Space Time Coding using Quaternion Algebra

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Approval

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Abstract

Code rate and decoder complexity are considered important performance metrics in space time block coding (STBC). After comprehensive study and analysis of STBCs over complex domain, quaternion orthogonal designs (QODs) have been explored that can provide less decoder complexity and higher code rate. However, some researchers argue that existing maximum likelihood (ML) decoding rule for QODs does not yield optimal performance for all QODs and therefore, the design of a generalized optimal decoupled decoder for QODs remains an open research problem of this field. To counter this observation, this study proposes a modified transmit beamforming-aided system model to realize an effective channel at the receiver that simplifies the maximum-likelihood (ML) quaternion-norm criterion and ultimately leads to low-complexity decoupled decoding for all QODs. In addition to this, this work also explores QODs of different categories. Various generalized QODs has been proposed in this work that can generate QODs for any number of transmit antennas. It is important to note that the code rate of many of these designs is higher than their respective complex orthogonal designs (CODs). Out of all the proposed generalized QODs, the conventional ML quaternion-norm-based decoder fails to provide decoupled decoding for most of them. As a solution, we apply the proposed norm criterion on the presented QODs and derive linear equation-based decoding solutions. Simulation results verify that the proposed decoder yields optimal decoupled decoding with remarkably low complexity.

Dedication

First of all, I would like to dedicate this thesis to the One Who bestows honor on whom He wills and in Whose hands lies the good. Moreover, I dedicate this thesis to my supervisor, Dr. Syed Ali Hassan. To all his efforts he put to make his students succeed.

Certificate of Originality

I hereby declare that this submission is my own work and to the best of my knowledge it contains no materials previously published or written by another person, nor material which to a substantial extent has been accepted for the award of any degree or diploma at NUST SEECS or at any other educational institute, except where due acknowledgement has been made in the thesis. Any contribution made to the research by others, with whom I have worked at NUST SEECS or elsewhere, is explicitly acknowledged in the thesis.

I also declare that the intellectual content of this thesis is the product of my own work, except for the assistance from others in the project's design and conception or in style, presentation and linguistics which has been acknowledged.

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Signature: ________________

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Chapter 1

Introduction

Surge of high speed and high data rate communication services have accelerated the demand for efficient data communication techniques having the potential to make reliable data transmissions without compromising on data rates. However, provision of significantly high data rates with restricted communication resources in a cost effective manner is considered a major research challenge of wireless communication field. Limited bandwidth resources and power limitations of user equipment (UE) and base station (BS) are one of the obstacles in the realization of efficient data communication techniques. Space time coding is one of these techniques.

1.1 Introduction of Space Time Block Codes

Space time coding requires multiple transmit antennas and one or more receive antennas. There are primarily two types of space time codes; space time trellis codes and space time block codes as represented in Fig. 1.1. Where space time trellis codes provide coding gain and transmit diversity gain, there decoders are computationally very complex. On the other hand, space time block codes provide transmit diversity with relatively low-complexity decoder, however, they do not yield coding gain [1]. In space time block codes, data is arranged in blocks to be transmitted through multiple transmit antenna streams in different timeslots [2], [3]. This method of transmission helps in eliminating the effects of fading and interference at the receiver end. Complex orthogonal space time block codes (COSTBCs) have been ex-

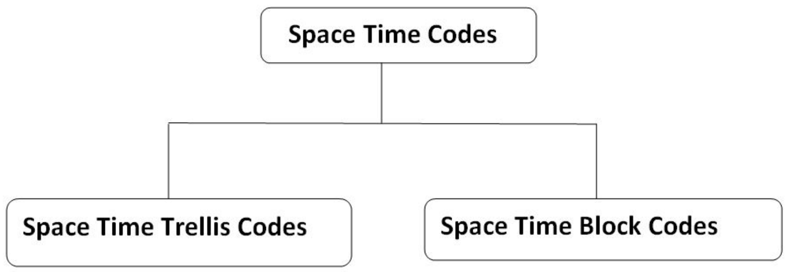

Figure 1.1: Types of Space Time Codes

tensively used in in the form of an Alamouti code in the 3G standard and wireless LANs IEEE 802.11n standard. One of the main motivations behind the use of COSTBCs has been their orthogonality property that simplifies the maximum-likelihood (ML) decoding rule by providing decoupled linear equations at the receiver end. To obtain this attractive feature of low-complexity decoder, coding matrices often compromises on the code rates, i.e., the ratio of the number of independent complex transmitted symbols and the number of total time slots taken to transmit a coding matrix. In [4], a general COD is designed for $l+1$ symbols embedded in a square matrix of order 2^l such that

$$\mathbf{A} = \begin{bmatrix} \mathbf{G}_{2^{l-1}}(z_1, z_2, .., z_l) & z_{l+1}\mathbf{I}_{2^{l-1}} \\ -z_{l+1}^{*}\mathbf{I}_{2^{l-1}} & \mathbf{G}_{2^{l-1}}^{H}(z_1, z_2, .., z_l) \end{bmatrix}, \tag{1.1}$$

where $l = \{1, 2, 3...\}$ and $\mathbf{G}_{2^{l-1}}(z_1, z_2, .., z_l)$ represents a COD of order $2^{l-1} \times 2^{l-1}$ defined on symbols $z_1, z_2, .., z_l$. For example, for $l = 1$, $\mathbf{G}_1(z_1) = [z_1]$

and

$$\mathbf{A} = \begin{bmatrix} z_1 & z_2 \\ -z_2^* & z_1^* \end{bmatrix}.$$

Likewise, for $l = 2$,

$$\mathbf{G}_2(z_1, z_2) = \begin{bmatrix} z_1 & z_2 \\ -z_2^* & z_1^* \end{bmatrix} \quad \textit{and} \quad \mathbf{A} = \begin{bmatrix} z_1 & z_2 & z_3 & 0 \\ -z_2^* & z_1^* & 0 & z_3 \\ -z_3^* & 0 & z_1^* & -z_2 \\ 0 & -z_3^* & z_2^* & z_1 \end{bmatrix}. \tag{1.2}$$

For $l = 3$,

$$\mathbf{G}_4(z_1, z_2, z_3) = \begin{bmatrix} z_1 & z_2 & z_3 & 0 \\ -z_2^* & z_1^* & 0 & z_3 \\ -z_3^* & 0 & z_1^* & -z_2 \\ 0 & -z_3^* & z_2^* & z_1 \end{bmatrix}. \tag{1.3}$$

$$\mathbf{A} = \begin{bmatrix} z_1 & z_2 & z_3 & 0 & z_4 & 0 & 0 & 0 \\ -z_2^* & z_1^* & 0 & z_3 & 0 & z_4 & 0 & 0 \\ -z_3^* & 0 & z_1^* & -z_2 & 0 & 0 & z_4 & 0 \\ 0 & -z_3^* & z_2^* & z_1 & 0 & 0 & 0 & z_4 \\ -z_4^* & 0 & 0 & 0 & z_1^* & -z_2 & -z_3 & 0 \\ 0 & -z_4^* & 0 & 0 & z_2^* & z_1 & 0 & -z_3 \\ 0 & 0 & -z_4^* & 0 & z_3^* & 0 & z_1 & z_2 \\ 0 & 0 & 0 & -z_4^* & 0 & z_3^* & -z_2^* & z_1^* \end{bmatrix}. \tag{1.4}$$

It is important to note that the full-rate COSTBCs only exist for 2 transmit antennas and maximum code rate of COSTBCs approaches half with the increase in the number of transmit antennas [4]. Various designs have been explored such as quasi-orthogonal space time block codes (QOSTBCs) to provide comparatively higher code rates but these designs result in either pair-wise or completely coupled ML decoding [5], [6].

1.2 Introduction of Quaternion Orthogonal Designs

To overcome the limited code rate performance barrier of COSTBCs, [7] proposed the use of quaternion orthogonal designs (QODs), which laid the foundation of orthogonal space time polarization block codes (OSTPBCs). It is expected that the use of STBCs alongwith other forms of diversity can further move the capacity of wireless communication systems. It is found that polarization diversity improves performance gain in terms of diversity gain by saving cost, time and space [8]- [9]. It can be nearly as efficient as spatial diversity for base station antennas without a noticeable increase in their dimensions [7]. Furthermore, it helps in realizing quaternionic structure in STBCs as a signal with two elements in two orthogonal polarizations can form a quaternion. Because of these substantial advantages, it has been utilized alongwith space and time diversity in OSTPBCs to meet higher data rate demands. Hence, space time coding using quaternion algebra has the potential to provide higher code rate designs, space and time saving benefits. In this direction, [7] proposed verious construction techniques of QODs. Out of all the construction techniques given by [7], we would be focusing on QOD construction based on CODs in this work, therefore, we would define QOD in terms of complex numbers only.

Definition 1 (QOD): A QOD $\mathbf{Q}$ on complex variables $\{z_1, z_2, ..., z_u\}$, is a $r \times n$ matrix which can have entries from the set $\{0, z_1, z_1^*, z_2, z_2^*, ..., z_u, z_u^*\}$ including possible multiplications on the left and/or right by quaternion elements $q \in \mathbf{Q}$, and satisfies the following condition,

$$\mathbf{Q}^Q\mathbf{Q} = \sum_{h=1}^{u}(s_h(|z_h|)^2)\mathbf{I}_n = \lambda\mathbf{I}_n. \tag{1.5}$$

where λ is a positive real number and $\mathbf{I}_{n\times n}$ is the $n \times n$ identity matrix.

Chapter 2

Literature Review

Recently, multiple transmit and receive antennas have been extensively exploited in the field of information theory and communication networks. In literature works [11]- [15], researchers have shown that multiple-input multiple-output (MIMO) systems have the potential to enhance the channel capacity significantly as compare to communication systems consisting of single-antennas having the same necessities of power and bandwidth. Given the significance of MIMO systems, different approaches applicable for the transmission of data through the channels of MIMO systems have been explored which includes unitary space time codes [16], [17], space time block codes [18], [19], [20], space time trellis codes [21], [22] etc. Works [23], [24] provide a detailed survey on the importance and applications of space time coding with different modulation schemes.

In addition to transmitter side where different coding algorithms are designed to achieve transmit diversity, decoding algorithms are also considered very critical for obtaining better performance results from MIMO systems. To design low-complexity decoder, Alamouti constructed a complex orthogonal code for two transmit antennas and derived linear equations based maximum-likelihood decoder for this design. Given the significance of this work, other researchers extended this approach to higher number of transmit antennas [20] and explored the advantages of these designs. Full transmit diversity and a low-complexity linear equations based ML decoder are considered the main advantages of complex orthogonal designs.

