A Method of Moments Estimator for Modulation Index of Continuous Phase Modulation

Conference Paper · February 2019

4 authors:

Ihtisham Khalid
National University of Sciences and Technology

Syed Asad Ullah
National University of Sciences and Technology

Sajid Saleem
National University of Sciences and Technology

Syed Ali Hassan
National University of Sciences & Technology

A Method of Moments Estimator for Modulation Index of Continuous Phase Modulation

Ihtisham Khalid, Syed Asad Ullah, Sajid Saleem, Syed Ali Hassan
School of Electrical Engineering & Computer Science (SEECS),
National University of Sciences & Technology (NUST), Islamabad, Pakistan
{14mseeikhalid, 14mseesullah, sajid.saleem, ali.hassan}@seecs.edu.pk

***Abstract*— A novel non data-aided estimator based upon the method of moments is proposed for estimating the modulation index of continuous phase modulation (CPM) schemes. Analytical expressions are derived for the asymptotic (large data record N) mean and variance of the estimator using truncated Taylor series. It is shown that the estimator is asymptotically unbiased and has variance that depends upon the modulation index, which decreases as $O(N^{-1})$. Numerical simulations are performed to study the convergence of mean and variance of the estimator to their respective asymptotic values. The performance comparison establishes that the proposed estimator outperforms state-of-the-art estimators in the low signal-to-noise ratio regime.**

***keywords*– modulation index, estimation, continuous phase modulation, statistical linearization.**

I. INTRODUCTION

CPM is known for its power and bandwidth efficiency owing to its desirable characteristics of constant envelope and phase continuity, respectively [1]. These characteristics make CPM a waveform of choice for several application areas such as cordless telephony [2], maritime navigation systems [3] and personal area communications.

One of the most important technical parameters of CPM is its modulation index that controls the peak excursion of phase per symbol interval. The modulation index allows a trade-off between the bandwidth of the transmitted signal and the receiver bit-error rate (BER) performance. The precise knowledge of the modulation index at the receiver is critical for implementation of the optimum receiver, i.e., the maximum likelihood sequence detection (MLSD) receiver, and also for miscellaneous sub-optimum techniques, especially the ones based upon Laurent decomposition of the CPM signal [4].

The modulation index is determined, at the transmitter, by the gain of the voltage controlled oscillator (VCO), whose precise control is hopeless for inexpensive analog implementations. For this reason, several communication standards (such as DECT ULE [2], AIS [3], Bluetooth etc), in interest of their wide adoption, do not specify an exact value of the modulation index, but instead allow for considerable tolerance around a nominal value. However, a greater tolerance allowed to the transmitter generates greater mismatch of the modulation index at the receiver, and consequently, less chances of successful demodulation of data. There could be other scenarios where the receiver may have no knowledge of the modulation index at all, e.g., automatic modulation recognition (AMR) by a universal receiver or passive listening from a non-cooperative transmitter. There are two different approaches presented in the literature to mitigate these issues. The first one focuses on the design of robust receivers in the presence of modulation index mismatch [5] - [6], and the second approach is to design an estimator of the modulation index at the receiver. The former approach, though, very effective in the presence of reasonable mismatch, has limited applicability for the AMR or passive listening applications.

The problem of modulation index estimation has been undertaken earlier in [7], [8], [9], [10] and [11]. The estimators proposed in [8] and [9] exhibit good performance but at the expense of pilot symbols and are based upon the (quasi) maximum likelihood choice among a finite set of possible modulation indices. A closed-form non data-aided estimator is proposed in [10], which is based upon higher-order statistics (HOS), specifically, a ratio of fourth-order cumulants. The HOS estimator exhibits reasonable performance at low signal-to-noise ratio (SNR) but its performance does not improve considerably at high SNR due to slower convergence of fourth-order moments. The work in [11] uses cyclo-stationary properties of the CPM signal and proposes a non data-aided iterative algorithm (NDA-CYC) that uses unwrapped phase to estimate the signal. However, the performance of NDA-CYC estimator is unacceptable at low SNR due to the phase unwrapping errors. Another estimator for partial response CPM signals is derived in [12].

