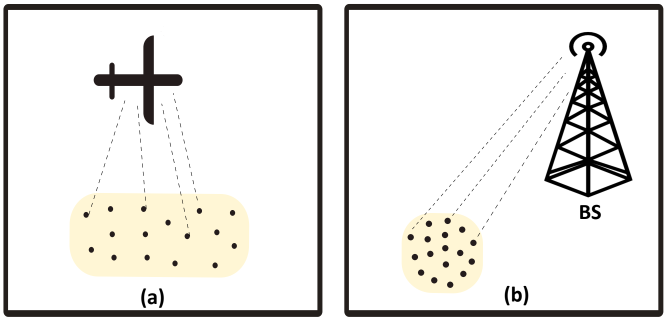

Fig. 1: (a) An airborne collector receiving the information from the sensors on ground (b). BS receiving the information from a co-located cluster of sensors

Under the proposed polling scheme, we assume our polling packet to be consistent with an OFDM symbol that contains K orthogonal sub-carriers. A sensor that has detected an event transmits in each of the K_1 orthogonal channels that compose the detection band, $\mathbb{D}$, whereas a sensor that has not detected the event transmits in each of a separate set of K_2 orthogonal channels that compose the non-detection band, $\mathbb{ND}$, such that $K_1 + K_2 = K$. The sensors' transmissions can be scheduled based on a trigger packet sent by the collector or BS [11] or scheduled for simultaneous transmission, if the network is synchronous. Let $|R_k|^2$ be the squared envelope of the received signal in orthogonal channel k, expressed as

$$|R_k|^2 = |G_k + W_k|^2, \tag{1}$$

where G_k is the sum of the complex gains of the sensors' signals in the k^{th} channel and W_k is the noise term for channel k. The elements of W_k are independent and identically distributed (i.i.d) complex Gaussian random variables (RVs) with zero mean and variance σ_n^2. Let $|R_D|^2$ be the sum of squared envelopes of the K_1 receiver branches in the detection band, and similarly $|R_{ND}|^2$ be the sum of the squared envelopes of the K_2 branches in the non-detection band, expressed as

$$|R_D|^2 = \sum_{k\in\mathbb{D}} |R_k|^2 \quad \text{and} \quad |R_{ND}|^2 = \sum_{k\in\mathbb{ND}} |R_k|^2. \tag{2}$$

After receiving the signals from the sensors, the collector makes a detection decision by comparing a threshold with the ratio of the likelihoods of $|R_D|^2$ and $|R_{ND}|^2$. We denote the probability of detection as P_D and the probability of false alarm as P_{FA}.

III. Statistics of the Received Signals

In this section, we derive the statistics of the received signals for both LOS and NLOS channels as given below.

A. Non Line-Of-Sight (NLOS) Channel

In the NLOS case, when S_1 sensors are transmitting in each sub-carrier, the G_k in (1) is modeled as a zero mean complex Gaussian RV, implying its squared envelope, $|G_k|^2$, is given as exponential with mean γS_1, i.e., $G_k \sim \mathcal{CN}(0, \gamma S_1)$, $|G_k|^2 \sim \exp(\gamma S_1)$, where γ denotes the signal power of each sensor. Similarly from (1), the squared envelope $|R_k|^2$ becomes an exponential RV, i.e., $|R_k|^2 \sim \exp(\gamma S_1 + \sigma_n^2)$, thereby implying a signal-to-noise ratio (SNR) of $\gamma S_1/\sigma_n^2$ for the kth channel. From (2), the squared envelopes $|R_D|^2$ and $|R_{ND}|^2$ are, therefore, gamma distributed each having shape parameters K_1 and K_2 with scale parameters $\sigma_1^2 = \gamma S_1 + \sigma_n^2$ and $\sigma_2^2 = \gamma S_2 + \sigma_n^2$, respectively. We assume that G_k and G_j for $j \neq k$, are i.i.d. This assumption is justified if the K_i sub-carriers have a minimum separation of at least the coherence bandwidth or, in a flat fading channel, if each sensor puts a random phase rotation on *each* of the sub-carriers it excites and there are a sufficient number of excited sub-carriers to assume a central limit theorem approximation.