Although orthogonal coding designs provide full transmit diversity and low-complexity decoder, but they compromise on bandwidth efficiency for higher order designs. Bandwidth efficiency can be defined as ratio of the number of transmitted information symbols in a block of channel uses to the length of the given block [4]. As number of transmit antennas increases, code rate which is defined as the ratio of number of transmitted symbols to the number of timeslots, decrease and number of zeros in code blocks increase. This leads to peak-to-average-power-ratio (PAPR) problem. In [4], authors highlighted that maximum code rate of complex orthogonal block codes approaches half with the increase in the number of transmit antennas. Since Isaeva and Sarytchev presented a way to represent a signal from a dual-polarized transmit antenna as a quaternion in [25], [7] laid the foundation of OSTPBCs that utilize polarization together with space and time diversity with the help of quaternion algebra. [7] presented various QOD construction techniques also shown in Fig 2.1 and explained with an example of 2×2 order QOD that quaternion decoding statistics can provide decoupled decoding for any QOD. Their subsequent works [26]- [27] used the same QOD and emphasized the similar postulate that quaternion ML norm can provide optimal decoupled decoding for any QOD construction. Later on, the authors corrected their decoding rule in [28] and highlighted that the proposed decoding rule does not yield optimal decoding for all QODs and therefore, the design of a semi-optimal or optimal low-complexity decoder for QODs

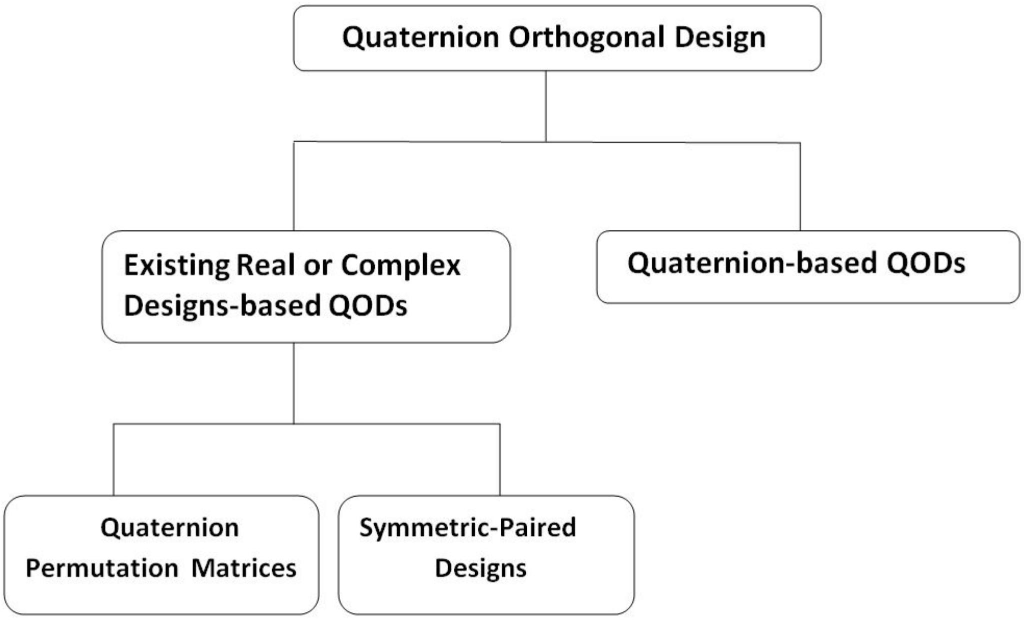

Figure 2.1: Different QOD Construction Techniques presented by [7]

remains an open research problem [29]. In addition to this, Calderbank *et al.* presented a new QOD construction method with a nonlinear technique that can achieve full rate and provides reduced complexity of ML decoding [30]. However, these approaches are restricted to small scale designs, where the number transmit and/or receive antennas is small. To construct higher order QODs, [31] has proposed quaternion coordinate interleaved orthogonal designs (QCIODs) and their decoding rule that constructs QODs for even number of antennas only. QCIODs are constructed using *isomorphism*, where smaller QODs are used to construct higher order QODs with the help of coordinate interleaved orthogonal designs (CIODs). It is important to note that there are many wireless communication applications for instance multiple-input multiple-output (MIMO) system, opportunistic relaying in wireless sensor networks etc., where number of transmit antennas may not always be an even number. As the previous works investigated even number of transmit antenna arrangements and small scale designs only, therefore, there is a need of a generalized construction technique given any number of antenna arrangement.

To address this issue, we, [32], explored the symmetric-paired QODs for higher order system designs and proposed some constraints in addition to the quaternion orthogonality and symmetricity that must be satisfied by the coding matrix to provide low-complexity decoding. However, decoupled decoding has been derived for either orthogonal or same vectors of existing STBCs in all of these works. Therefore, there has been a need of a generalized low-complexity ML decoder that can work for all QODs [29].

Given the progress on QODs, this work is the first work that seeks to solve the open research problem of decoupled decoding of QODs [29] by proposing a generalized optimal decoder. We begin by presenting various construction techniques of QODs [34], mostly the ones that do not have optimal linear decoding solutions with the previous ML decoding schemes [7], [32]. Then in the next chapter, we provide linear equations-based decoupled decoding solutions for these codes with the proposed scheme. An attractive feature of some of the presented examples of QODs is higher code rate, i.e., 1 or more than 1 code rate for 4 single-polarized transmit antennas. Besides this, we highlight the primary reason of failure of applicability of quaternion orthogonality constraint on the quaternion ML norm to provide a decoupled decoding for any QOD. As a solution, we propose a modified system model that uses the concepts from transmit beamforming with QODs to obtain a low-complexity decoder. Transmit beamforming is a multifaceted data transmission technique that enhances the signal power and signal-to-noise ratio (SNR) at the intended user and reduces interference to other users [35]. In our work, it helps in modifying the impact of a dual-polarized transmission channel at the receiver side in such a way that ML quaternion norm criterion simplifies to a decoupled linear equation-based decoding solutions which reduces the decoder complexity significantly.

Chapter 3

Quaternion Orthogonal Designs

In this chapter, we present the system model based on QODs. This includes the preliminary description of quaternion algebra and symmetric-paired designs. In addition to this, three construction techniques of generalized quaternion orthogonal designs are also provided.

3.1 System Model based on QODs

We begin with the description of some important properties of quaternions that are significant for the explanation of our work. Basically, a quaternion consists of four real numbers or two complex numbers, for example, $q = q_0 + q_1 i + q_2 j + q_3 k = z_1 + z_2 j$, where $q_o \in \mathbb{R}$; $o = \{0, 1, 2, 3\}$ and $z_p \in \mathbb{C}$; $p = \{1, 2\}$. Moreover, $i^2 = j^2 = k^2 = ijk = -1$ and $ij = k, jk = i, ki = j$ [33]. The quaternion conjugate of q can be written as $q^Q = z_1^* - jz_2^*$ and multiplication of q^Q with q gives $|q|^2$. Similarly, quaternion transpose of a matrix $\mathbf{A} = [a_{r\times n}]$ produces $\mathbf{A}^Q = [a^Q_{n\times r}]$, where $r \times n$ is the dimension of matrix $\mathbf{A}$. It is noted that the non-commutative nature of quaternions makes it difficult to generalize CODs to QODs. To construct QODs from CODs in a consistent and transparent manner, we need to define *symmetric-paired* and *complex amicable designs* defined on complex variables.

Definition 3.1. (Symmetric-Paired Designs) Two CODs $\mathbf{A}$ and $\mathbf{B}$ based on complex variables $\{z_1, z_2, ..., z_u\}$ form a symmetric-paired design $(\mathbf{A} + \mathbf{B}j)$ provided $\mathbf{A}^H\mathbf{B}$ or $\mathbf{B}^H\mathbf{A}$ is symmetric.

Definition 3.2. (Complex Amicable Designs) Two CODs $\mathbf{A}$ and $\mathbf{B}$ based on complex variables $\{z_1, z_2, ..., z_u\}$ are called complex amicable designs if $\mathbf{A}^H\mathbf{B} = \mathbf{B}^H\mathbf{A}$ and/or $\mathbf{A}\mathbf{B}^H = \mathbf{B}\mathbf{A}^H$.

We consider a MIMO transmission system with N_t dual-polarized antennas at the transmitter and one dual-polarized antenna at the receiver. The transmitting antennas transmit data using OSTPBCs, which are received by the receiving antenna in the form of quaternion vectors. The channel matrix is represented as $\mathbf{H}^{(m)} = \begin{bmatrix} h_{11}^{(m)} & h_{12}^{(m)} \\ h_{21}^{(m)} & h_{22}^{(m)} \end{bmatrix}$, where $m = \{1, 2, ..., N_t\}$. We assume a Rayleigh fading channel, therefore, each term of channel gain matrix represents a complex Gaussian random variable (RV) with zero mean and unit variance. For this $N_t \times 1$ antenna arrangement, the received signal $\mathbf{R}$ can be written as

$$\mathbf{R} = \mathcal{C}(\mathbf{Q}) \begin{bmatrix} \mathbf{H}^{(1)} \\ \vdots \\ \mathbf{H}^{(N_t)} \end{bmatrix} + \begin{bmatrix} n_{11} & n_{12} \\ \vdots & \vdots \\ n_{N_t1} & n_{N_t2} \end{bmatrix}, \tag{3.1}$$

where the entries $n_{k_1k_2}\ \forall\ k_1 = \{1, 2, ..., N_t\}, k_2 = \{1, 2\}$ denote the elements of white noise as two dimensional independent and identically distributed (i.i.d.) complex Gaussian RVs with zero mean and identical variance per dimension, while $\mathcal{C}(\mathbf{Q})$ is a complex code matrix containing transmitted symbols. $\mathbf{R}$ represents a quaternion vector, i.e., $\mathbf{R} = \mathcal{C}^{-1}\{\mathcal{C}(\mathbf{Q})\mathbf{H} + \mathbf{N}\}$, where $\mathbf{N}$ represents the noise matrix. The operator $\mathcal{C}^{-1}\{.\}$ and its counterpart $\mathcal{C}(.)$ operates on complex numbers z_1, z_2 as $\mathcal{C}^{-1}\{z_1, z_2\} = z_1 + z_2j$ and $\mathcal{C}(z_1 + z_2j) = [z_1, z_2]$.

In this paper, we will be using $\mathcal{C}^{-1}\{\mathbf{Q}\}$ and $\mathbf{Q}$ interchangeably. Moreover, $\mathcal{C}\{\mathbf{Q}\}$ and $\mathbf{C}_q$ are also used interchangeably

3.2 Construction Techniques of QODs

In this section, we present three generalized construction techniques of symmetric-paired designs.

3.2.1 Symmetric-Paired Design 1:

According to the Definition 1.2, a symmetric-paired design $\mathbf{A} + \mathbf{B}j$ yields a valid QOD only if CODs $\mathbf{A}$ and $\mathbf{B}$ satisfy the symmetry property. Symmetric-paired design 1 construction technique covers those symmetric-paired designs in which COD $\mathbf{B}$ is obtained from $\mathbf{A}$ through permutation of columns, where permutation operation on two columns i and j results in swapping the positions of these two columns with each other. It is important to note that as the dimension of COD matrix gets larger, not every permutation of columns of a COD $\mathbf{B}$ yields a valid QOD.

