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On Decoupled Decoding of Quasi-Orthogonal STBCs Using Quaternion Algebra

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***Abstract*—Receiver complexity is an important performance metric in space time block coding (STBC). After comprehensive study and analysis of STBCs over complex domain, quaternion orthogonal designs (QODs) have been explored that can provide less decoder complexity. However, some researchers argue that existing maximum likelihood (ML) decoding rule for QODs does not yield optimal performance for quasi-orthogonal STBC-based QODs and, therefore, the design of optimal decoder for such designs remains a big challenge of this field. To counter this observation, we modify the system model slightly. In addition to this, we present a new generalized QOD construction that can generate QODs for any number of transmit antennas. These codes are quasi-orthogonal in complex domain and provide code rates higher than their respective complex orthogonal designs (CODs). We show that the proposed ML decoder provides optimal linear decoupled decoding solution for these codes because of their distinctive construction. Simulation results verify the optimal performance of the ML decoder for the proposed codes and also show that the diversity of the proposed quaternion-based codes is same as CODs, however, they provide better code rates.**

***Index Terms*—Maximum-likelihood (ML) decoder, quaternion orthogonal designs (QODs), quasi space time block codes (STBC).**

I. Introduction

SPACE time block codes (STBCs) have been employed in various communication standards and have successfully elevated the capacity of communication systems. More specifically, the complex orthogonal designs (CODs), a class of STBCs, have been extensively used because of their potential to provide full transmit diversity and simplified decoupled maximum-likelihood (ML) decoding. However, their code rates, i.e., the ratio of the number of independent complex transmitted symbols and the number of total time slots taken to transmit a COD block, become smaller as the number of transmit antennas increases [1]. To overcome this performance barrier and further elevate the capacity of communication systems, the work in [2] proposed the use of quaternion orthogonal designs (QODs), which laid the foundation of orthogonal space time polarization block codes. One of the main motivations behind these codes is to obtain larger transmit diversity gains and better code rates.

To realize quaternionic structures in multiple-input multiple-output (MIMO) systems, Seberry *et al.* in [2] proposed a dual-polarized antenna-based system model and used two transmit (Tx) and one receive (Rx) antenna configuration, i.e., a 2×1 order QOD model for data transmission. The authors obtained decoupled decoding for this specific QOD and conjectured that the optimal decoupled decoding can be achieved for any QOD construction. Their subsequent works [3]–[4] used the same QOD and, therefore, remained limited to two dual-polarized transmit antenna based configuration. These initial works on QODs emphasized the same claim that optimal decoupled decoding can be achieved for any QOD construction. Later on, Wysocki *et al.* corrected their decoding rule in [5] and highlighted that the proposed decoding decoding rule does not yield optimal decoding for any QOD [6]. A recent work, [7], extended the QOD construction given in [2] to higher dimensions, where the proposed QODs are comprised of orthogonal complex vectors and, therefore, a decoupled decoding can be obtained. Until now, decoupled decoding has been derived for either orthogonal or same vectors of existing STBCs or QODs. Therefore, there has been a dire need to exploit quaternionic structure of QODs to obtain optimal or semioptimal decoupled decoding for higher order quasi-orthogonal STBC-based QODs [6].

Given the amount of work on QODs, a major contribution of this paper is that it is the first paper, to the best of our knowledge, that proposes an optimal solution to the problem highlighted by Seberry *et al.* of the work on QODs, [2], i.e., the issue of decoupled decoding [6]. For this purpose, we present a quaternionic structure-based system model that exploits space, time, and polarization diversity jointly. We propose a generalized QOD construction technique and generate codes for data transmission on the quaternionic system model. We also derive a decoupled ML decoding rule for these codes. The proposed decoding rule also provides an optimal linear decoding solution for the QOD presented in [6] as a problem statement. In this paper, we also highlight the necessary constraints to achieve simplified decoupled decoding for any QOD or quasi-orthogonal STBC. In addition to this, we also compare the simulation results of the proposed construction to CODs and verify that for the given system model, both the designs perform same in terms of diversity. However, the proposed design provides better code rate as compared to their COD counterparts.

