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Development of a 2 Degrees of Freedom Tracking System Part III: System Modeling

Syed Ali Hassan, Irtaza Askari, M. Salman Fareed, Khalid Munawar, M. Bilal Malik.

College of Electrical and Mechanical Engineering. Rawalpindi. Pakistan
alihassan_20002000@yahoo.com, irtazaaskari@yahoo.com

Abstract

The problem of tracking moving objects using a moving camera in real time is a complex one which requires a highly stable platform and a robust control algorithm. This paper is a continuation of our previous papers in which we discussed the design and digital control routine of a 2DOF stable platform with coupled joint. In the previous paper we implemented a non-model based PI controller that proved sufficient for static targets, however for image based tracking, a number of non-linear forces are affecting the system and such scenarios require model-based controllers. The design of model based controller starts with model selection and then the estimation of its parameters. In this paper we deal with the modeling and parameters estimation of a 2 DOF platform using the least squares method.

Keywords: 2DOF platform, System Identification, Parameter Estimation.

1. Introduction

The model-based approach for the development of a control scheme relies on the use of explicit models in the design process, if this model faithfully represents the components of the system; it is used to predict the dynamic behavior of the system, and the system is controlled accordingly. In this paper we describe the modeling of a 2 DOF platform. The system modeling process starts with the study of the system at hand, to determine the factors affecting the system, with the knowledge of the system a model is selected that can cater for the forces acting on the system. Then the parameters of this model are adjusted using parameter estimation techniques. Finally this model is validated, if the response of the model and the system match the model fully represents the system in that operating range and a model based controller can be designed else another model has to be chosen that better represents the system. Before going in details of modeling lets have a brief description of the mechanical structure of platform.

This fundamental design has two degrees of freedom; the azimuth and the elevation (pitch). These two degrees of freedom are sufficient to scan almost a spherical workspace [1].

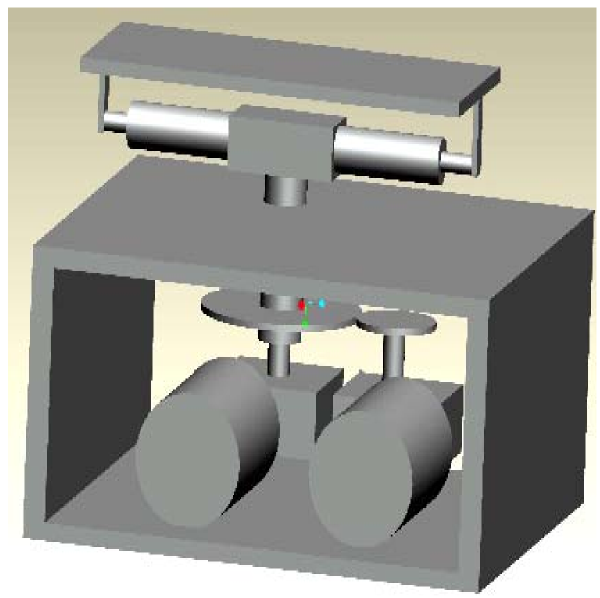

Fig. 1. Mechanical structure of the platform

The design criteria are the following:

- The azimuth for the horizontal span of 360 degrees.
- The elevation (pitch) for the vertical motion of 180 degrees span.

Two motors are used to drive the elevation and the azimuth, however the system was designed in such a manner that the azimuth and the elevation links were coupled to each other i.e. drive applied to the azimuth caused motion in the elevation along with the azimuth.

The platform has been controlled using a PI controller implemented on a PIC-17C44 micro-controller to achieve reasonable performance [2], however certain non-linearities are present in the system that have to be catered for to further improve the system's performance, for which it is necessary to have a platform model. This paper gives a detail description of the model building process for the 2 DOF platform.

2. System Modeling:

This section describes the modeling of platform for both azimuth and elevation.

2.1 Basic Steps:

The basic steps used for model building are:

1. Model selection
2. Parameter Estimation
3. Model validation

These steps are used iteratively until an appropriate model for the data has been developed.