Essentially, $\mathbf{A}^H\mathbf{B} = (\mathbf{A}^H\mathbf{B})^T$ translates to the condition that in matrix $\mathbf{A}^H\mathbf{B}$, each $\alpha_{i,j} = \alpha_{j,i}$, where $\alpha_{i,j}$ denotes the element present at the ith row and jth column of matrix $\mathbf{A}^H\mathbf{B}$. By the definition of COD, we know that $\mathbf{A}^H\mathbf{A} = \lambda\mathbf{I}$. Therefore, if COD $\mathbf{B}$ is same as COD $\mathbf{A}$, then $\mathbf{A}^H\mathbf{B} = \lambda\mathbf{I}$. However, if permutation operation is performed on columns j and k of COD $\mathbf{B}$, then both entries, i.e., $\alpha_{j,k}$ and $\alpha_{k,j}$ of $\mathbf{A}^H\mathbf{B}$ will become λ. Afterwards, if column k gets permuted with any other column, then $\alpha_{j,k}$ and $\alpha_{k,j}$ will no longer be equal and consequently, will not satisfy symmetry property. That is why, only one permutation per column is allowed. As the diversity order of all valid designs constructed from the permutation of columns of a COD remains the same, therefore, any one of them can be picked to give a closed-form of symmetric-paired QODs. In [4], a general COD is designed for $l+1$ symbols embedded in a square matrix of order 2^l such that

$$\mathbf{A} = \begin{bmatrix} \mathbf{G}_{2^{l-1}}(z_1, z_2, .., z_l) & z_{l+1}\mathbf{I}_{2^{l-1}} \\ -z_{l+1}^*\mathbf{I}_{2^{l-1}} & \mathbf{G}_{2^{l-1}}^H(z_1, z_2, .., z_l) \end{bmatrix}, \quad (3.2)$$

where $l = \{1, 2, 3...\}$ and $\mathbf{G}_{2^{l-1}}(z_1, z_2, .., z_l)$ represents a COD of order $2^{l-1} \times 2^{l-1}$ defined on symbols $z_1, z_2, .., z_l$. For example, for $l = 1$, $\mathbf{G}_1(z_1) = [z_1]$. Moreover, $\mathbf{G}_2(z_1, z_2)$, $\mathbf{G}_3(z_1, z_2, z_3)$ and $\mathbf{G}_4(z_1, z_2, z_3, z_4)$ are given in [4]. We now use the above scheme to generate square QODs. In this construction, columns $1, 2, .., N_t/2$ of matrix $\mathbf{A}$ are swapped with $(N_t/2) + 1, (N_t/2) + 2, .., N_t$ columns, respectively, to give matrix $\mathbf{B}$, where N_t represents the number of antennas of COD on which permutation is performed. We now arrive at the following result where we have omitted the redundant argument in $\mathbf{G}$.

Theorem 3.1. *For a given COD* $\mathbf{A}$ *in (3.2), a complex amicable and symmertic-paired design can be constructed such that the following realization* $\mathbf{Q}_{2^l}(z_1, z_2, .., z_{l+1}) = \mathbf{A} + \mathbf{B}j =$

$$\begin{bmatrix} \mathbf{G}_{2^{l-1}} + z_{l+1}\mathbf{I}_{2^{l-1}}j & z_{l+1}\mathbf{I}_{2^{l-1}} + \mathbf{G}_{2^{l-1}}j \\ -z_{l+1}^*\mathbf{I}_{2^{l-1}} + \mathbf{G}_{2^{l-1}}^H j & \mathbf{G}_{2^{l-1}}^H - z_{l+1}^*\mathbf{I}_{2^{l-1}}j \end{bmatrix}, \quad (3.3)$$

provides a QOD of dimension $2^l \times 2^l$ *with rate* $(l+1)/2^l$.

Proof. We begin by proving that both $\mathbf{A}$ and $\mathbf{B}$ in (3.11) satisfy the symmetry and amicability property. We first observe that

$$\mathbf{A}_{2^l}^H \mathbf{B}_{2^l} = \begin{bmatrix} \mathbf{O}_{2^{l-1}} & 2\lambda_1\mathbf{I}_{2^{l-1}} \\ 2\lambda_1\mathbf{I}_{2^{l-1}} & \mathbf{O}_{2^{l-1}} \end{bmatrix}, \quad (3.4)$$

where $\lambda_1 \mathbf{I}_{2^{l-1}} = \mathbf{G}_{2^{l-1}}^H \mathbf{G}_{2^{l-1}} + z_{l+1} z_{l+1}^* \mathbf{I}_{2^{l-1}}$ and $\mathbf{O}_{2^{l-1}}$ is a null matrix of order $2^{l-1} \times 2^{l-1}$. It can seen from Eq. (3.4) that $\mathbf{A}^H \mathbf{B}$ is symmetric as its transpose will not change this expression. Moreover, it also satisfies complex amicable design condition as $\mathbf{B}^H \mathbf{A}$ is equal to $\mathbf{A}^H \mathbf{B}$ given in (3.4). Now, to prove quaternion orthogonality, quaternion conjugate of (3.11) can be written as $\mathbf{Q}_{2^l}^Q(z_1, z_2, .., z_{l+1}) =$

$$\begin{bmatrix} \mathbf{G}_{2^{l-1}}^H - j z_{l+1}^* \mathbf{I}_{2^{l-1}} & -z_{l+1} \mathbf{I}_{2^{l-1}} - j \mathbf{G}_{2^{l-1}} \\ z_{l+1}^* \mathbf{I}_{2^{l-1}} - j \mathbf{G}_{2^{l-1}}^H & \mathbf{G}_{2^{l-1}} + j z_{l+1} \mathbf{I}_{2^{l-1}} \end{bmatrix}. \tag{3.5}$$

Following the definition of QOD, multiplication of (3.5) with (3.11) provides

$$\mathbf{Q}_{2^l}^Q \mathbf{Q}_{2^l} = \begin{bmatrix} 2\lambda_1 \mathbf{I}_{2^{l-1}} & \mathbf{O}_{2^{l-1}} \\ \mathbf{O}_{2^{l-1}} & 2\lambda_1 \mathbf{I}_{2^{l-1}} \end{bmatrix} = \lambda \mathbf{I}_{2^l}, \tag{3.6}$$

where $\lambda = 2\lambda_1$. Hence, Theorem 3.1 is proved. □

In order to illustrate the construction, we present the following example.

Example 1. Suppose we have a rate one Alamouti code $\mathbf{G}_2 = \begin{bmatrix} z_1 & z_2 \\ -z_2^* & z_1 \end{bmatrix}$, then using (3.2) we first obtain a COD of order 4. The matrix $\mathbf{B}$ is generated using (3.11) and finally we get the following QOD for 4 dual-polarized antennas

$$\mathbf{Q} = \begin{bmatrix} z_1 + z_3 j & z_2 & z_3 + z_1 j & z_2 j \\ -z_2^* & z_1^* + z_3 j & -z_2^* j & z_3 + z_1^* j \\ -z_3^* + z_1^* j & -z_2 j & z_1^* - z_3^* j & -z_2 \\ z_2^* j & -z_3^* + z_1 j & z_2^* & z_1 - z_3^* j \end{bmatrix}.$$

With this QOD, 3 complex symbols z_1, z_2 and z_3 can be transmitted in four time slots and therefore, it provides 3/4 code rate. It is important to note that (3.11) formulates QODs for only those systems where number of dual-polarized transmit antennas can be written in a power of 2. For other dual-polarized transmit antenna arrangements, truncation of unnecessary columns provides the desired QOD.

3.2.2 Symmetric-Paired Design 2:

In the first construction scheme, the matrix **B** is obtained from **A** by permuting its columns. However, a general mechanism is required to generate quaternion designs from two arbitrary CODs. We propose a rather simple construction to generate such designs and prove that they perform relatively better than the designs obtained from the first technique. To the best knowledge of the authors, this construction has not been developed before. It maps existing CODs to QODs directly.

Lemma 3.1. *For a given square COD* $\mathbf{G}_{2^{l-1}}(z_1, z_2, .., z_{l+1})$*, the matrix* $\boldsymbol{Q}_{2^1 \times 2^{l-1}}(z_1, z_2, .., z_{l+1}) =$

$$\begin{bmatrix} \mathbf{G}_{2^{l-1}}(z_1, z_2, .., z_l) + z_{l+1}\mathbf{I}_{2^{l-1}}j \\ -z_{l+1}^*\mathbf{I}_{2^{l-1}} + \mathbf{G}_{2^{l-1}}^H(z_1, z_2, .., z_l)j \end{bmatrix} \tag{3.7}$$

provides a quaternion design of order $2^l \times 2^{l-1}$*, with rate* $(l+1)/2^l$*.*

Proof. We first note that for this design, 0 is the only element present in $\mathbf{A}^H\mathbf{B}$ as each column of matrix **A** is orthogonal to matrix **B** and vice versa. Therefore, it is a symmetric-paired and a complex amicable design. Following the steps of proof of Theorem 3.1, it is straightforward to show that

$$\mathbf{Q}_{2^{l-1}\times 2^l}^Q \mathbf{Q}_{2^l \times 2^{l-1}} = 2\lambda_1 \mathbf{I}_{2^{l-1}} = \lambda \mathbf{I}_{2^{l-1}}. \tag{3.8}$$

which complete the proof of Lemma 3.1. □

Hence, this construction also provides valid QODs as is also evident from the following example.

Example 2. Using (3.7), we get the following QOD for 2 dual-polarized antennas

$$\mathbf{Q} = \begin{bmatrix} z_1 + z_3 j & z_2 \\ -z_2^* & z_1^* + z_3 j \\ -z_3^* + z_1^* j & -z_2 j \\ z_2^* j & -z_3^* + z_1 j \end{bmatrix}.$$

With this QOD, 3 complex symbols z_1, z_2 and z_3 can be transmitted in four time slots and therefore, it provides 3/4 data rate. It is important to note that symmetric-paired design 1 provides minimum decoding delay as compare to symmetric-paired design 2. That is because design 2 transmits data in comparatively more number of time slots, for example, for 2 dual-polarized antennas, design 2 provides a QOD of order 4×2 and design 1 yields a QOD of order 2×2. Expression (3.7) generates QODs for system arrangements in which number of transmit antennas can be represented in a power of 2 and for other arrangements, truncation of unnecessary columns results in the desired QOD.

3.2.3 Symmetric-Paired Design 3:

Before providing the main construction, consider the following lemma.

Lemma 3.2 *For two generalized CODs* $\mathbf{G}_{2^l}(z_1, z_2, .., z_{l+1})$ *and* $\mathbf{L}_{2^l}(z_{l+2}, z_{l+3}, .., z_{2l+2})$ *with same structure, which are constructed on the COD formulation (3.2), it follows that*

$$\mathbf{G}_{2^l}^H \mathbf{L}_{2^l} + \mathbf{L}_{2^l}^H \mathbf{G}_{2^l} = \mathbf{G}_{2^l} \mathbf{L}_{2^l}^H + \mathbf{L}_{2^l} \mathbf{G}_{2^l}^H = \gamma \mathbf{I}_{2^l}, \tag{3.9}$$

where $\gamma = 2\Re\left(\sum_{k=1}^{l+1} z_k^* z_{l+1+k}\right)$ *and* $\Re(.)$ *represents the real part of a complex number.*