In this paper, we propose a non data-aided estimator for the modulation index using the method of moments approach. We also use statistical linearization to compute the asymptotic mean and variance of the proposed estimator. A comparison of the mean-squared error (MSE) performance reveals that the proposed estimator outperforms both non-data aided estimators, i.e., HOS and NDA-CYC at low SNRs. The vectors and matrices are represented by boldface small and capital letters, respectively, the matrix transpose by $(.)^T$, and $\mathbb{E}[.]$ represents the expectation of a random variable.

The rest of the paper is organized as follows. In Sec. II, we discuss the format of CPM signal, followed by the proposed method of moments estimator in Sec. III. The approximations of mean and variance of the proposed estimator are derived in Sec. IV, while the numerical results are presented in Sec. V. The conclusions are drawn in Sec. VI.

II. CPM Signal Description

The complex envelope of a CPM signal is expressed as

$$s(t) = \sqrt{\frac{E}{T}} e^{j(\phi(t,\alpha)+\phi_0)}, \quad (1)$$

where E is the energy per symbol, T is the symbol duration and ϕ_0 is the constant phase term. The excess phase $\phi(t, \alpha)$ is defined as

$$\phi(t,\alpha) = 2\pi \sum_{i=0}^{m} \alpha_i h_{(i)_H} q(t - iT), \text{ for } t \le mT, \quad (2)$$

where the data symbols represented as α_i are components of the data sequence α and belong to an M-ary Pulse-amplitude modulation constellation, h is the modulation index, and $q(t)$ is the phase shaping function. The derivative of $q(t)$ is called as the frequency shaping function and is denoted by $g(t)$. The function $g(t)$ has a support of L symbol intervals and an underlying area of 1/2. A CPM signal with $L = 1$ is known as full response, while it is partial response for $L > 1$. In this work, we focus only upon the binary full-response modulation schemes. The samples of the CPM signal in (1) are defined as

$$s[n] := s(t)|_{t=nT_s} = Ae^{j\phi[n,\alpha]}, \quad (3)$$

where T_s is the sampling interval and $\phi[n, \alpha] := \phi(t, \alpha)|_{t=nT_s}$ are the samples of the excess phase. The samples are considered for $n \in \{0, 1, ..., NN_s - 1\}$, where $N_s := T/T_s$ is the number of samples per symbol and N is the number of observed symbols. The samples of the excess phase are represented as

$$\phi[n,\alpha] = 2\pi h \sum_{i=0}^{m} \alpha_i q_{n-iN_s}, \quad (4)$$

where $m = \lfloor \frac{n}{N_s} \rfloor$ is the symbol index and $\lfloor . \rfloor$ represents the floor operation. The received signal $x[n]$ observed in additive white Gaussian noise (AWGN) of variance σ^2 is given as

$$x[n] = s[n] + w[n]. \quad (5)$$

III. Method of Moments Estimator

This estimator is designed by computing the auto-correlation function of the received CPM signal at two different lags and then solving the resulting equations simultaneously. In this paper, since we are focusing on binary full-response single-h CPM, the auto-correlation function of these schemes for $0 \le \tau \le T$ is defined from [13] as

$$R(\tau) = \left(1 - \frac{\tau}{2T}\right) \cos\left(\frac{\pi h \tau}{T}\right) + \frac{1}{2\pi h} \sin\left(\frac{\pi h \tau}{T}\right). \quad (6)$$

Evaluating (6) at $\tau = T/2$ and $\tau = T$, we get

$$\gamma_1 = \left(\frac{3}{4}\right) \cos\left(\frac{\pi h}{2}\right) + \frac{\sin\left(\frac{\pi h}{2}\right)}{2\pi h} \quad (7)$$

and

$$\gamma_2 = \left(\frac{1}{2}\right) \cos(\pi h) + \frac{\sin(\pi h)}{2\pi h}, \quad (8)$$

respectively. Simultaneously solving these two equations along with use of double angle identities and substituting $y := \cos(\frac{\pi h}{2})$ help us express method of moments estimator as

$$\hat{h} = \frac{2}{\pi} \cos^{-1}\left(2\hat{\gamma}_1 - \sqrt{4\hat{\gamma}_1^2 - 2\hat{\gamma}_2 - 1}\right) \quad (9)$$