B. Line-Of-Sight (LOS) Channel

In the LOS case, we assume that all the S_1 signals in one sub-carrier are i.i.d complex Gaussian with the same non zero mean. Note that the i.i.d assumption is justified if the excited sub-carriers are separated by at least the coherence bandwidth and if each sensor adjusts its carrier phase to ensure the phase of the LOS component at the receiver is the same for all sensors. Hence, G_k becomes a complex Gaussian RV, i.e., $G_k \sim \mathcal{CN}(\mu S_1, \gamma S_1)$, where μ denotes the mean of a signal per sensor. It follows that $|G_k|$ is a Ricean RV with κ factor, $\kappa = S_1\mu^2/\gamma$. Since $W_k \sim \mathcal{CN}(0, \sigma_n^2)$, the $|R_k|^2$ becomes a non-central Chi-squared RV, with the following parameters,

$$\alpha = 2S_1\gamma - 3\Delta^2 + \Lambda + 2\sigma_n^2 + S_1^2\mu^2 - \Phi, \tag{3}$$

and

$$\beta = 3\Delta^2 - S_1\gamma - \Lambda - \sigma_n^2 - \frac{1}{2}S_1^2\mu^2 + \Phi, \tag{4}$$

where $\Delta = \frac{1}{2}(S_1\gamma + \sigma_n^2)$, $\Lambda = \frac{1}{4}\left(\frac{1}{2}S_1^2\mu^2 + S_1\gamma + \sigma_n^2\right)^2$ and $\Phi = \frac{1}{16}S_1^4\mu^4 + \frac{3}{2}S_1^2\mu^2\Delta$. It can be shown that the mean and the variance of this non-central Chi-squared RV can be given as $\alpha + \beta$ and $2(\alpha + 2\beta)$, respectively. The mean can be represented in a simplified form as $\frac{1}{2}S_1^2\mu^2 + S_1\gamma + \sigma_n^2$. From (2), the squared envelopes $|R_D|^2$ and $|R_{ND}|^2$ are each sums of non-central Chi-squared random variables. It is well known that the non-central Chi-squared distribution involves a non-linear Bessel function. Therefore, the distribution of the sum of non-central Chi-squared RVs becomes prohibitive analytically [12]. In the sequel, we approximate this sum distribution to a gamma distribution using the method of moments approach and also verify the approximation using Kolmogorov-Smirnov (K-S) test [13].

Lemma 1: *Let $U = X_1 + X_2 + ... + X_n$ be the sum of n i.i.d non-central Chi-squared RVs where each RV X_i, $\forall i \in \{1, 2, ..., n\}$ has identical parameters α and β, then U can be approximated by a gamma RV with shape parameter $\lambda = \frac{n(\alpha+\beta)^2}{2(\alpha+2\beta)}$ and scale parameter $\theta = \frac{2(\alpha+2\beta)}{\alpha+\beta}$, with distribution,*

$$f_U(u) = \frac{u^{\lambda-1}exp(\frac{-u}{\theta})}{\theta^\lambda\Gamma(\lambda)}, \tag{5}$$

Proof: The mean and variance of the sum of i.i.d. non-central Chi-squared RVs can easily be found as $n(\alpha + \beta)$

and $2n(\alpha+2\beta)$, respectively [14]. For the moment matching approach, we equate two moments of both the distributions as

$$\mathbb{E}[|R_k|^2] = n(\alpha+\beta) = \lambda\theta = \mathbb{E}[U], \quad (6)$$

$$Var[|R_k|^2] = 2n(\alpha+2\beta) = \lambda\theta^2 = Var[U]. \quad (7)$$

Solving the above equations provide us the shape parameter λ and scale parameter θ for RV U as

$$\lambda = \frac{n(\alpha+\beta)^2}{2(\alpha+2\beta)} \quad \text{and} \quad \theta = \frac{2(\alpha+2\beta)}{\alpha+\beta}. \quad (8)$$