Proof. It is straightforward to show that

$$\mathbf{G}_{2^l}^H \mathbf{L}_{2^l} + \mathbf{L}_{2^l}^H \mathbf{G}_{2^l} = \mathbf{G}_{2^l} \mathbf{L}_{2^l}^H + \mathbf{L}_{2^l} \mathbf{G}_{2^l}^H = \begin{bmatrix} \mathbf{M}_{2^{l-1}} & \mathbf{O}_{2^{l-1}} \\ \mathbf{O}_{2^{l-1}} & \mathbf{N}_{2^{l-1}} \end{bmatrix}, \tag{3.10}$$

where $\mathbf{M}_{2^{l-1}} = \mathbf{E}^H\mathbf{F} + \mathbf{F}^H\mathbf{E} + (z_{2l+2}z_{l+1}^* + z_{2l+2}^* z_{l+1})\mathbf{I}$, $\mathbf{N}_{2^{l-1}} = \mathbf{E}\mathbf{F}^H + \mathbf{F}\mathbf{E}^H + (z_{2l+2}z_{l+1}^* + z_{2l+2}^* z_{l+1})\mathbf{I}$ and, $\mathbf{E}_{2^{l-1}}(z_1, z_2, .., z_l)$ and $\mathbf{F}_{2^{l-1}}(z_{l+2}, z_{l+3}, .., z_{2l+1})$ are the CODs used in the formulation of $\mathbf{G}_{2^l}(z_1, z_2, .., z_{l+1})$ and $\mathbf{L}_{2^l}(z_{l+2}, z_{l+3}, .., z_{2l+2})$, respectively, such as $\mathbf{G}_{2^{l-1}}$ is used for the construction of COD $\mathbf{A}$ given in (3.2). Moreover, $\mathbf{O}$ represents a null matrix. Now to simplify the diagonal terms of the expression (3.10), we observe that $\mathbf{E}^H\mathbf{F} + \mathbf{F}^H\mathbf{E} = \mathbf{E}\mathbf{F}^H + \mathbf{F}\mathbf{E}^H$. As both matrices $\mathbf{E}$ and $\mathbf{F}$ are constructed on the same format but on different complex numbers, therefore, for any vector $\mathbf{a}_i$ of COD $\mathbf{E}$ and its corresponding vector $\mathbf{b}_i$ of COD $\mathbf{F}$, where i denotes the index of column, we get $\mathbf{a}_i^H\mathbf{b}_i + \mathbf{b}_i^H\mathbf{a} = \sum_{k=1}^{l}(z_k)^*(z_{l+1+k}) + \sum_{k=1}^{l}(z_k)(z_{l+1+k})^*$. Consequently, diagonal terms of (3.10) equate to $\gamma\mathbf{I}_{2^l}$, hence, Lemma 3.2 is proved. □

It is important to mention that the above lemma does not hold true for two general CODs. For example, two Almouti codes with different structure $\begin{bmatrix} z_1 & z_2 \\ z_2^* & -z_1^* \end{bmatrix}$ and $\begin{bmatrix} z_3 & z_4 \\ -z_4^* & z_3^* \end{bmatrix}$, fail to satisfy it while they can be used effectively in generating a consistent COD of the form (3.2). By using the above lemma, we prove the following theorem.

Theorem 3.2. *For generalized CODs* $\mathbf{G}_{2^{l-1}}(z_1, z_2, .., z_l)$ *and* $\mathbf{L}_{2^{l-1}}(z_{1+2}, z_2, .., z_{2l+2})$, *a symmetric-paired design,*

$$\mathbf{Q}_{2^{l+1}\times 2^l}(z_1, .., z_{2(l+1)}) = \begin{bmatrix} \mathbf{G}_{2^l} + \mathbf{L}_{2^l} j \,, \\ \mathbf{L}_{2^l} + \mathbf{G}_{2^l} j \end{bmatrix}, \tag{3.11}$$

is a QOD of dimension $2^{l+1} \times 2^l$ *with rate* $(l+1)/2^l$.

Proof. In order to prove quaternion orthogonality, the quaternion conjugate of (3.11) can be expressed as $\mathbf{Q}_{2^l\times 2^{l+1}}^Q = \begin{bmatrix} \mathbf{G}_{2^l}^H - j\mathbf{L}_{2^l}^H & \mathbf{L}_{2^l}^H - j\mathbf{G}_{2^l}^H \end{bmatrix}$. We now need to multiply it with (3.11) to satisfy condition (1.5), where the first term in the product is

$$(\mathbf{G}_{2^l}^H - j\mathbf{L}_{2^l}^H)(\mathbf{G}_{2^l} + \mathbf{L}_{2^l} j) = \mathbf{G}_{2^l}^H \mathbf{G}_{2^l} + \mathbf{L}_{2^l}^H \mathbf{L}_{2^l}, \tag{3.12}$$

and where we have used the fact that $\mathbf{L}_{2^l}^H \mathbf{G}_{2^l}$ is the Hermitian conjugate of $\mathbf{G}_{2^l}^H \mathbf{L}_{2^l}$ and $zj = jz^* \; \forall \; z \in \mathcal{C}$, therefore $\mathbf{L}_{2^l}^H \mathbf{G}_{2^l} j - j\mathbf{G}_{2^l}^H \mathbf{L}_{2^l} = \mathbf{O}$. Using expression (3.12) and basic quaternion arithmetic, we obtain $\mathbf{Q}_{2^l \times 2^{l+1}}^Q \mathbf{Q}_{2^{l+1} \times 2^l} = \lambda \mathbf{I}_{2^l}$, where $\lambda_1 \mathbf{I}_{2^l} = \mathbf{G}_{2^l}^H \mathbf{G}_{2^l} + \mathbf{L}_{2^l}^H \mathbf{L}_{2^l}$ and $\lambda = 2\lambda_1$. This completes the proof of Theorem 3.2. $\square$

To elaborate the generalized construction technique, we present a QOD of rate 1, where the COD $\mathbf{A}$ contains symbols z_1 and z_2, while the COD $\mathbf{B}$ contains independent symbols z_3 and z_4, respectively.

Rate 1 QOD: Using (3.11), we obtain the following symmetric-paired design of order 4×2 with a code rate of 1, i.e.,

$$\mathbf{Q}_1 = \begin{bmatrix} z_1 + z_3 j & z_2 + z_4 j \\ z_2^* + z_4^* j & -z_1^* - z_3^* j \\ z_3 + z_1 j & z_4 + z_2 j \\ z_4^* + z_2^* j & -z_3^* - z_1^* j \end{bmatrix}, \tag{3.13}$$

where $l = 1$, $\mathbf{G}_1 = [z_1]$, and $\mathbf{L}_1 = [z_3]$ from (3.11).

It is worth-noticing that (3.11) generates QODs for only those antenna arrangements where dual-polarized transmit antennas are in powers of two, e.g., $2, 4$ and 8 etc. For other antenna arrangements, we can simply omit extra columns in the given QOD to obtain the desired code.

The above procedure of obtaining quasi-orthogonal codes through QODs is not unique and we can also generate Almouti-type codes as

$$\tilde{\mathbf{C}}_q(z_1, .., z_{2(l+1)}) = \begin{bmatrix} \mathbf{G}_{2^l} & \mathbf{L}_{2^l} \\ \mathbf{L}_{2^l}^H & -\mathbf{G}_{2^l}^H \end{bmatrix}, \tag{3.14}$$

using symmetric-paired designs $\mathbf{G}$ and $\mathbf{L}$, arising from an equivalent QOD $\tilde{\mathbf{Q}}_{2^{l+1}\times 2^l}(z_1,..,z_{2(l+1)}) = \begin{bmatrix} \mathbf{G}_{2^l} + \mathbf{L}_{2^l}j\,, \\ \mathbf{L}_{2^l}^H - \mathbf{G}_{2^l}^H j \end{bmatrix}$. This completes the QOD construction techniques chapter.

Chapter 4

Maximum Likelihood Decoder

For QODs, ML decoding rule corresponds to the minimization of either the norm $\|\mathbf{R} - \mathcal{C}^{-1}\{\mathcal{C}(\mathbf{Q})\mathbf{H}\}\|$ or its square for finding the transmitted symbols. This chapter presents the expansion and application of conventional quaternion norm-based ML decoder. Given the limitation of this decoder, transmit beamforming aided ML decoder is also provided that can work for any QOD.

4.1 Quaternion Norm-based ML Decoder

For QODs, ML decoding rule corresponds to the minimization of either the norm $\|\mathbf{R} - \mathcal{C}^{-1}\{\mathcal{C}(\mathbf{Q})\mathbf{H}\}\|$ or its square for finding the transmitted symbols. The squared norm can be given as,

$$
\begin{aligned}
&\min\big(\|\mathbf{R} - \mathcal{C}^{-1}\{C(\mathbf{Q})\mathbf{H}\}\|^2\big) = \\
&\min tr\big((\mathbf{R} - \mathcal{C}^{-1}\{\mathcal{C}(\mathbf{Q})\mathbf{H}\})^Q(\mathbf{R} - \mathcal{C}^{-1}\{\mathcal{C}(\mathbf{Q})\mathbf{H}\})\big) = \\
&\min \underbrace{tr\big(\mathbf{R}^Q\mathbf{R}\big)}_{\Gamma^{(1)}} \underbrace{-2\Re\Big(tr\big(\mathbf{R}^Q\big(\mathcal{C}^{-1}(\mathcal{C}(\mathbf{Q})\mathbf{H})\big)\big)\Big)}_{\Gamma^{(2)}} \\
&\underbrace{+tr\big(\big(\mathcal{C}^{-1}\big(\mathcal{C}(\mathbf{Q})\mathbf{H}\big)\big)^Q\big(\mathcal{C}^{-1}\big(\mathcal{C}(\mathbf{Q})\mathbf{H}\big)\big)\big)}_{\Gamma^{(3)}},
\end{aligned}
\tag{4.1}
$$

where $\Re(.)$ and $tr(.)$ signify the real part of a complex number and trace operator, respectively. In (4.1), the superscript k of $\Gamma^{(k)}$ represents the index of corresponding term in the decoding statistics, respectively. Our aim is to simplify (4.1) to design low complexity decoder. This is proven in the following lemma.

Lemma 4.1 *The ML decoding rule derived for any QOD, comprises of three real valued elements* $\Gamma^{(1)}$, $\Gamma^{(2)}$ *and* $\Gamma^{(3)}$ *corresponding to the norms of received signal vector* $\mathbf{R}$*, mixed transmitted symbol and channel coefficients vector* $\mathcal{C}^{-1}\{\mathcal{C}(\mathbf{Q})\mathbf{H}\}$*, respectively*

$$\Gamma^{(1)} = \sum_{i=1}^{t}|r_i|^2,\ \Gamma^{(3)} = \sum_{i=1}^{t}\sum_{k=1}^{2N_t}|\alpha_{ik}(\beta_{k1} + \beta_{k2}j)|^2, \tag{4.2}$$

$$\Gamma^{(2)} = -2\Re\Big(\sum_{i=1}^{t}\sum_{k=1}^{2N_t}(r_i^Q\alpha_{ik}(\beta_{k1} + \beta_{k2}j)\Big), \tag{4.3}$$

where t *refers to time slots. Moreover,* r_x*,* α_{xy} *and* β_{xy} *represent elements present in received quaternion vector, complex code matrix and channel matrix at row* x *and column* y*, respectively.*

Proof. The term $\mathcal{C}(\mathbf{Q})\mathbf{H}$ can be obtained by simple matrix multiplication and result of it will always be a matrix of $N_t \times 2$ dimension for our system model. Application of $\mathcal{C}^{-1}\{.\}$ operator on this expression converts it to a vector of order $N_t \times 1$. Likewise, it can be seen from Eq. (4.17) that $\mathbf{R}$ is a quaternion vector of length N_t and its quaternion transpose changes its order to $1 \times N_t$. Thus, multiplication of $\mathbf{R}^Q$ with $\mathcal{C}^{-1}\{\mathcal{C}(\mathbf{Q})\mathbf{H}\}$ provides only one quaternion element. With the help of basic quaternion arithmetic, it can be seen that the expression (4.3) represents the simplified form of that quaternion element. According to quaternionic algebra, multiplication of $\mathbf{x}^Q$ with $\mathbf{x}$ is equal to the sum of norm squared of each quaternion element present in the quaternion vector $\mathbf{x}$. With this rule, $\Gamma^{(1)}$ and $\Gamma^{(3)}$ come out to be the sum of norm squared of each quaternion element present in $\mathbf{R}$ and $\mathcal{C}^{-1}\{\mathcal{C}(\mathbf{Q})\mathbf{H}\}$, respectively, as given in (4.2). $\square$

Now, we derive low-complexity decoder for the symmetric-paired design constructions presented in (3.11) and (3.7).