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II. Quaternion Orthogonal Designs

A QOD $\mathbf{Q}$ on complex variables $\{z_1, z_2, \ldots, z_u\}$ of type $\{s_1, s_2, \ldots, s_u\}$ is a $r \times n$ matrix, which can have entries from the set $\{0, z_1, z_1^*, z_2, z_2^*, \ldots, z_u, z_u^*\}$ including possible multiplications on the left and/or right by quaternion elements $q \in \mathbf{Q}$, and satisfying the following condition:

$$\mathbf{Q}^Q\mathbf{Q} = \sum_{h=1}^{u}(s_h(|z_h|)^2)\mathbf{I}_n = \lambda\mathbf{I}_n \quad (1)$$

where λ is a positive real number and $\mathbf{I}_{n\times n}$ is the $n \times n$ identity matrix. Moreover, the operator $(.)^Q$ denotes the quaternion transpose of a matrix and we refer [10] for the explanation of operations of quaternion algebra. Among the various proposed QOD construction techniques, [2] used symmetric-paired design to construct a 2×1 QOD and derived its decoding statistics. According to symmetric-paired design, two CODs $\mathbf{A}$ and $\mathbf{B}$ based on complex variables $\{z_1, z_2, \ldots, z_u\}$ form a QOD $\mathbf{A} + \mathbf{B}j$, provided $\mathbf{A}^H\mathbf{B}$ or $\mathbf{B}^H\mathbf{A}$ is symmetric, where $(.)^H$ indicates the Hermitian operator.

Since [2] employed a 2×1 order symmetric-paired design only, we propose a generalized construction technique that can generate QODs for any number of transmit antennas. It is important to note that in this construction, the COD $\mathbf{B}$ is not merely a column permutation of COD $\mathbf{A}$ as given in [7]. This generalization is based on one of the three proposed generic COD construction methods in [1], namely, Adams–Lax–Phillips, Józefiak and Wolfe constructions. We can apply the proposed procedure of generating QODs on all of them; however, in this study, we demonstrate it using Józefiak construction. A general COD is designed for $l + 1$ symbols embeded in a square matrix of order 2^l such that

$$\mathbf{A} = \begin{bmatrix} \mathbf{G}_{2^{l-1}}(z_1, z_2, \ldots, z_l) & z_{l+1}\mathbf{I}_{2^{l-1}} \\ -z_{l+1}^*\mathbf{I}_{2^{l-1}} & \mathbf{G}_{2^{l-1}}^H(z_1, z_2, \ldots, z_l) \end{bmatrix} \quad (2)$$

where $l = \{1, 2, 3 \ldots\}$ and $\mathbf{G}_{2^{l-1}}(z_1, z_2, \ldots, z_l)$ represents a COD of order $2^{l-1} \times 2^{l-1}$ defined on symbols $\{z_1, z_2, \ldots, z_l\}$. For example, for $l = 1$, $\mathbf{G}_1(z_1) = [z_1]$. We will use this construction of generalized CODs in the subsequent lemmas and prove the existence of generalized QODs in which the matrix $\mathbf{B}$ is totally different from matrix $\mathbf{A}$ to construct a QOD. A remarkable feature of this construction is that we attain decoupled decoding contrary to what was previously thought that it is only possible for the construction in which the COD $\mathbf{B}$ is the permutation of columns of $\mathbf{A}$. Before providing the main construction, consider the following lemma.

Lemma 2.1: For two generalized CODs, $\mathbf{G}_{2^l}(z_1, z_2, \ldots, z_{l+1})$ and $\mathbf{L}_{2^l}(z_{l+2}, z_{l+3}, \ldots, z_{2l+2})$ with same structure, which are constructed on the COD formulation (2), it follows that

$$\mathbf{G}_{2^l}^H\mathbf{L}_{2^l} + \mathbf{L}_{2^l}^H\mathbf{G}_{2^l} = \mathbf{G}_{2^l}\mathbf{L}_{2^l}^H + \mathbf{L}_{2^l}\mathbf{G}_{2^l}^H = \gamma\mathbf{I}_{2^l} \quad (3)$$

where $\gamma = 2\Re(\sum_{k=1}^{l+1} z_k^* z_{l+1+k})$ and $\Re(.)$ represents the real part of a complex number.