The second step involves performing the K-S test for the goodness of fit of the distributions. The K-S test can be used to compare the sample distribution with some reference distribution [15]. The test is performed by taking the samples $(\zeta_1, \zeta_2, ...\zeta_\Omega)$ from both the distributions and the maximum distance between the cumulative distribution function (CDF) $F_1(\zeta)$ of reference distribution and empirical distribution function $F_0(\zeta)$ of sample distribution is found using these samples. The test is performed by making two hypotheses: $\mathcal{H}_0$ (null hypothesis) and $\mathcal{H}_1$ (reject hypothesis). The null hypothesis says that the samples of both the distributions are from the same distribution, i.e.,

$$\mathcal{H}_0 : F_1 = F_0, \quad (9)$$

whereas the hypothesis $\mathcal{H}_1$ rejects the null hypothesis. The maximum difference between the CDFs is given as

$$\hat{D}_f = \max_i |F_1(\zeta_i) - F_0(\zeta_i)|. \quad (10)$$

The K-S test also depends on the significance level, ε, of the test, which is defined as the probability of rejecting $\mathcal{H}_0$ given that the two distributions are the same, i.e.,

$$\varepsilon = \mathbb{P}(\hat{D}_f \geq c|\mathcal{H}_0), \quad (11)$$

where c is the critical value and is dependent on number of samples Ω as well as ε. The null hypothesis is accepted only if $\hat{D}_f \leq c$.

The K-S tests performed for different number of samples, Ω, and for sums of 100 and 1000 non-central Chi-squared RVs with $\varepsilon = 0.05$, $S_1 = 100$, $\gamma = 2$, $\mu = 4$ and $\sigma_n^2 = 1$ are given in Table I. The values of c can be found from the table given in [15]. The findings in Table I show that the value of $\hat{D}_f$ is always less than the value of c, which implies that the sum of non-central Chi-squared RVs can be well approximated as a gamma RV. Hence the distributions of $|R_D|^2$ and $|R_{ND}|^2$ are approximated as gamma distribution, with shape and scale parameters $\lambda_i = \frac{K_i(\alpha_i+\beta_i)^2}{2(\alpha_i+2\beta_i)}$, and $\theta_i = \frac{2(\alpha_i+2\beta_i)}{\alpha_i+\beta_i}$ for $i = \{1, 2\}$, respectively. ■

TABLE I: K-S test for CDF approximations.

No: of Samples (Ω)	Value of c	$\hat{D}_f$, $n = 100$	$\hat{D}_f$, $n = 1000$
3000	0.024	0.0146	0.0218
4000	0.021	0.0180	0.0103
5000	0.019	0.0172	0.0083
6000	0.017	0.0111	0.0104

IV. Neyman-Pearson Detection Tests

In this section, we perform the NP tests for both LOS as well as NLOS channels as given below.

A. NP Test for the NLOS Channel

For detection, the Neyman-Pearson method is employed, which states that the alternative hypothesis is decided if

$$\frac{p(x; \mathcal{H}_1)}{p(x; \mathcal{H}_0)} > \tau. \quad (12)$$

In (12), $\mathcal{H}_0$ is the null hypothesis and $\mathcal{H}_1$ is the alternative hypothesis and τ is the threshold. From the previous section, $p(x; \mathcal{H}_0) \sim Gamma(K_2, \sigma_2^2)$ and $p(x; \mathcal{H}_1) \sim Gamma(K_1, \sigma_1^2)$. Hence, (12) can be written as

$$\frac{\frac{x^{K_1-1}\exp\left(\frac{-x}{\sigma_1^2}\right)}{\sigma_1^{2K_1}\Gamma(K_1)}}{\frac{x^{K_2-1}\exp\left(\frac{-x}{\sigma_2^2}\right)}{\sigma_2^{2K_2}\Gamma(K_2)}} > \tau. \quad (13)$$

Further simplification provides

$$\frac{x^{K_1-1}\exp\left(\frac{-x}{\sigma_1^2}\right)\sigma_2^{2K_2}\Gamma(K_2)}{x^{K_2-1}\exp\left(\frac{-x}{\sigma_2^2}\right)\sigma_1^{2K_1}\Gamma(K_1)} > \tau. \quad (14)$$