for $h \in [0, 1]$, where

$$\hat{\gamma}_1 = \frac{1}{N - \frac{N_s}{2}} \sum_{n=0}^{N-\frac{N_s}{2}-1} x[n]x\left[n + \frac{N_s}{2}\right] \quad (10)$$

and

$$\hat{\gamma}_2 = \frac{1}{N - N_s} \sum_{n=0}^{N-N_s-1} x[n]x[n + N_s] \quad (11)$$

are the consistent estimators for γ_1 & γ_2, respectively.

IV. Statistical Evaluation of the Proposed Estimator

In a non-Bayesian framework, an unbiased estimator having a minimum variance is desirable. Therefore, the performance evaluation of an estimator entails the computation of the mean and variance of the estimator. However, the method of moments approach, as observed in the previous section, generally results in estimators that are non-linear in the observations and it is not always easy to evaluate the mean and the variance of these estimators. For example, the proposed estimator $\hat{h}$ as expressed in (9) is a non-linear function of $\hat{\gamma}_1$ and $\hat{\gamma}_2$. Let $\hat{h} = g(\boldsymbol{T})$, where $\boldsymbol{T} = [\hat{\gamma}_1, \hat{\gamma}_2]^T$ is the statistics vector employed by the proposed estimator. In this situation, we can apply the statistical linearization approach using Taylor series of the estimator and then compute its approximate mean and variance [14]. In this method of statistical linearization, we compute the first-order Taylor series of the estimator about the true mean of the statistics under consideration, i.e., $\hat{\gamma}_1$ and $\hat{\gamma}_2$. Thus expanding Taylor series about the $\mathbb{E}(\boldsymbol{T}) = \boldsymbol{\mu}$, we obtain

$$\hat{h} = g(\boldsymbol{T}) \approx g(\boldsymbol{\mu}) + \sum_{k=1}^{2} \left.\frac{\partial g}{\partial \hat{\gamma}_k}\right|_{\hat{\gamma}_k = \gamma_k} (\hat{\gamma}_k - \gamma_k), \quad (12)$$

where $\boldsymbol{\mu} = [\gamma_1, \gamma_2]^T$ is the vector of true moments. As N increases, the probability density functions of the statistics concentrate about their respective means and the linear approximation in (12) becomes a better representation of the non-linear estimator. Now the mean of the estimator can be approximated as $\mathbb{E}(\hat{h}) \approx g(\boldsymbol{\mu})$ and similarly,

$$Var(\hat{h}) \approx \left.\frac{\partial g}{\partial \boldsymbol{T}}\right|_{\boldsymbol{T}=\boldsymbol{\mu}}^{T} \boldsymbol{C_T} \left.\frac{\partial g}{\partial \boldsymbol{T}}\right|_{\boldsymbol{T}=\boldsymbol{\mu}}, \quad (13)$$

where $\boldsymbol{C_T} = \mathbb{E}\left[(\boldsymbol{T} - \boldsymbol{\mu})(\boldsymbol{T} - \boldsymbol{\mu})^T\right]$ is the covariance matrix of $\boldsymbol{T}$. Using the above relations, we derive the mean and variance of $\hat{h}$ for full response CPM waveform.

A. Asymptotic Mean of Estimator

Under the large N approximation, we have that $\mathbb{E}(\hat{h}) \approx g(\gamma_1, \gamma_2)$. Let

$$u = 2\gamma_1 - \sqrt{4\gamma_1^2 - 2\gamma_2 - 1}, \tag{14}$$

be the inner part of $\cos^{-1}(.)$ from (9). Its radical part, after simplification, becomes a complete square as follows:

$$\sqrt{4\gamma_1^2 - 2\gamma_2 - 1} = \sqrt{\left(\frac{\cos\theta}{2} + \frac{\sin\theta}{2\theta}\right)^2}, \tag{15}$$

where $\theta := \frac{\pi h}{2}$. Thus (14) becomes

$$u = 2\gamma_1 - \sqrt{4\gamma_1^2 - 2\gamma_2 - 1} = \cos\theta. \tag{16}$$

Putting this value of u in (9), we get

$$\mathbb{E}[\hat{h}] \approx g(\gamma_1, \gamma_2) = \frac{2}{\pi}\cos^{-1}(\cos\theta) = \frac{2}{\pi}(\theta) = h, \tag{17}$$

implying that the estimator is asymptotically unbiased.