Proof: It is straightforward to show that

$$\mathbf{G}_{2^l}^H\mathbf{L}_{2^l} + \mathbf{L}_{2^l}^H\mathbf{G}_{2^l} = \mathbf{G}_{2^l}\mathbf{L}_{2^l}^H + \mathbf{L}_{2^l}\mathbf{G}_{2^l}^H = \begin{bmatrix} \mathbf{M}_{2^{l-1}} & \mathbf{O}_{2^{l-1}} \\ \mathbf{O}_{2^{l-1}} & \mathbf{N}_{2^{l-1}} \end{bmatrix} \quad (4)$$

where $\mathbf{M}_{2^{l-1}} = \mathbf{E}^H\mathbf{F} + \mathbf{F}^H\mathbf{E} + (z_{2l+2}z_{l+1}^* + z_{2l+2}^* z_{l+1})\mathbf{I}$, $\mathbf{N}_{2^{l-1}} = \mathbf{E}\mathbf{F}^H + \mathbf{F}\mathbf{E}^H + (z_{2l+2}z_{l+1}^* + z_{2l+2}^* z_{l+1})\mathbf{I}$ and, $\mathbf{E}_{2^{l-1}}(z_1, z_2, \ldots, z_l)$ and $\mathbf{F}_{2^{l-1}}(z_{l+2}, z_{l+3}, \ldots, z_{2l+1})$ are the CODs used in the formulation of $\mathbf{G}_{2^l}(z_1, z_2, \ldots, z_{l+1})$ and $\mathbf{L}_{2^l}(z_{l+2}, z_{l+3}, \ldots, z_{2l+2})$, respectively, such as $\mathbf{G}_{2^{l-1}}$ is used for the construction of COD $\mathbf{A}$ given in (2). Moreover, $\mathbf{O}$ represents a null matrix. Now to simplify the diagonal terms of the expression (4), we observe that $\mathbf{E}^H\mathbf{F} + \mathbf{F}^H\mathbf{E} = \mathbf{E}\mathbf{F}^H + \mathbf{F}\mathbf{E}^H$. As both matrices $\mathbf{E}$ and $\mathbf{F}$ are constructed on the same format but on different complex numbers, therefore, for any vector $\mathbf{a}_i$ of COD $\mathbf{E}$ and its corresponding vector $\mathbf{b}_i$ of COD $\mathbf{F}$, where i denotes the index of column, we get $\mathbf{a}_i^H\mathbf{b}_i + \mathbf{b}_i^H\mathbf{a} = \sum_{k=1}^{l}(z_k)^*(z_{l+1+k}) + \sum_{k=1}^{l}(z_k)(z_{l+1+k})^*$. Consequently, diagonal terms of (4) equate to $\gamma\mathbf{I}_{2^l}$, hence, Lemma 2.2 is proved. ■

It is important to mention that the above lemma does not hold true for two general CODs. For example, two Almouti codes with different structure $\left[\begin{smallmatrix} z_1 & z_2 \\ z_2^* & -z_1^* \end{smallmatrix}\right]$ and $\left[\begin{smallmatrix} z_3 & z_4 \\ -z_4^* & z_3^* \end{smallmatrix}\right]$, fail to satisfy it while they can be used effectively in generating a consistent COD of the form (2). By using the above lemma, we prove the following theorem.

Theorem 2.1: For generalized CODs $\mathbf{G}_{2^{l-1}}(z_1, z_2, \ldots, z_l)$ and $\mathbf{L}_{2^{l-1}}(z_{1+2}, z_2, \ldots, z_{2l+2})$, a symmetric-paired design

$$\mathbf{Q}_{2^{l+1}\times 2^l}(z_1, \ldots, z_{2(l+1)}) = \begin{bmatrix} \mathbf{G}_{2^l} + \mathbf{L}_{2^l}j, \\ \mathbf{L}_{2^l} + \mathbf{G}_{2^l}j \end{bmatrix} \quad (5)$$

is a QOD of dimension $2^{l+1} \times 2^l$ with rate $(l + 1)/2^l$.