Lemma 4.2: *The ML decoding rule for the symmetric-paired design constructions given in (3.11) and (3.7), simplifies to the following form,*

$$min\left(\|\boldsymbol{R} - \mathcal{C}^{-1}\{\mathcal{C}(\boldsymbol{Q})\boldsymbol{H}\}\|^2\right) = min\Big(\lambda\ tr(\tilde{\mathbf{H}}) + \Gamma^{(2)}\Big), \tag{4.4}$$

where $\tilde{\mathbf{H}}$ *contains mixed combinations of complex channel coefficients only.*

Proof. $\Gamma^{(1)}$ is independent of transmitted symbols and does not contribute in the norm calculation, hence, it can be neglected. Similarly, $\Gamma^{(2)}$ is a linear combination of transmitted symbols and can be decoupled easily. However, there is a need to expand $\Gamma^{(3)}$ to analyze its impact on decoding statistics. Let $\mathbf{h}_1$ and $\mathbf{h}_2$ denote the column vectors of the channel matrix, then the term $\Gamma^{(3)}$ can be simplified

$$\begin{aligned}
\Gamma^{(3)} &= tr\Big((\mathcal{C}(\mathbf{Q})\mathbf{h}_1 + \mathcal{C}(\mathbf{Q})\mathbf{h}_2 j)^Q(\mathcal{C}(\mathbf{Q})\mathbf{h}_1 + \mathcal{C}(\mathbf{Q})\mathbf{h}_2 j)\Big)\\
&\overset{(a)}{=} tr\Big(\mathbf{h}_1^H(\mathcal{C}(\mathbf{Q}))^H\mathcal{C}(\mathbf{Q})\mathbf{h}_1\Big) + tr\Big(\mathbf{h}_2^H(\mathcal{C}(\mathbf{Q}))^H\mathcal{C}(\mathbf{Q})\mathbf{h}_2\Big)\\
&\overset{(b)}{=} \lambda tr\Big(\mathbf{h}_1^H\mathbf{E}\mathbf{h}_1 + \mathbf{h}_2^H\mathbf{E}\mathbf{h}_2\Big) = \lambda\ tr(\tilde{\mathbf{H}}).
\end{aligned} \tag{4.5}$$

It is easy to prove that amicability property implies $(\mathcal{C}(\mathbf{Q}))^H\mathcal{C}(\mathbf{Q}) = \lambda\mathbf{E}$ in symmetric-paired QODs, where $\mathbf{E}$ is a symmetric matrix and contains zeros and ones only. It may not necessarily be an identity matrix. (a) follows from commutativity of complex numbers and (b) is the result of application of amicability property of the proposed symmetric-paired construction techniques. Hence, this simplification leads to the decoding rule (4.4). $\square$

The following corollary further reduces the number of calculations of the decoding rule (4.4).

Corollary 4.1. *Use of quadrature phase shift keying (QPSK) modulation for the transmission of the symmetric-paired design constructions given in (3.11) and (3.7) further simplifies the decoding rule (4.4) to*

$$\min\left(\|\mathbf{R} - \mathcal{C}^{-1}\{\mathcal{C}(\mathbf{Q})\mathbf{H}\}\|^2\right) = \min(\Gamma^{(2)}), \tag{4.6}$$

where (4.6) comes from the fact that the symbols are of equal energy and that the minimization is not affected by channel coefficients.

Now, we will use decoding rule (4.6) to decode the transmitted symbols of QODs given in Example 1 and 2.

Corollary 4.2. *The decoupled decoding statistics for each transmitted symbol z_1, z_2 and z_3 for QOD in Example 1 is*

$$\begin{aligned}
&\min_{z_1} -2\Re\{r_1^Q z_1 g_{12} + r_2^Q z_1^* g_{34} + r_3^Q z_1^* g_{56} + r_4^Q z_1 g_{78}\},\\
&\min_{z_2} -2\Re\{r_1^Q z_2 g_{34} - r_2^Q z_2^* g_{12} - r_3^Q z_2 g_{78} + r_4^Q z_2^* g_{56}\},\\
&\min_{z_3} -2\Re\{r_1^Q z_3 g_{56} + r_2^Q z_3 g_{78} - r_3^Q z_3^* g_{12} - r_4^Q z_3^* g_{34}\},
\end{aligned} \tag{4.7}$$

respectively, in which $g_{mn} = g_m + g_n j$ for all m, n. Furthermore, $g_1 = h_{11}^{(1)} + h_{21}^{(3)}$, $g_2 = h_{12}^{(1)} + h_{22}^{(3)}$, $g_3 = h_{11}^{(2)} + h_{21}^{(4)}$, $g_4 = h_{12}^{(2)} + h_{22}^{(4)}$, $g_5 = h_{21}^{(1)} + h_{11}^{(3)}$, $g_6 = h_{22}^{(1)} + h_{12}^{(3)}$, $g_7 = h_{21}^{(2)} + h_{11}^{(4)}$ and $g_8 = h_{22}^{(2)} + h_{12}^{(4)}$.

Corollary 4.3. *The decoupled decoding statistics for each transmitted symbol z_1, z_2 and z_3 for QOD in Example 2 is*

$$\begin{aligned}
&\min_{z_1} -2\Re\{r_1^Q z_1 e_1 + r_2^Q z_1^* e_2 + r_3^Q z_1^* e_3 + r_4^Q z_1 e_4\},\\
&\min_{z_2} -2\Re\{r_1^Q z_2 e_2 - r_2^Q z_2^* e_1 - r_3^Q z_2 e_4 + r_4^Q z_2^* e_3\},\\
&\min_{z_3} -2\Re\{r_1^Q z_3 e_3 + r_2^Q z_3 e_4 - r_3^Q z_3^* e_1 - r_4^Q z_3^* e_2\}
\end{aligned} \tag{4.8}$$

respectively, where $e_1 = h_{11}^{(1)} + h_{12}^{(1)} j$, $e_2 = h_{11}^{(2)} + h_{12}^{(2)} j$, $e_3 = h_{21}^{(1)} + h_{22}^{(1)} j$ and $e_4 = h_{21}^{(2)} + h_{22}^{(2)} j$.

4.2 Transmit beamforming aided ML Decoder

As a pioneering work QODs, [7] proposed that a decoupled decoding can be obtained for any QOD even if the channel gain matrix of a dual-polarized transmission system is not modeled by a single quaternion gain. Later on, they corrected their decoding rule and clarified that the decoupled decoding can be achieved for certain QODs only. However, the properties of those QODs are not elaborated in their work. We would like to highlight that while investigating the ML-decoding criterion we are confronted with three main terms

$$\|\mathbf{R} - \mathcal{C}^{-1}\{\mathbf{CH}\}\|^2 = tr\big(\mathbf{R}^Q\mathbf{R}\big) - 2\Re\Big(tr\big(\mathbf{R}^Q\mathcal{C}^{-1}\{\mathbf{CH}\}\big)\Big) + \\ + tr\Big(\big(\mathcal{C}^{-1}\{\mathbf{CH}\}\big)^Q\big(\mathcal{C}^{-1}\{\mathbf{CH}\}\big)\Big), \tag{4.9}$$

of which the first term is a constant while the second term is already decoupled. The last term, however, can be simplified to $tr\Big(\mathbf{H}^H\mathbf{C}^H\mathbf{CH}\Big)$, which in the case of symmetric-paired quaternion designs gets fully decoupled due to the identity that $\mathbf{C}^H\mathbf{C} = \lambda\mathbf{E}$.

Remark 1. It is evident from the simplification of the last term that it is still dependent on the complex orthogonality constraint $\mathbf{C}^H\mathbf{C}$ and therefore, the property $\mathbf{Q}^Q\mathbf{Q} = \lambda\mathbf{I}$ of QODs is not directly applicable to the quaternion ML norm.

Remark 2. The existence of identity $\mathbf{C}^H\mathbf{C} = \lambda\mathbf{E}$, in the case of symmetric-paired designs indicates that QODs are also effective in case of single-polarized antenna arrangements, which is not a surprise. This observation can also be verified through the simulation results of [26]- [27], in which the diversity order of the proposed 2×2 QOD is exactly same as that of 2×4 CODs.

As is clear from the above discussion that the operation $\mathcal{C}$, which transforms a QOD into a complex matrix is indeed causing the main problem. Besides the channel matrix is also complex which is why it is not possible to fully decouple the main term in ML-norm. On the other hand, it is easy to realize that one possible remedy of the problem is to convert either one or both of them into quaternion domain. In particular, the term $(\mathbf{QH})^Q = \mathbf{H}^Q\mathbf{Q}^Q$, will precisely give us a decoupled solution, in sharp contrast to $\mathcal{C}(\mathbf{Q})$ which fails to yield it. Below, we propose one possible physical realization to achieve it using the idea of transmit beamforming.

4.2.1 Proposed Transmitter

In order to employ $\mathbf{Q}^Q\mathbf{Q} = \lambda\mathbf{I}$ in ML decoding rule, we propose a redefined channel matrix based on the dual-polarized transmission channel at the receiver end as illustrated in Fig. 1. To realize this change of channel matrix, we use the concept of transmit beamforming with QODs. Transmit beamforming is a versatile data transmission technique that assigns different amplitudes and phases to the transmitted data to minimize interference at the non-intended users [35]. It also helps in increasing the signal power at the target destination. In this work, we utilize transmit beamforming to modify the transmitted signal in such a way that the desired channel-coefficients embedded signals are received.

We now illustrate the significance in modifying the channel coefficients to resolve the issue of coupled decoding in QODs. For a given QOD $\mathbf{Q}$ of order $T \times N_t$, the channel gains can be realized in terms of quaternion as follows

$$\tilde{\mathbf{H}} = [\tilde{\mathbf{H}}^{(1)} \quad \tilde{\mathbf{H}}^{(2)} \cdots \tilde{\mathbf{H}}^{(N_t)}]^T, \tag{4.10}$$

where $\tilde{\mathbf{H}}^{(m)} = \left[h_{11}^{(m)} + h_{12}^{(m)} j\right]$, therefore, order of $\tilde{\mathbf{H}}$ is $N_t \times 1$. Channel gains in terms of quaternions are realizable in the form of (4.10) given that in the original channel matrix $\mathbf{H}^{(m)}$, $h_{21}^{(m)} = -h_{12}^{*(m)}$ and $h_{22}^{(m)} = h_{11}^{*(m)}$. This provides the required decoupled solution as given in the following lemma. For clarity and without the loss in generality, we omit the superscript (m) from the channels.

Lemma 4.3 *For dual-polarized antenna configurations, the transmission channel can be realized in terms of quaternionic channel gains (4.10) which decouples the term* $tr\big(\mathcal{C}^{-1}(\mathbf{CH})^{Q}\mathcal{C}^{-1}(\mathbf{CH})\big)$, *in (4.9) for all the transmitted symbols.*

Proof. It is easy to see that $\mathcal{C}^{-1}(\mathbf{CH})$, is equal to the product $\mathbf{Q}\tilde{\mathbf{H}}$, which was not possible earlier and consequently we get

$$\begin{aligned}\mathcal{C}^{-1}(\mathbf{CH})^{Q}\mathcal{C}^{-1}(\mathbf{CH}) =& (\mathbf{Q}\tilde{\mathbf{H}})^{Q}(\mathbf{Q}\tilde{\mathbf{H}}) = \tilde{\mathbf{H}}^{Q}\mathbf{Q}^{Q}\mathbf{Q}\tilde{\mathbf{H}}, \\ =& \lambda\tilde{\mathbf{H}}^{Q}\mathbf{I}\tilde{\mathbf{H}} = \lambda\tilde{\mathbf{H}}^{Q}\tilde{\mathbf{H}}, \end{aligned} \tag{4.11}$$

which completes the proof. □

Provided the significance of the transformation of the channel matrix required at the receiver, we vary the amplitude and the phase of each transmitted symbol depending on its position in the coding matrix. We assume that perfect channel state information (CSI) is available at both the transmitter and the receiver.