2.2 Model Selection

The general equation of a Newton's robotic manipulator is given as:

$$D(q)\ddot{q} + h(q,\dot{q}) + c(q) + B(q,\dot{q})\dot{q} = \tau \qquad (1)$$

where

$y = q$ ___ Output of the system $(n \times 1)$
$D(q)$ ___ Inertial matrix $(n \times n)$
$h(q,\dot{q})$ ___ Centrigugal forces $(n \times 1)$
$c(q)$ ___ Gravitational forces $(n \times 1$
$B(q,\dot{q})$ ___ Frictional loss matrix (ususlly diagonal) $(n \times n)$
τ ___ Applied force/torque vector $(n \times 1)$

Equation (1) represents an n-input and n-output system. Basically the platform should be modeled as a MIMO (Multiple input multiple output) system, as it has two inputs i.e. the elevation and azimuth torque and two outputs i.e. the angular change in the elevation and the azimuth, having the inputs and the outputs linked with coupled joints, but as the coupling ratio is already known (computed with the gear ratios) the system can be modeled as two separate SISO systems. In both cases the output of the system is the angle θ. So

$$\theta = \begin{bmatrix} \theta_E \\ \theta_A \end{bmatrix}$$

where

$\theta_E =$ output of the elevation platform
$\theta_A =$ output of the azimuth platform

The system can be described by the following kinematics:

$$\begin{bmatrix} \theta_E \\ \theta_A \end{bmatrix} = \begin{bmatrix} 1/N_1 & N_2/N_1 \\ 0 & 1/N_2 \end{bmatrix} \begin{bmatrix} \tau_E \\ \tau_A \end{bmatrix}$$

N_1 = Gear ratio for Elevation

N_2 = Gear ratio for Azimuth

τ_E = Torque applied to elevation platform

τ_A = Torque applied to azimuth platform

3. Elevation Modeling

Following are the steps for model building process for elevation.

3.1. Behavior of Parameters

For the platform, in the elevation the significant forces playing effect are gravitational force, frictional force, inertia and applied torque. Of all these forces, applied torque is the input that is known and rest are the parameters that have to be modeled.
A rough sketch of the elevation model can be viewed as:

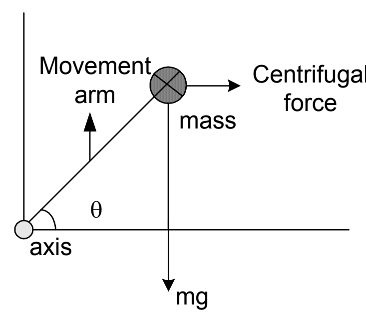

Fig. 2. Elevation Model

The platform is raised in elevation from the pivoting points (axis). The stick and ball diagram of figure 2 shows that gravity affects the applied Torque in a non- linear manner similar to a cosine function. This fact is clearly illustrated in fig 3.

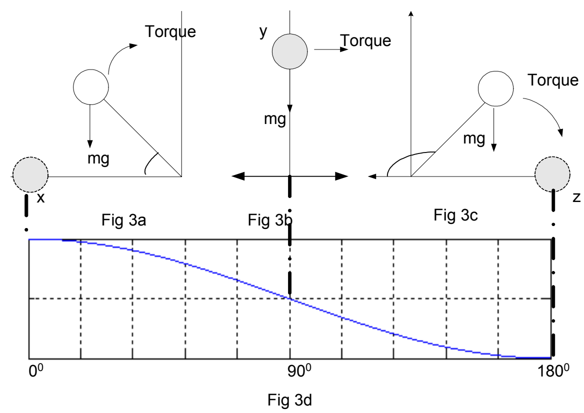

Fig. 3. Modeling of Gravitational force

In figure 3a, it is shown that Gravity opposes the applied torque, in 3b the two forces are orthogonal to each other and didn't interfere and in figure 3c the gravitation forces adds in applied force. So this force is modeled as cosine where the positions x, y and z are marked by the sinusoid in figure 3d.

For platform having multiple joints, the effects of motion or position of one joint on the other are modeled by centrifugal forces. For this system, the movement arm of the elevation is short and the operating speed of the azimuth is low, therefore the effect of centrifugal force is nominal and can be ignored.