Solving and rearranging (14), we get

$$(K_1 - K_2)ln(x) + \left(\frac{\sigma_1^2 - \sigma_2^2}{\sigma_1^2\sigma_2^2}\right)x > ln(\tau) + K_1 ln(\sigma_1^2) + ln\Gamma(K_1) - K_2 ln(\sigma_2^2) - ln\Gamma(K_2). \quad (15)$$

General Case: Solving for x, we get the general solution as

$$x > \frac{A\mathcal{W}\left(BeA^C\right)}{B} := t_1, \quad (16)$$

where $\mathcal{W}(.)$ is Lambert-W function, $A = (K_1 - K_2)$, $B = \left(\frac{\sigma_1^2-\sigma_2^2}{\sigma_1^2\sigma_2^2}\right)$ and $C = ln(\tau) + K_1 ln(\sigma_1^2) + ln\Gamma(K_1) - K_2 ln(\sigma_2^2) - ln\Gamma(K_2)$. The above equation is true only for $\sigma_1^2 > \sigma_2^2$.

Case 1a : In (15), when $K_1 = K_2$, (19) simplifies to

$$x > \frac{ln(\tau) + K ln(\sigma_1^2) - K ln(\sigma_2^2)}{\left(\frac{\sigma_1^2-\sigma_2^2}{\sigma_1^2\sigma_2^2}\right)} := t_2. \quad (17)$$

Case 1b : Similarly in (15), when $\sigma_1^2 = \sigma_2^2$, we will have a different solution for x as

$$x > \exp\left(\frac{ln(\tau) - (K_2 - K_1)ln(\sigma_1^2) + \psi}{K_1 - K_2}\right) := t_3, \quad (18)$$

where $\psi = ln\Gamma(K_1) - ln\Gamma(K_2)$. The probability of false alarm P_{FA} and the probability of detection P_D for each case can now be determined as

$$P_{FA} = \int_{t_i}^{\infty} p(x; \mathcal{H}_0)dx, \quad i \in \{1, 2, 3\} \quad (19)$$

and

$$P_D = \int_{t_i}^{\infty} p(x; \mathcal{H}_1)dx \quad i \in \{1, 2, 3\}. \quad (20)$$

B. NP Test for the LOS Channel

In the LOS case, the distributions of $|R_D|^2$ and $|R_{ND}|^2$ are approximated as gamma distributions; therefore, the Neyman-Pearson test is conducted in the same manner as in the NLOS case and follows the same steps as done in (12) to (15), where we can replace the σ_1^2 and σ_2^2 with λ_1 and λ_2 and and K_1 and K_2 with θ_1 and θ_2, respectively.

V. Results and Discussions

This section presents the numerical results obtained through NP tests for both LOS and NLOS channels. Fig. 2 provides the ROC curves for P_D versus P_{FA} for equal number of sensors transmitting in the detection and non-detection bands, or $S_1 = S_2$ such that $S_1 + S_2 = 40$, while the number of sub-carriers in the detection band is larger than or equal to the number of sub-carriers in the non-detection band, keeping the total number of diversity channels equal to 30, i.e., $K_1 \geq K_2$ and $K_1 + K_2 = 30$. If $K_1 = K_2$ and $S_1 = S_2$, the detector performance is shown by the linear line at 45^0, which is intuitive since there is no majority in the sensors' reports. However, it can be seen that if the diversity channels for detection band are increased, then we achieve a higher P_D. In other words, the graph provides an insight into the 'distortion of truth' if we increase the diversity channels only in the detection band. A Monte-Carlo Simulation test is also conducted to prove the theoretical model. It can be seen that the theoretical results have a close match with the simulation results, thereby providing the accuracy of our proposed model.