In order to find the weight vector of complex numbers encoded in a QOD, a simple mechanism can be developed based on the following result. We then briefly describe its implementation through a relatively simple example.

Lemma 4.4. *The weight vector* $[w_{z_1}\ w_{z_2}]^T$, *of two complex symbols* z_1 *and*

z_2 *transmitted at some instant through a dual polarized antenna, is given by*

$$\begin{bmatrix} w_{z_1} \\ w_{z_2} \end{bmatrix} = \begin{bmatrix} z_1 h_{11} & z_2 h_{21} \\ z_1 h_{12} & z_2 h_{22} \end{bmatrix}^{-1} \begin{bmatrix} z_1 h_{11} - z_2 h_{12}^* \\ z_1 h_{12} + z_2 h_{11}^* \end{bmatrix}. \tag{4.12}$$

Proof. To find the weight vector for a quaternion symbol q_1 having $\mathcal{C}^{-1}(q_1) = [z_1 \quad z_2]$ to be transmitted through dual-polarized antenna 1, we equate the required $[z_1 \quad z_2] \begin{bmatrix} h_{11}^{(1)} & h_{12}^{(1)} \\ -h_{12}^{*(1)} & h_{11}^{*(1)} \end{bmatrix}$ to its actual (but containing weighted transmitted complex symbols) $\mathbf{CH}$, i.e., $[z_1 w_{z_1} \quad z_2 w_{z_2}]\mathbf{H}^{(1)}$, where w_{z_1} and w_{z_2} are the weight coefficients. Eventually, we obtain a system of two complex algebraic equations to be solved for two unknowns. As the determinant of coefficient matrix is non-zero, therefore, a unique solution (4.12) is attained. Hence proved. □

The beamforming Equation (4.12) can be brought to a simplified form as

$$w_{z_1} = 1 - \frac{(h_{21}^{(1)} h_{11}^{*(1)} + h_{22}^{(1)} h_{12}^{*(1)}) z_2}{(h_{11}^{(1)} h_{22}^{(1)} - h_{12}^{(1)} h_{21}^{(1)}) z_1}, \tag{4.13}$$

$$w_{z_2} = \frac{h_{11}^{(1)} h_{11}^{*(1)} + h_{12}^{(1)} h_{12}^{*(1)}}{h_{11}^{(1)} h_{22}^{(1)} - h_{12}^{(1)} h_{21}^{(1)}}, \tag{4.14}$$

where the first weight w_{z_1} turns out to be a linear function of the ratio of transmitted symbols, i.e., z_2/z_1, while the other weight contains purely channel elements. The above weight coefficients can be generalized to any quaternion symbol of a given QOD, i.e.,

$$w_{s_1} = 1 - \frac{(h_{21}^{(t)} h_{11}^{*(t)} + h_{22}^{(t)} h_{12}^{*(t)}) s_2}{(h_{11}^{(t)} h_{22}^{(t)} - h_{12}^{(t)} h_{21}^{(t)}) s_1}, \tag{4.15}$$

$$w_{s_2} = \frac{h_{11}^{(t)} h_{11}^{*(t)} + h_{12}^{(t)} h_{12}^{*(t)}}{h_{11}^{(t)} h_{22}^{(t)} - h_{12}^{(t)} h_{21}^{(t)}}. \tag{4.16}$$

where w_{s_1} and w_{s_2} are the weight coefficients of the first and the second complex symbols, i.e., s_1 and s_2, of the quaternion symbol $q = s_1 + s_2 j$, respectively. The superscript t of the channel coefficients represents the dual-polarized antenna number through which the corresponding quaternion symbol q will be transmitted. It is important to note that besides the channel coefficients, complex symbols s_1 and s_2 also contribute to the weight coefficients, thereby, making use of a modified form of transmit beamforming. Incorporation of this modified transmit beamforming would change the conventional system model (4.17) to the following form.

$$\mathbf{R} = \tilde{\mathbf{C}}\mathbf{H} + \mathbf{N}, \tag{4.17}$$

where $\tilde{\mathcal{C}}$ is the weighted complex code matrix of the QOD $\mathbf{Q}$.

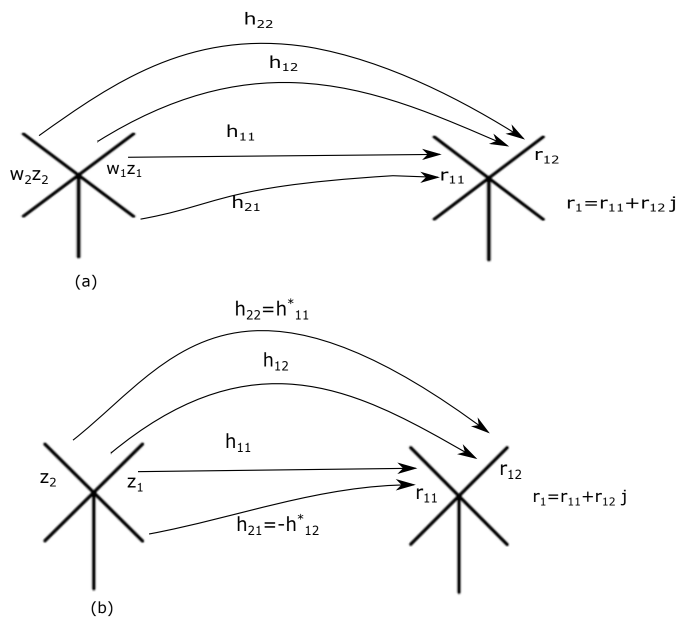

Figure 4.1: Two different realizations of the proposed system model, where (a) and (b) are equivalent to one another and provide the same received vector r_1.

4.2.2 Proposed Low-Complexity Decoder

We employ ML decoder based on quaternion norm criterion, i.e., $\min_{z_u}\left(\|\mathbf{R} - \mathcal{C}^{-1}\{\mathbf{CH}\}\|^2\right)$, to decode the coded matrix $\mathbf{C}$ at the receiver end. However, we know that with the proposed system model, $\tilde{\mathbf{C}}\mathbf{H}$ gets changed to $\mathbf{Q}\tilde{\mathbf{H}}$ at the receiver side. The squared norm criterion (4.9), upon using the identity (4.11), finally assumes a decoupled form given by

$$\|\mathbf{R} - \mathcal{C}^{-1}\{\mathbf{C}\mathbf{H}\}\|^2 = -2\Re\big(tr\big(\mathbf{R}^Q\mathbf{Q}\tilde{\mathbf{H}}\big)\big) + \lambda tr\Big(\tilde{\mathbf{H}}^Q\tilde{\mathbf{H}}\Big). \tag{4.18}$$

It is important to note that the actual channel matrix is $\mathbf{H}$ but we employ $\tilde{\mathbf{H}}$ at the receiver. Moreover, it can be easily verified that for all the QODs presented in Section II, the conventional quaternion-norm-based ML norm criterion fails to provide decoupled decoding rule because $(\mathcal{C}(\mathbf{Q})^H\mathcal{C}(\mathbf{Q})) \neq \lambda\mathbf{I}$, for these QODs except $\mathbf{Q}_0$ and $\mathbf{Q}_1$. Now, we apply this proposed decoder on the different QODs to highlight that it works for all categories of the QODs. Here we use short hand notation for channel coefficients as $h^{(m)} = h_{11}^{(m)} + h_{12}^{(m)}j$ and $\gamma_1 = \sum_{k=1}^{2} \|h_{11}^{(k)}\|^2 + \|h_{12}^{(k)}\|^2$. First we list five different QODs and then present their decoupled decoding equations.

The first QOD $\mathbf{Q}_1$ is of rate 3/4 as it transmits 3 complex symbols in 4 time-slots by two dual-polarized transmit antennas.

$$\mathbf{Q}_1 = \begin{bmatrix} z_1 + z_3 j & z_2 \\ -z_2^* & z_1^* + z_3 j \\ -z_3^* + z_1^* j & -z_2 j \\ z_2^* j & -z_3^* + z_1 j \end{bmatrix}. \tag{4.19}$$

The following QOD $\mathbf{Q}_2$ provides rate 1.

$$\mathbf{Q}_2 = \mathbf{A} + \mathbf{B}j = \begin{bmatrix} z_1 + z_3 j & z_2 + z_4 j \\ z_2^* + z_4^* j & -z_1^* - z_3^* j \\ z_3 + z_1 j & z_4 + z_2 j \\ z_4^* + z_2^* j & -z_3^* - z_1^* j \end{bmatrix} \tag{4.20}$$

. Interestingly, a non-trivial extension of standard Alamouti scheme in the quaternion domain can be found using real variables. Indeed such a design was formed in [29] with the combination of five reals $\{x_0, x_1, x_2, x_3, x_4\}$, providing us five complex numbers $z_1 = x_3+x_4 i, z_2 = x_0+x_2 i, z_3 = x_1+x_2 i, z_4 = -x_1 + x_2 i, z_5 = x_0 - x_2 i$, encoded in a QOD of order 2

$$\mathbf{Q}_3 = \begin{bmatrix} z_1 + z_2 j & z_1 + z_3 j \\ z_1 + z_4 j & -z_1 + z_5 j \end{bmatrix}, \tag{4.21}$$

which has rate 5/4. Note that $z_4 = -z_3^*$ and $z_5 = z_2^*$, which clearly limits its realization in encoding.

Another example of this construction is given below that provides a code rate of 2,

$$\mathbf{Q}_4 = \begin{bmatrix} z_1 + z_3 j & z_2 + z_4 j \\ z_2^* - z_4^* j & -z_1^* + z_3^* j \end{bmatrix}. \tag{4.22}$$

Next, we would include the following pure quaternion-based QOD example, given in [7], which is based on two quaternion elements a and b

$$\mathbf{Q}_5 = \begin{bmatrix} a & jb \\ ib & -ka \end{bmatrix}. \tag{4.23}$$

This QOD satisfies quaternion orthogonality given that $ab^Q = ba^Q$. To realize this constraint, it is assumed that $a = b = x_0 + x_1 i + x_2 j + x_3 k$, where x_0, x_1, x_2 and x_3 denote real numbers. However, this assumption limits the rate of this QOD to 1.

In the following corollaries, we provide the linear decoupled decoding equations for the above QODs.