The system model for elevation is of one input and one output system. Dynamic equation of this system is therefore given as following: -

$$J\ddot{q}+\alpha\cos(q)+\mathrm{K}\dot{q}=\tau \qquad (2)$$

Where
J= Inertia matrix
α = Gravitational force
K =Frictional force
τ =Applied input torque

Let the initial state q is x_1 and $\dot{q}$ is x_2 then the equation can be written as:

$$\dot{x}_1 = x_2$$

$$x_2 = -\frac{\alpha}{J}\cos x_1 - \frac{K}{J}x_2 + \frac{1}{J}\tau$$

The equation becomes:

$$\ddot{x} = -\frac{\alpha}{J}\cos x - \frac{K}{J}\dot{x} + \frac{1}{J}\tau \qquad (3)$$

3.2. Discrete form of the Elevation Model

Since the model is in continuous domain and input and output data is in sampled form, the parameters have to be estimated for discrete model. Therefore the next step is the discretization of the model. Since the system is causal, therefore it is suitable to use the backward difference approximation for first and second order equations sshown [3]:

$$\dot{x} \cong \frac{x[(kT)]-x[(k-1)T]}{T}$$

$$\ddot{x} \cong \frac{x[(kT)]-2x[(k-1)T]+x[(k-2)T]}{T^2}$$

so the discrete form of the model for (3) becomes:

$$\frac{x[k]-2x[k-1]+x[k]}{T^2} = -\frac{\alpha}{J}\cos x[k] - \frac{k}{J}\left[\frac{x[k]-x[k-1]}{T}\right] + \frac{\tau[k]}{J}$$

$$x[k]-2x[k-1]+x[k-2] = -\frac{\alpha}{J}\cos x[k]T^2 - \frac{k}{J}x[k]T - \frac{k}{J}x[k-1]T + \frac{\tau[k]T^2}{J}$$

$$x[k]\left[J+kT\right] = -\alpha T^2\cos x[k] - x[k-1]\left[kT+2J\right] + Jx[k-2] - kT^2\tau[k]$$

$$x[k] = -\frac{\alpha T^2}{J+kT}\cos x[k] - \frac{2J+kT}{J+kT}k[k-1] + \frac{J}{J+kT}x[k-2] - \frac{T^2}{J+kT}\tau[k]$$

Where T is the sampling interval

3.3. Parameter Estimation

After selecting the basic form of the functional part of the model, the next step in the model-building process is estimation of the unknown parameters in the function. The universal indicator of goodness of fit is to use the sum of the squares of the deviations, which is the "method of least squares". The problem is to find the set of constants that minimize the sum of the squares of the deviations between the measured values and the calculated values.

In least squares (LS) estimation, the unknown values of the parameters, $\beta_0, \beta_1, \ldots,$ in the function, $f(\vec{x} : \vec{\beta})$, are estimated by finding numerical values for the parameters that minimize the sum of the squared deviations between the observed responses and the functional portion of the model. [4]

Mathematically, the least (sum of) squares criterion that is minimized to obtain the parameter estimates is

$$Q = \sum_{i=1}^{n}[y_i - f(\vec{x}_i : \vec{\beta})]^2$$

3.4. Parameter estimation in Elevation Model

The impact of system on any given input is to induce a change in the amplitude and phase for a linear system whereas frequency is also affected in case of non-linear system. Since the 2DOF platform has been designed for tracking purposes from tracking a football in the ground so as per the specifications the maximum frequency that should be tracked is at least 2Hz.So a persistently exciting chirp signal was generated in the micro-controller and applied to the system and the system's response was recorded for further calculations. The general expression of the chirp signal i.e. a sinusoid of increasing frequency is given as:

$\tau = A\sin(.01t)^2$ a chirp signal

In a chirp signal the frequency varies linearly with time, the relation becomes:

$$\tau = A\sin(2\pi ft) = A\sin(2\pi tt) = A\sin(2\pi t^2)$$

To decrease the rate of change of frequency with time, the constant term has to be decreased. So the chirp applied as input can be written as: $\tau = A\sin(.01t)^2$

Applying the Least Square algorithm in MATLAB on the input and output of the system, the parameters of the model are calculated such that the error between the model and the plant's output is minimized. The parameters thus created for the elevation are:

α = 2.1081
J = 0.3427
K = 3.7180

3.5. Model Validation

For model validation a similar model of the system with above parameters was simulated in SIMULINK and both the simulink model and the actual model were excited by the same chirp input and response was calculated. The dynamical model as simulated in SIMULINK is shown in fig 4:

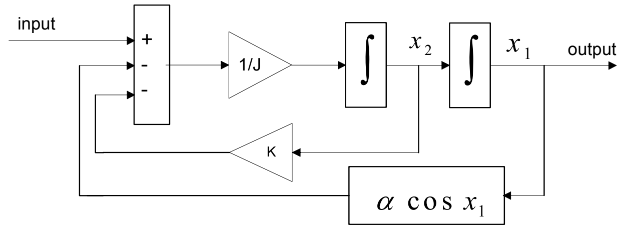

Fig. 4. Block Diagram of Elevation in Simulink

The estimate is approximately close to the actual values of the parameters as shown by the figure 5.