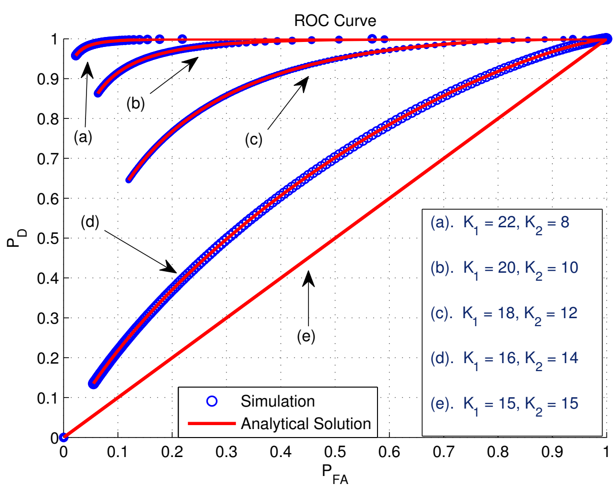

Fig. 2: The ROC curves for different values of K_1 and K_2 but for same values of S_1 and S_2, i.e., $S_1 = S_2 = 20$, at 10dB of SNR with NLOS channel

Fig. 3 represents the ROC performance for equal number of sub-carriers in both detection as well as non-detection bands, however, varying the number of sensors that report the event happening. The total number of sensors in the area are kept at 100. It can observed that as the number of sensors transmitting in the detection band increases, the P_D increases for a fixed P_{FA}. This phenomenon can be attributed to the 'majority voting' in a binary polling system where

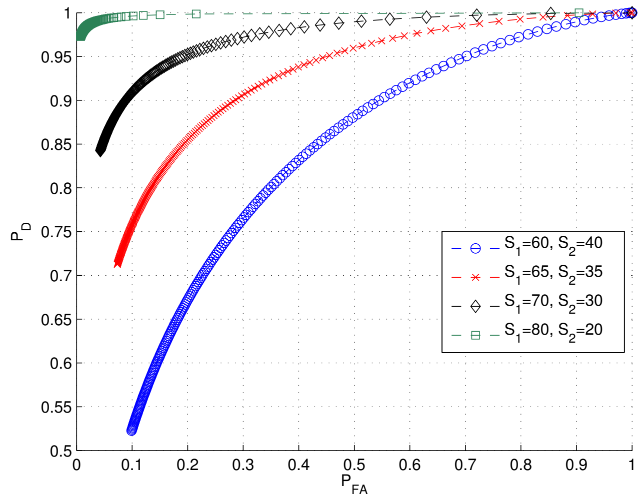

Fig. 3: The ROC curves for $K_1 = K_2 = 10$ for varying values of S_1 and S_2 at SNR of 10dB in a NLOS channel.

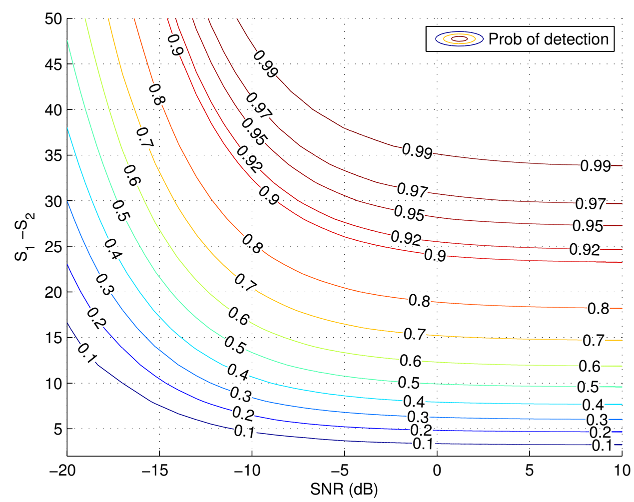

Fig. 4: Contour plot of probability of detection for varying SNR; $K_1 = K_2 = 10$, and $P_{FA} = 0.05$

the detection probability increases as the number of sensors increases.

Fig. 4 shows a contour plot of the probability of detection for a case where $K_1 = K_2 = 10$, and the total number of sensors are 50, i.e., $S_1 + S_2 = 50$, however, their difference is plotted against the SNR. The P_{FA} is kept fixed at 0.05. It can be seen that a high P_D can be achieved by keeping the difference of S_1 and S_2 large, however, operating the system at low SNRs. This is because when many sensors are transmitting in the detection band, the power gain (or array gain) improves the detection probability. Similarly, a high P_D also results when the SNR of the system is high although the margin of the majority, i.e., $S_1 - S_2$, is small. Note that the top right corner of the plot depicts unit P_D.