Corollary 4.4. *The decoupled decoding statistics for each transmitted symbols z_1, z_2 and z_3 for QOD $\mathbf{Q}_1$ is*

$$\min_{z_1} (1+\gamma_1)|z_1|^2 - 2\Re\{r_1^Q z_1 h^{(1)} + r_2^Q z_1^* h^{(2)} + r_3^Q z_1^* j h^{(1)} + r_4^Q z_1 j h^{(2)}\},$$

$$\min_{z_2} (1+\gamma_1)|z_2|^2 - 2\Re\{r_1^Q z_2 h^{(2)} - r_2^Q z_2^* h^{(1)} - r_3^Q z_2 j h^{(2)} + r_4^Q z_2^* j h^{(1)}\},$$

$$\min_{z_3} (1+\gamma_1)|z_3|^2 - 2\Re\{r_1^Q z_3 j h^{(1)} + r_2^Q z_3 j h^{(1)} - r_3^Q z_3^* h^{(1)} - r_4^Q z_3^* h^{(2)}\}.$$

Corollary 4.5. *The transmitted symbols z_1, z_2, z_3 and z_4 in QOD $\mathbf{Q}_2$ can be decoded using*

$$\min_{z_1} (1+\gamma_1)|z_1|^2-2\Re\{r_1^Q z_1 h^{(1)}-r_2^Q z_1^* h^{(2)}+r_3^Q z_1 jh^{(1)}-r_4^Q z_1^* jh^{(2)}\},$$

$$\min_{z_2} (1+\gamma_1)|z_2|^2-2\Re\{r_1^Q z_2 h^{(2)}-r_2^Q z_2^* h^{(1)}+r_3^Q z_2 jh^{(2)}+r_4^Q z_2^* jh^{(1)}\},$$

$$\min_{z_3} (1+\gamma_1)|z_3|^2-2\Re\{r_1^Q z_3 jh^{(1)}-r_2^Q z_3^* jh^{(2)}+r_3^Q z_3 h^{(1)}-r_4^Q z_3^* h^{(2)}\},$$

$$\min_{z_4} (1+\gamma_1)|z_4|^2-2\Re\{r_1^Q z_4 jh^{(2)}+r_2^Q z_4^* jh^{(1)}+r_3^Q z_4 h^{(2)}+r_4^Q z_4^* h^{(1)}\}.$$

Unlike the decoding of QODs $\mathbf{Q}_1$ and $\mathbf{Q}_2$, in which we decode complex numbers, the same can not be done for QOD $\mathbf{Q}_3$. It is not a surprise that QOD $\mathbf{Q}_3$, contains complex numbers of which two are conjugates which make it difficult to be extracted individually. However, the decoupled decoding of the same QOD is obtainable for the reals on which those complex symbols were constructed.

Corollary 4.6. *For QOD $\mathbf{Q}_3$, the simplified ML norm-based decoder for each real transmitted symbols x_0, x_1, x_2, x_3 and x_4 is*

$$\min_{x_0} (1+\gamma_1)|x_0|^2-2\Re\{r_1^Q x_0 jh^{(1)}+r_2^Q x_0 jh^{(2)}\},$$

$$\min_{x_1} (1+\gamma_1)|x_1|^2-2\Re\{r_1^Q (x_1 jh^{(2)})+r_2^Q(-x_1 jh^{(1)})\},$$

$$\min_{x_2} (1+\gamma_1)|x_2|^2-2\Re\{r_1^Q x_2 k(h^{(1)}+h^{(2)})+r_2^Q x_2 k(h^{(1)}-h^{(2)})\},$$

$$\min_{x_3} (1+\gamma_1)|x_3|^2-2\Re\{r_1^Q x_3(h^{(1)}+h^{(2)})+r_2^Q x_3(h^{(1)}-h^{(2)}\},$$

$$\min_{x_4} (1+\gamma_1)|x_4|^2-2\Re\{r_1^Q x_4 i(h^{(1)}+h^{(2)})+r_2^Q x_4 i(h^{(1)}-h^{(2)}\}.$$

Corollary 4.7. *The decoupled ML norm for each transmitted symbols z_1, z_2, z_3 and z_4 for QOD $\mathbf{Q}_4$ is*

$$\min_{z_1}\,(1+\gamma_1)|z_1|^2-2\Re\{r_1^Q z_1 h^{(1)}-r_2^Q z_1^* h^{(2)}\},$$

$$\min_{z_2}\,(1+\gamma_1)|z_2|^2-2\Re\{r_1^Q z_2 h^{(2)}+r_2^Q z_2^* h^{(1)}\},$$

$$\min_{z_3}\,(1+\gamma_1)|z_3|^2-2\Re\{r_1^Q z_3 j h^{(1)}+r_2^Q z_3^* j h^{(2)}\},$$

$$\min_{z_4}\,(1+\gamma_1)|z_4|^2-2\Re\{r_1^Q z_4 j h^{(2)}-r_2^Q z_4^* j h^{(1)}\}.$$

As in the case of QOD $\mathbf{Q}_3$, the decoupled decoding for QOD $\mathbf{Q}_5$ is only obtainable for real numbers only.

Corollary 4.8. *The decoupled ML norm for each real transmitted symbols x_0, x_1, x_2 and x_3 of a quaternion symbols $a = x_0 + x_1 i + x_2 j + x_3 k$ for QOD $\mathbf{Q}_5$ is*

$$\min_{x_0}\,(1+\gamma_1)|x_0|^2-2\Re\{r_1^Q(x_0 h^{(1)}+x_0 j h^{(2)})+r_2^Q(x_0 i h^{(1)}-x_0 k h^{(2)})\},$$

$$\min_{x_1}\,(1+\gamma_1)|x_1|^2-2\Re\{r_1^Q(x_1 i h^{(1)}-x_1 k h^{(2)})+r_2^Q(-x_1 h^{(1)}-x_1 j h^{(2)})\},$$

$$\min_{x_2}\,(1+\gamma_1)|x_2|^2-2\Re\{r_1^Q(x_2 j h^{(1)}-x_2 h^{(2)})+r_2^Q(x_2 k h^{(1)}+x_2 i h^{(2)})\},$$

$$\min_{x_3}\,(1+\gamma_1)|x_3|^2-2\Re\{r_1^Q(x_3 k h^{(1)}+x_3 i h^{(2)})+r_2^Q(-x_3 j h^{(1)}+x_3 h^{(2)}\}.$$

Chapter 5

Simulation and Results

For simulations, we have considered QPSK constellation. The channel coefficients are assumed perfectly known at the receiver, whereas, white noise is added in each polarization uniformly.

5.1 Simulation Results of Symmetric-Paired Design 1 and 2

In this study, we have compared the performance of both the designs by constructing QODs for 2, 3 and 4 dual polarized transmit antennas that include both even and odd antenna arrangements. Their decoding statistics are derived using Lemma 4.2 and are completely decoupled. The rate of design 1 for these antenna arrangements is 1, 3/4 and 3/4, respectively. Likewise, code rate of design 2 is 3/4, 1/2 and 1/2, respectively.

Fig. 5.1 presents the simulation results of QOD for 2, 3 and 4 dual-polarized transmit antennas for both the designs. As symmetric-paired design 1 provides a QOD similar to the one presented in [7], therefore, its bit error rate (BER) curve is identical to [7]. This similarity hints the accuracy of our work. Overall, system's diversity gain increases with each added antenna dimension. For example, for symmetric-paired design 1, 10^{-5} BER is achieved by 2Tx/1Rx, 3Tx/1Rx and 4Tx/1Rx antenna arrangements at signal-to-noise ratio (SNR) values 16, 11.8 and 9.4 dB, respectively. Similarly, for symmetric-paired design 2, the same BER is achieved by 2T/1Rx, 3Tx/1Rx and 4Tx/1Rx antenna arrangements at 12.2, 9.8 and 8 dB SNR values, respectively. Furthermore, impact of the transmission rate of codes can also be observed from Figure 1. For both designs, the SNR gain from 2Tx/1Rx to 3Tx/1Rx is larger than from 3Tx/1Rx to 4Tx/1Rx. This is because for the former case, rate has dropped from 1 to 3/4 for design 1 and from 3/4 to 1/2 for design 2. However, in the latter scenario, the rate remains the same for both designs.

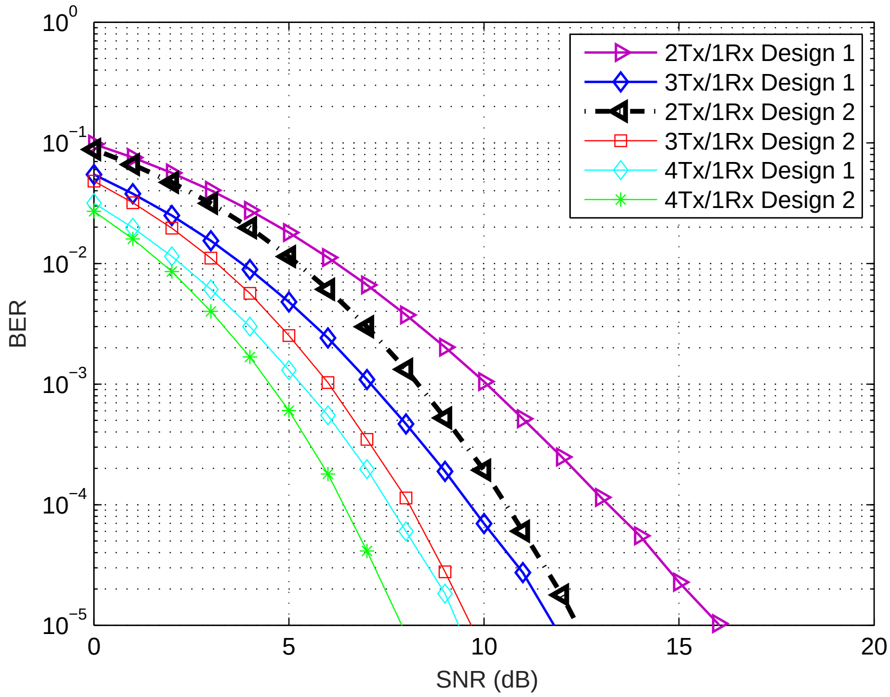

Figure 5.1: BER vs. SNR for OSTPBCs with 2, 3 and 4 Tx antennas.

In a nutshell, the trend of BER curves indicates that design 2 performs better than design 1 in terms of diversity gain because each BER curve of design 2 provides lesser BER as compared to its respective design 1. However, as antenna dimensions increase, this improvement starts decreasing. That is because for higher order designs, symmetric-paired design 2 provides a larger QOD comprising of more number of zeros than its respective symmetric-paired design 1. For example, for 5Tx/1Rx case, symmetric-paired design 2 produces a QOD of order 16×5 which transmits 5 symbols in 16 time slots whereas symmetric-paired design 2 generates a QOD of order 8×5 which transmits 4 symbols in 8 timeslots. This shows that for higher order designs, former design performance is better in terms of rate and decoding delay with a slight compromise on diversity. On the other hand, symmetric-paired design 2 is suitable for small scale designs only because for higher order designs, its rate falls below half and therefore, its fails to provide high code rates.

5.1.1 Comparison of QCIODs with Symmetric-Paired Design 1:

In Figure 5.2, we compare symmetric-paired design 1 with QCIODs to explain diversity-multiplexing tradeoff (DMT). It can be seen that symmetric-paired design 1 obtains 10^{-5} (bit error probability) BER at 9 and 7 dB SNR for 4Tx/1Rx and 6Tx/1Rx, respectively. However, in QCIODs case, by increasing transmit antennas in even number, such as $\{4, 6, 8, ..\}$ Tx/1Rx, performance of QCIODs remains constant. It shows that this construction fails to provide reasonable performance for high order designs and the problem of using small QODs to construct higher order QODs remains unsolved. Moreover, symmetric-paired design 1 outperforms QCIODs remarkably.