B. Asymptotic Variance of Estimator

The variance of the proposed estimator can be approximated using (13), which can be re-expressed as

$$Var(\hat{h}) \approx \begin{bmatrix} \frac{\partial g}{\partial \gamma_1} & \frac{\partial g}{\partial \gamma_2} \end{bmatrix} \begin{bmatrix} Var(\hat{\gamma_1}) & Cov(\hat{\gamma_1}, \hat{\gamma_2}) \\ Cov(\hat{\gamma_2}, \hat{\gamma_1}) & Var(\hat{\gamma_2}) \end{bmatrix} \begin{bmatrix} \frac{\partial g}{\partial \gamma_1} \\ \frac{\partial g}{\partial \gamma_2} \end{bmatrix}. \tag{18}$$

Below we discuss the computation of all the terms in the above equation. Firstly, the partial derivatives of the estimator (given in (18)) with respect to each statistic are computed. Using the chain rule, we obtain

$$\frac{\partial g}{\partial \gamma_1} = \frac{2}{\pi}\left(\frac{-1}{\sqrt{1-u^2}}\frac{\partial u}{\partial \gamma_1}\right), \tag{19}$$

which simplifies to

$$\frac{\partial g}{\partial \gamma_1} = \frac{16h\cos\left(\frac{\pi h}{2}\right)}{\pi h \sin(\pi h) + 4\sin^2\left(\frac{\pi h}{2}\right)}. \tag{20}$$

Similarly,

$$\frac{\partial g}{\partial \gamma_2} = \frac{-8h}{\pi h \sin(\pi h) + 4\sin^2\left(\frac{\pi h}{2}\right)}. \tag{21}$$

Secondly, the $Var(\hat{\gamma_1}) := \mathbb{E}[(\hat{\gamma_1} - \mathbb{E}[\hat{\gamma_1}])^2]$ requires the mean of $\hat{\gamma_1}$, which can be computed from (10) and reduces to

$$\mathbb{E}[\hat{\gamma_1}] = \frac{1}{N - \frac{N_s}{2}} \sum_{n=0}^{N-\frac{N_s}{2}-1} s[n]s\left[n + \frac{N_s}{2}\right]. \tag{22}$$

Using (10) and (22), the $Var(\hat{\gamma_1})$ simplifies to the following form from the one mentioned at the bottom of this page:

$$Var(\hat{\gamma_1}) = \frac{\sigma^2}{\left(N - \frac{N_s}{2}\right)^2}\Bigg[2\left(N - \frac{N_s}{2}\right) + \sigma^2\left(N - \frac{N_s}{2}\right) + 2(N - N_s)R(T)\Bigg], \tag{23}$$

where $R(T)$ is the autocorrelation function at lag T. Similarly, the $Var(\hat{\gamma_2})$ simplifies to the following form from the one mentioned at the top of next page:

$$Var(\hat{\gamma_2}) = \frac{\sigma^2}{(N - N_s)^2}\Big[2(N - N_s) + \sigma^2(N - N_s) + 2(N - 2N_s)R(2T)\Big], \tag{24}$$

where $R(2T)$ is the autocorrelation function at lag $2T$. Fourthly, the computation of the covariance of $\hat{\gamma_1}$ & $\hat{\gamma_2}$ defined as

$$Cov(\hat{\gamma_1}, \hat{\gamma_2}) = \mathbb{E}([\hat{\gamma_1} - \mathbb{E}(\hat{\gamma_1})][\hat{\gamma_2} - \mathbb{E}(\hat{\gamma_2})]) \tag{25}$$

requires $\hat{\gamma_1}$, $\hat{\gamma_2}$, $\mathbb{E}[\hat{\gamma_1}]$ and $\mathbb{E}[\hat{\gamma_2}]$, which have already been computed. Using these terms, $Cov(\hat{\gamma_1}, \hat{\gamma_2})$ is as shown at the bottom of next page and it further simplifies to

$$Cov(\hat{\gamma_1}, \hat{\gamma_2}) = \frac{2\sigma^2}{(N - N_s)\left(N - \frac{N_s}{2}\right)}\Bigg[(N - N_s)R\left(\frac{T}{2}\right) + \left(N - \frac{3N_s}{2} + 1\right)R\left(\frac{3T}{2}\right)\Bigg]. \tag{26}$$