Fig. 5 shows the diversity effects on the performance of detection when the number of sub-carriers across both

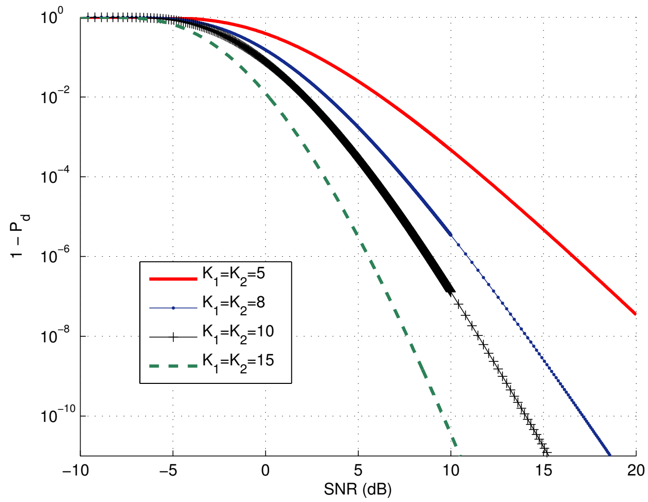

Fig. 5: The error performance vs. the SNR for varying values of K_1 and K_2; $S_1 = 5$, and $S_2 = 0$

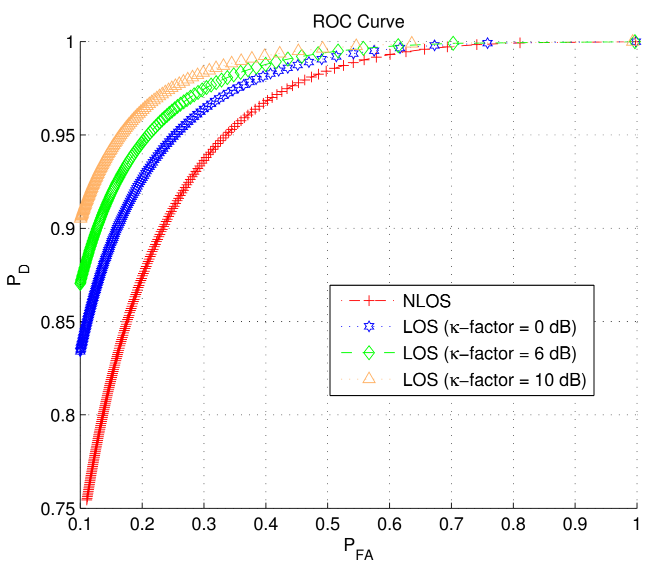

Fig. 6: The ROC curves for both LOS and NLOS channels at $K_1 = 20$, $K_2 = 10$, and $S_1 = S_2 = 200$.

detection and non-detection bands are the same, however, there are only event reporting sensors in the area. In symbols, this implies $K_1 = K_2$, $S_1 = 5$ and $S_2 = 0$. The plot shows that to get a low detection error, one must resort to increase the number of diversity channels so that the diversity gain starts to play its role. We observe the increasing slopes' characteristic of increasing diversity. For example, the limiting slope of $K_1 = K_2 = 10$ is twice that of $K_1 = K_2 = 5$.

Finally, we also provide results of ROCs for both LOS and NLOS channels at an SNR of 0dB. It can be seen from Fig. 6 that the detection performance increases as we increase the κ-factor for the LOS channels.

VI. Conclusion

This paper has explored the role of diversity in a one-shot polling scheme, given certain numbers of sensors reporting detections and non-detections, respectively. A Neyman-Pearson test has been derived assuming independent fading diversity channels with or without the line-of-sight. For example, the technique could be implemented with OFDM and could be used to reduce the latency and overhead when polling a collection of sensors. Diversity order has been shown to play a strong role, enhancing the probability of detecting the signals sent by the sensors. Ideas for future work include practical implementation in OFDM, and consideration of prior probabilities for sensor detection.

VII. ACKNOWLEDGEMENTS

The authors gratefully acknowledge the partial support of this research by the U.S. National Science Foundation (NSF) under Grant CNS-1513884 and 1343256.

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