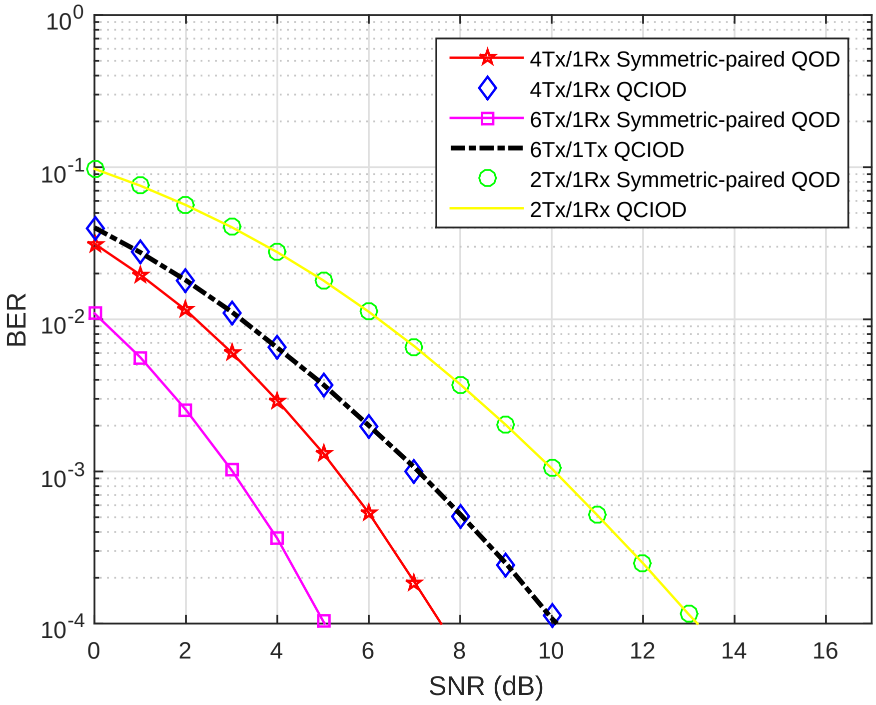

Figure 5.2: Bit error probability (BER) versus signal-to-noise ratio (SNR) comparison of symmetric-paired orthogonal space-time polarization block codes OSTPBCs with quaternion coordinate interleaved orthogonal designs (QCIODs) for 4 and 6 transmit antennas.

5.2 Simulation Results of Symmetric-Paired Design 3

For simulations of this design, we develop QODs for 2, 3 and 4 dual-polarized transmit antennas with code rates 1, 3/4 and 3/4 while their complex counterparts have rates 3/4, 1/2 and 1/2, respectively. The transmit beamforming based ML decoupled decoding rule has been used to decode the transmitted data. Moreover, zero cross polarization with two dual-polarized receiving antennas has been assumed.

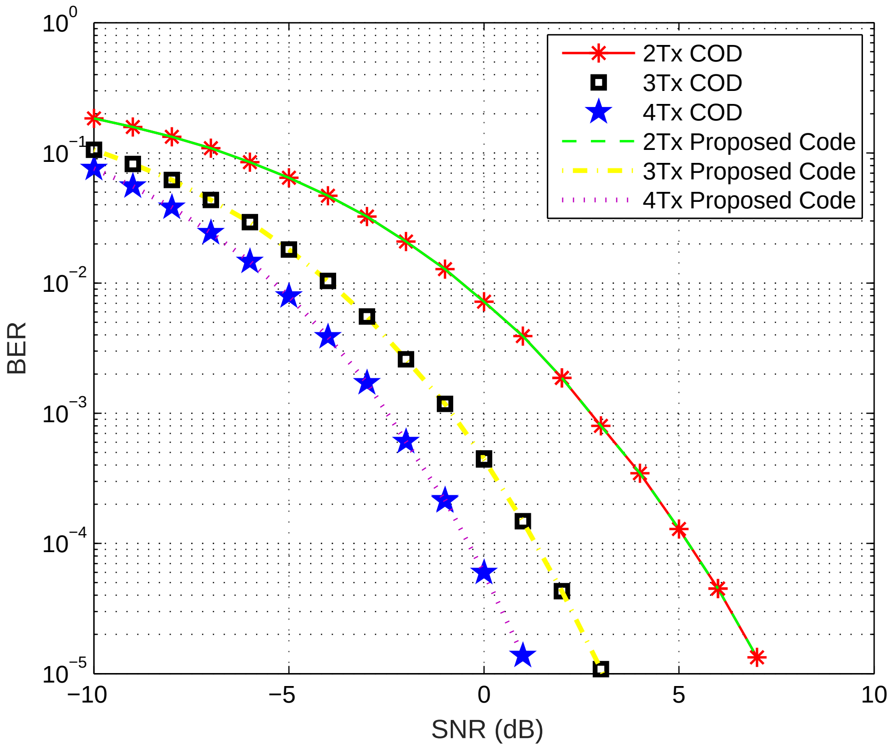

Figure 5.3: BER vs. SNR comparison of the ML coupled and decoupled decoding for 2, 3 and 4 dual-polarized transmit antennas.

Fig. 5.3 demonstrates the optimal performance of the proposed low-complexity ML decoder. All the BER curves are derived using transmit beamforming aided ML decoupled decoder where it can be seen that performance of the proposed decoder is optimal for smaller as well as higher order QODs. In addition to this, the BER curves also exhibit the phenomenon of diminishing returns with the increase in the number of transmit antennas. Both codes, i.e., the proposed design and the CODs, for 2Tx/2Rx, 3Tx/2Rx and 4Tx/2Rx antenna arrangements yield the value of 7, 3 and 1 dB signal-to-noise ratio (SNR) at a target BER of 10^{-5}, respectively. Hence, for zero-cross polarization environment, the proposed construction outperforms CODs in terms of code rates.

5.3 Simulation Results of various other QODs

Fig. 5.4 demonstrates the optimal performance of the proposed generalized low-complexity ML decoder for different categories of QODs. It shows the coupled and decoupled decoding rule match for different QODs, i.e., $\mathbf{Q}_1, \mathbf{Q}_2, \mathbf{Q}_3, \mathbf{Q}_4$ and $\mathbf{Q}_5$, presented in Chapter 6. It is important to note that only $\mathcal{C}(\mathbf{Q}_1)$ is a valid COD and $\mathcal{C}(\mathbf{Q}_2), \mathcal{C}(\mathbf{Q}_3), \mathcal{C}(\mathbf{Q}_4)$ and $\mathcal{C}(\mathbf{Q}_5)$ are quasi-orthogonal STBCs. Despite this comprise on complex orthogonality, we have obtained decoupled linear equation-based decoding solutions for all these QODs. Furthermore, $\mathbf{Q}_1$, i.e., a pure COD-based QOD, outperforms all other QODs for 2 transmit and 1 receive dual-polarized antenna-based arrangements from diversity perspective. However, as the code rate and number of transmitted symbols of these QODs are different, therefore, it would not be a fair comparison in terms of diversity. Overall, these curves demonstrate the phenomenon of diversity-multiplexing tradeoff, i.e., diversity of coded matrices decreases with the increase in the code rate. Fig. 7.4 also illustrates the efficiency of pure QOD designs as pure QOD-based design $\mathbf{Q}_5$ of code rate 1 provide the same diversity in two timeslots as compare to COD-based non-permuted design $\mathbf{Q}_2$ of the same code rate that uses 4 time slots. This implies that $\mathbf{Q}_5$ has half the decoding delay, i.e., the number of timeslots receiver has to wait to receive a complete coded matrix, as compare to $\mathbf{Q}_2$.

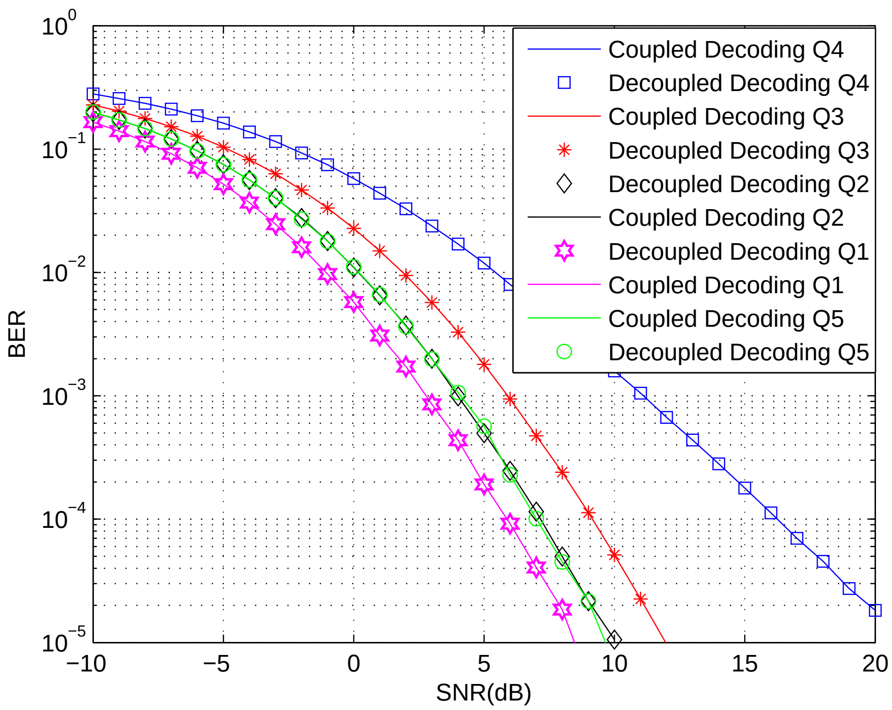

Figure 5.4: BER vs. SNR comparison of the ML coupled and decoupled decoding for different QODs for 2 transmit and 1 received dual-polarized antenna arrangement.

Chapter 6

Conclusions and Future Works

In this work, we have presented three generalized construction techniques of so called OSTPBCs for data transmission in MIMO systems comprising dual-polarized antennas. These generalized schemes can construct QOD for any number of transmit antennas. Symmetric-paired design 1 and 3 provides code rate higher than their respective CODs. On the other hand, symmetric-paired design 2 provide comparatively higher diversity. In addition to these QOD construction techniques, we have explored conventional quaternion norm based ML decoder and presented the constraints that are important to obtain decoupled decoding using this decoder. However, there can be some valid QODs that may not satisfy these constraints which necessitates the need of a generalized low-complexity decoder. Therefore, in this study, we have provided a solution to the issue of decoupled decoding by deriving a generalized low-complexity decoder that can work for any QOD. This implies that a decoupled decoding solution can be obtained even for quasi-orthogonal or non-orthogonal coding matrices provided that the codes satisfy only the basic orthogonality constraint of QOD. We have presented different QODs examples and most of the presented examples of QODs provide high code rate. We have applied the proposed generalized low-complexity decoder on them to validate its optimality. To achieve this objective, we have utilized the concept of transmit beamforming that helped us to modify a dual-polarized channel and consequently, simplified the quaternionic ML norm criterion. Optimality of the proposed decoder has been verified through simulation results. Overall, the symmetric-paired design 2 outperforms the first one in terms of transmit diversity. On the other hand, symmetric-paired design 1 performs better than design 2 in terms of data rate. Hence, there is a tradeoff between data rate and transmit diversity. Moreover, simulation results also indicate that the proposed symmetric-paired design 3 and CODs provide same diversity for zero-cross polarization based system model with two receiving dual-polarized antennas, however, design 3 provides better code rates. In sum, proposed generalized decoder can work for any type of QOD and reduces the computational complexity significantly. However, transmit diversity depends on code construction and usually decreases by increasing code rate.

In future, we intend to explore pure quaternion-based QOD design constructions. Quaternions do not satisfy commutative property and therefore, design of a generalized QOD construction based on pure quaternion elements is a challenging work. Moreover, we would like to explore QODs providing higher transmit diversity to multiply the benefits of QODs. To increase transmit diversity, constellation rotation can be incorporated in QODs. In addition to this, we also aim to investigate the design of quasi-QODs and a low-complexity ML decoder for these designs. Quasi-QODs would further increase the code rate, which is considered a desirable feature of efficient codes.

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