With the analytic expressions for $\frac{\partial g}{\partial \gamma_1}$, $\frac{\partial g}{\partial \gamma_2}$, $Var(\hat{\gamma_1})$, $Var(\hat{\gamma_2})$ and $Cov(\hat{\gamma_1}, \hat{\gamma_2})$ derived, we can find the expression for $Var(\hat{h})$ by putting these terms in (18).

V. SIMULATIONS

In this section, we perform numerical simulations to assess the performance of the proposed estimator in terms of its bias and variance. We also compare the bias and variance of the proposed estimator with their approximate versions based upon the Taylor series expansion. The simulations are performed for the binary full-response CPM signals received in AWGN.

The approximate variance of the estimator depends upon $Var\left(\hat{\gamma_1}\right)$, $Var\left(\hat{\gamma_2}\right)$, $Cov\left(\hat{\gamma_1}, \hat{\gamma_2}\right)$ and the partial derivatives of the estimator, as apparent from (18). In Fig. 1, we observe

$$Var(\hat{\gamma_1}) = \mathbb{E}\left[\sum_{n=0}^{N-\frac{N_s}{2}-1} s[n]w\left[n + \frac{N_s}{2}\right] \sum_{m=0}^{N-\frac{N_s}{2}-1} s[m]w\left[m + \frac{N_s}{2}\right]\right] + \mathbb{E}\left[\sum_{n=0}^{N-\frac{N_s}{2}-1} s\left[n + \frac{N_s}{2}\right]w[n] \sum_{m=0}^{N-\frac{N_s}{2}-1} s\left[m + \frac{N_s}{2}\right]w[m]\right]$$
$$+ \mathbb{E}\left[\sum_{n=0}^{N-\frac{N_s}{2}-1} w[n]w\left[n + \frac{N_s}{2}\right] \sum_{m=0}^{N-\frac{N_s}{2}-1} w[m]w\left[m + \frac{N_s}{2}\right]\right] + 2\mathbb{E}\left[\sum_{n=0}^{N-\frac{N_s}{2}-1} s[n]w\left[n + \frac{N_s}{2}\right] \sum_{m=0}^{N-\frac{N_s}{2}-1} s\left[m + \frac{N_s}{2}\right]w[m]\right]$$

$$Var(\hat{\gamma}_2)=\mathbb{E}\left[\sum_{n=0}^{N-N_s-1} s[n]w\left[n+N_s\right]\sum_{m=0}^{N-N_s-1} s[m]w\left[m+N_s\right]\right]+\mathbb{E}\left[\sum_{n=0}^{N-N_s-1} s\left[n+N_s\right]w[n]\sum_{m=0}^{N-N_s-1} s\left[m+N_s\right]w[m]\right]$$
$$+\mathbb{E}\left[\sum_{n=0}^{N-N_s-1} w[n]w\left[n+N_s\right]\sum_{m=0}^{N-N_s-1} w[m]w\left[m+N_s\right]\right]+2\mathbb{E}\left[\sum_{n=0}^{N-N_s-1} s[n]w\left[n+N_s\right]\sum_{m=0}^{N-N_s-1} s\left[m+N_s\right]w[m]\right]$$

the second order statistics of $\hat{\gamma}_1$ and $\hat{\gamma}_2$ as analytically expressed in (23), (24) and (26) as a function of the modulation index. It is observed that the simulated and analytical results are in close agreement for $h < 0.7$, while the convergence is relatively slower for $h \in [0.7, 1.0]$.