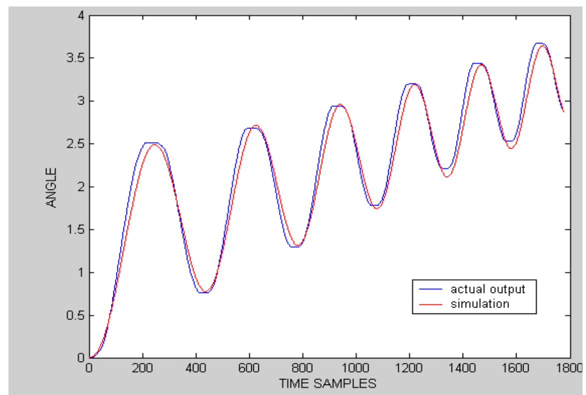

Fig. 5. Comparison of Actual and Simulated Output

4. Azimuth Modeling

For the azimuth motion of platform there are only two forces that are affecting the plant, the inertial force and the frictional forces. Since the azimuth platform is not affected by gravity during its motion so the model for azimuth becomes:

$$J\ddot{q} + K\dot{q} = \tau \qquad (4)$$

Denoting the parameter as previously the azimuth model can be written as:

$$\ddot{x} = -\frac{K}{J}\dot{x} + \frac{1}{J}\tau \qquad (5)$$

Using the same equations as previous the discrete model of the system for azimuth becomes:

$$x[k] = \frac{-J}{J+KT}x[k-2] + \frac{2J+KT}{J+KT}x[k-1] + \frac{T^2}{J+KT}\tau[k]$$

4.1. Parameter estimation and model validation

Using the same parameter estimation method i.e. LEAST SQUARES as in elevation we can find the azimuth parameters. The modeling in the simulink yield the block diagram as follows:

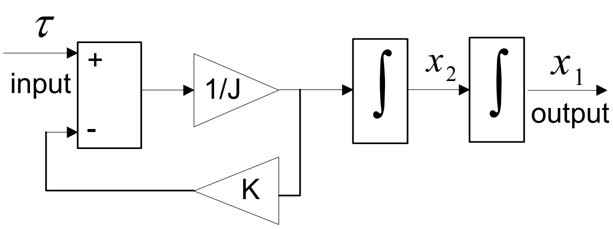

Fig. 6. Block Diagram of azimuth in Simulink

The estimate of the model parameters are as following:

J = 0.2617
K =3.4193
The estimates are close enough as the real output as shown in figure 7.

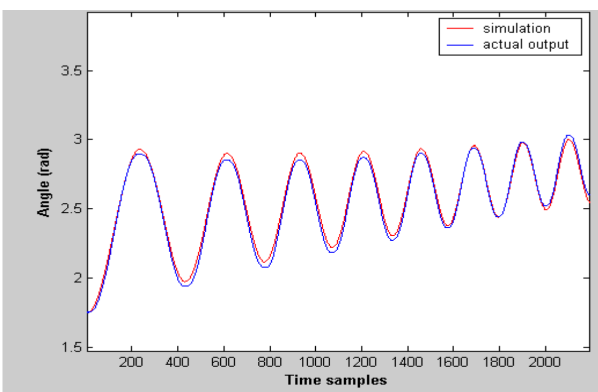

Fig. 7. Comparison of Actual and Simulated Output

6 CONCLUSION AND RECOMMENDATION

In this paper, the proper modeling of the platform has been discussed in details. Implementing better controllers is one of the major requirements for stabilizing and controlling any control system. However, without a good model no controller will perform adequately! The majority of effort (and software coding) for any advanced controller projects is in the model identification. Thus by modeling the 2 DOF platform over the desired range of operation and estimating the model parameters, we can easily apply a model based robust controller for the plant. This approach also saves time and money because it requires less extensive plant testing.

As the state space control is basically the model based control of a system, so the modeling of the plant has enabled us to have a state space control on the system. This provides insight into process dynamics and system interactions. It also provides meaningful model fidelity and system analysis tests to be done for more stability.

By modeling the system, we can form observers, which can be used to define the intermediate states of the system and other such controls, which can be used in tracking control of the platform.

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