Fig. 2 shows the variance of $\hat{h}$ for minimum shift keying (MSK), i.e., $h = 0.5$, as a function of the number of observed symbols N for fixed $N_s = 4$. We see that both the simulated and analytical results are very close, and due to the law of large numbers, the variance decreases with increasing N. The reduction in variance is also significant with increase in SNR. A similar decreasing trend (not shown here for lack of space) is obtained for increasing N_s with N fixed, though, the decrease in variance with N is more dramatic as compared to that with N_s.

Fig. 3 shows the dependence of the estimator MSE (and its analytical approximation) on the modulation index h itself, for fixed $N = 30000$. The performance of the estimator is satisfactory in the region $h \in [0.2, 0.8]$ where most of the practical modulation schemes belong, e.g., the variants of Bluetooth, GMSK, etc. However, it slightly deteriorates when the modulation index is close to the endpoints of the interval $[0, 1]$. It can also be observed that the approximation to the variance of (18) is also less accurate at the endpoints. Similarly, the convergence is slower for h close to 1 owing to the large value of the $Var(\hat{\gamma}_1)$.

Bias of the proposed estimator reduces with increasing number of symbols N as depicted in Fig. 4. It can also be seen that the bias of the estimator shows a convergence trend similar to variance vis-á-vis the modulation index, i.e., the estimator is least biased for values of h close to 0.5.

Fig. 5 presents MSE versus SNR-based performance comparison of the proposed method of moments estimator with HOS and NDA-CYC estimators of [10] and [11], respectively. The performance of the NDA-CYC is poor at low SNR due to the phase-unwrapping errors. The MSE of the HOS based estimator is larger due to the slow convergence of the fourth-order cumulants. The proposed estimator is based upon the auto-correlation function and outperforms

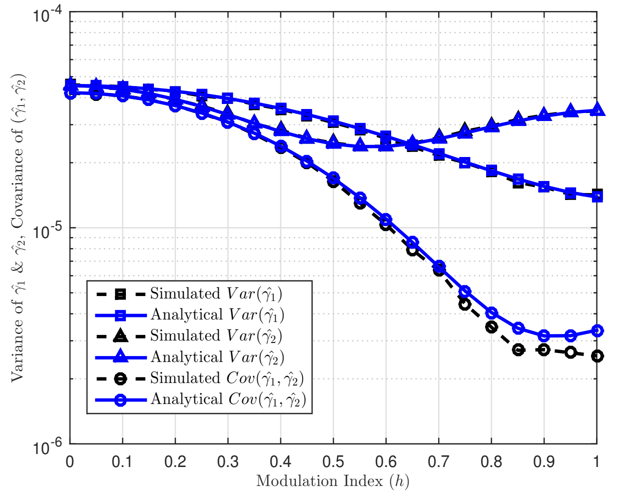

Fig. 1. Simulated and analytical results for variance and covariance of $\hat{\gamma}_1$ and $\hat{\gamma}_2$ vs. modulation index with $N = 30000$, $N_s = 4$ and SNR = 10 dB.

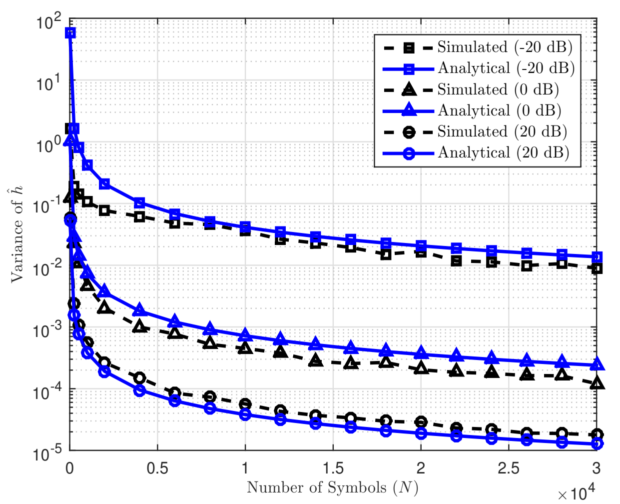

Fig. 2. Simulated and analytical variance of the proposed estimator vs. N with $h = 0.5$ and $N_s = 4$.

$$Cov(\hat{\gamma}_1,\hat{\gamma}_2)=\mathbb{E}\left[\sum_{n=0}^{N-\frac{N_s}{2}-1} s[n]w\left[n+\frac{N_s}{2}\right]\sum_{m=0}^{N-N_s-1} s[m]w\left[m+N_s\right]\right]+\mathbb{E}\left[\sum_{n=0}^{N-\frac{N_s}{2}-1} s[n]w\left[n+\frac{N_s}{2}\right]\sum_{m=0}^{N-N_s-1} s\left[m+N_s\right]w[m]\right]$$
$$+\mathbb{E}\left[\sum_{n=0}^{N-\frac{N_s}{2}-1} s\left[n+\frac{N_s}{2}\right]w[n]\sum_{m=0}^{N-N_s-1} s[m]w\left[m+N_s\right]\right]+\mathbb{E}\left[\sum_{n=0}^{N-\frac{N_s}{2}-1} s\left[n+\frac{N_s}{2}\right]w[n]\sum_{m=0}^{N-N_s-1} s\left[m+N_s\right]w[m]\right]$$

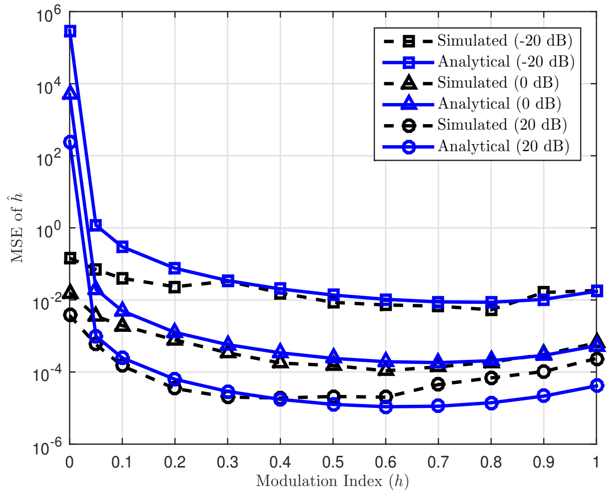

Fig. 3. Simulated and analytical MSE of the proposed estimator vs. modulation index with $N = 30000$ and $N_s = 4$.

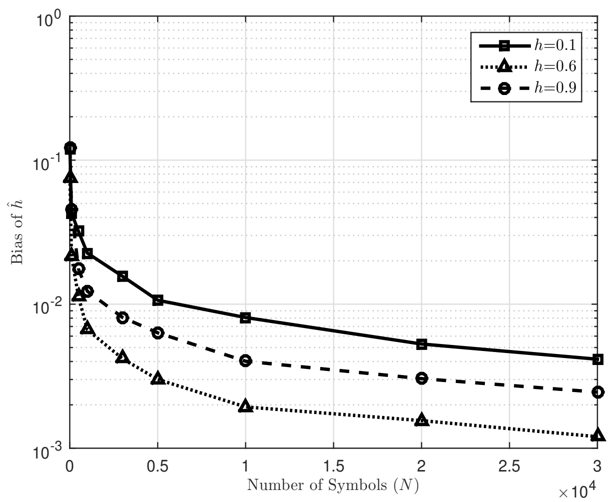

Fig. 4. Bias of the proposed estimator vs. N with SNR $= 10$ dB and $N_s = 4$.

the previously proposed non data-aided estimators in the low SNR scenario. Specifically, when SNR is less than 10 dB, the performance is better than both HOS and NDA-CYC estimators.

VI. Conclusion

In this paper, we have proposed a novel closed-form estimator for the modulation index based upon the auto-correlation function of the CPM signal. Analytical expressions are derived for the asymptotic mean and variance of the proposed estimator using statistical linearization approach. Numerical analysis showed the convergence to asymptotic mean-square error performance for large data records. A comparison of the performance with existing non data-aided methods showed that the proposed estimator performs better than the prior art at low signal-to-noise ratio. Although, the

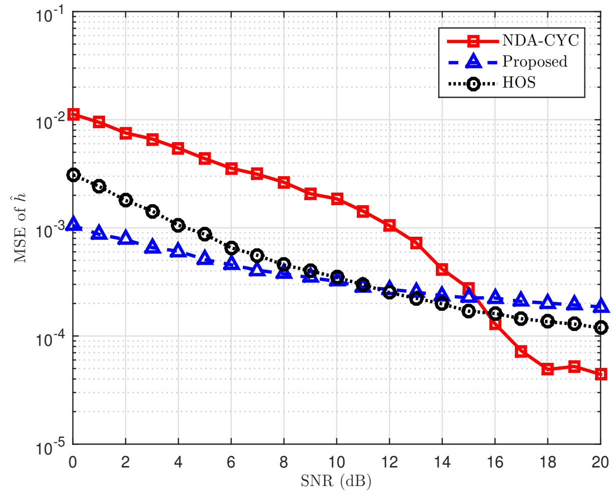

Fig. 5. Performance comparison of the proposed estimator with higher order statistics (HOS)-based estimator of [10] and non data-aided cyclic (NDA-CYC) estimator of [11]. The MSE of these estimators is presented vs. SNR for $h = 0.6$, $N = 1000$ and $N_s = 4$.

closed-form solution is presented for full-response and binary CPM signals, but using the same principles, a numerical algorithm can be used to estimate the modulation index for partial response or non-binary CPM signals.

References

[1] C. E. Sundberg, "Continuous phase modulation," *IEEE Commun. Mag.*, vol. 24, no. 4, pp. 25–38, 1986.
[2] ESTI, Digital Enhanced Cordless Telecommunications (DECT); Ultra Low Energy (ULE); Machine to Machine Communications; Part 1: Home Automation Network (phase 1), 2013.
[3] D. Bonacci, R. Prévost, J.-P. Millerioux, and J. LeMaitre, "Advanced concepts for satellite reception of AIS messages," in *Proc. Toulouse Space Show*, pp. 25–28, 2012.
[4] G. K. Kaleh, "Simple coherent receivers for partial response continuous phase modulation," *IEEE J. Sel. Areas Commun.*, vol. 7, no. 9, pp. 1427–1436, 1989.
[5] M. Messai, G. Colavolpe, K. Amis, and F. Guilloud, "Binary continuous phase modulations robust to a modulation index mismatch," *IEEE Tran. Commun.*, vol. 63, no. 11, pp. 4267–4275, 2015.
[6] M. Messai, G. Colavolpe, K. Amis, and F. Guilloud, "Robust detection of binary CPMs with unknown modulation index," *IEEE Commun. Lett.*, vol. 19, no. 3, pp. 339–342, 2015.
[7] S. A. Ullah, I. Khalid, S. Saleem, and S. A. Hassan, "On the estimation of modulation index for binary full response CPM signals," *IEEE Communications Letters*, vol. 22, pp. 918–921, May 2018.
[8] M. Messai, F. Guilloud, and K. Amis, "A low complexity coherent CPM receiver with modulation index estimation," in *Proc. 22nd Eur. Signal Process. Conf. (EUSIPCO)*, pp. 979–983, IEEE, 2014.
[9] D. Xu and Y. Zhang, "Estimation of the modulation index of CPM signals based on Laurent's decomposition," *IEEE Trans. Wireless Commun.*, vol. 12, no. 12, pp. 6268–6280, 2013.
[10] J. R. Fonollosa and J. A. R. Fonollosa, "Estimation of the modulation index of CPM signals using higher-order statistics," in *proc. ICASSP*, vol. 4, pp. 268–271, IEEE, 1993.
[11] P. Bianchi, P. Loubaton, and F. Sirven, "Non data-aided estimation of the modulation index of continuous phase modulations," *IEEE Trans. Signal Process.*, vol. 52, no. 10, pp. 2847–2861, 2004.
[12] T. Munawar, S. Saleem, S. A. Hassan, and S. M. H. Zaidi, "Estimation of modulation index for partial response CPM signal," *IEEE Access*, vol. 6, pp. 7664–7674, 2018.
[13] F. Xiong, *Digital modulation techniques*. Artech House, 2006.
[14] S. M. Kay, "Fundamentals of Statistical Signal Processing, volume I: Estimation Theory," 1993.