Proof: In order to prove quaternion orthogonality, the quaternion conjugate of (5) can be expressed as $\mathbf{Q}_{2^l\times 2^{l+1}}^Q = [\,\mathbf{G}_{2^l}^H - j\mathbf{L}_{2^l}^H \quad \mathbf{L}_{2^l}^H - j\mathbf{G}_{2^l}^H\,]$. We now need to multiply it with (5) to satisfy condition (1), where the first term in the product is

$$\left(\mathbf{G}_{2^l}^H - j\mathbf{L}_{2^l}^H\right)\left(\mathbf{G}_{2^l} + \mathbf{L}_{2^l}j\right) = \mathbf{G}_{2^l}^H\mathbf{G}_{2^l} + \mathbf{L}_{2^l}^H\mathbf{L}_{2^l} \quad (6)$$

and where we have used the fact that $\mathbf{L}_{2^l}^H\mathbf{G}_{2^l}$ is the Hermitian conjugate of $\mathbf{G}_{2^l}^H\mathbf{L}_{2^l}$ and $zj = jz^* \;\forall z \in \mathcal{C}$, therefore, $\mathbf{L}_{2^l}^H\mathbf{G}_{2^l}j - j\mathbf{G}_{2^l}^H\mathbf{L}_{2^l} = \mathbf{O}$. Using expression (6) and basic quaternion arithmetic, we obtain $\mathbf{Q}_{2^l\times 2^{l+1}}^Q\mathbf{Q}_{2^{l+1}\times 2^l} = \lambda\mathbf{I}_{2^l}$, where $\lambda_1\mathbf{I}_{2^l} = \mathbf{G}_{2^l}^H\mathbf{G}_{2^l} + \mathbf{L}_{2^l}^H\mathbf{L}_{2^l}$ and $\lambda = 2\lambda_1$. This completes the proof of Theorem 2.1. ■

To elaborate the generalized construction technique, we present a QOD of rate 1, where the COD $\mathbf{A}$ contains symbols z_1 and z_2, while the COD $\mathbf{B}$ contains independent symbols z_3 and z_4, respectively.

Rate 1 QOD: Using (5), we obtain the following symmetric-paired design of order 4×2 with a code rate of 1, i.e.,

$$\mathbf{Q}_1 = \begin{bmatrix} z_1 + z_3 j & z_2 + z_4 j \\ z_2^* + z_4^* j & -z_1^* - z_3^* j \\ z_3 + z_1 j & z_4 + z_2 j \\ z_4^* + z_2^* j & -z_3^* - z_1^* j \end{bmatrix} \quad (7)$$

where $l = 1$, $\mathbf{G}_1 = [z_1]$, and $\mathbf{L}_1 = [z_3]$ from (5).

It is worth-noticing that (5) generates QODs for only those antenna arrangements where dual-polarized transmit antennas are in powers of two, e.g., 2, 4, 8, etc. For other antenna arrangements, we can simply omit extra columns in the given QOD to obtain the desired code. The above construction in complex domain generates quasi-orthogonal STBCs in three main ways and pairwise ML decoding is employed at the receiver [11]. First, we obtain a complex code $\mathbf{C}_Q$ from a QOD $\mathbf{Q}$ in (5) by decomposing it as follows. The subscript Q indicates that we obtain a quasi complex code in this way. The first column of $\mathbf{G}$ and $\mathbf{L}$ in a QOD (5) becomes the first two columns of $\mathbf{C}_Q$. In this way, the odd columns in $\mathbf{C}_Q$ represent signals transmitted through one polarization, whereas even columns contain signals through orthogonal polarization. Such codes satisfy following corollary.

Corollary 2.2: $\mathbf{C}_Q(z_1,\ldots,z_{2(l+1)})$, a complex STBC, of the QOD construction given in (5) yields

$$\mathbf{C}_q^H\mathbf{C}_q = \lambda_1\mathbf{I} + \gamma\mathbf{E} \tag{8}$$

where $\lambda_1 = \sum_{k=1}^{2(l+1)} |z_k|^2$, γ is given in Lemma 2.1 and $\mathbf{E}$ is a symmetric matrix containing zeros and ones only. Another way to obtain a quasi-orthogonal STBC from QOD (5) is

$$\mathbf{C}_q(z_1,\ldots,z_{2(l+1)}) = \begin{bmatrix} \mathbf{G}_{2^l} & \mathbf{L}_{2^l} \\ \mathbf{L}_{2^l} & \mathbf{G}_{2^l} \end{bmatrix} \tag{9}$$

where subscript q indicates that it is a complex quasi-orthogonal STBC. The above procedure of obtaining quasi-orthogonal codes through QODs is not unique and we can also generate Almouti-type codes as

$$\tilde{\mathbf{C}}_q(z_1,\ldots,z_{2(l+1)}) = \begin{bmatrix} \mathbf{G}_{2^l} & \mathbf{L}_{2^l} \\ \mathbf{L}_{2^l}^H & -\mathbf{G}_{2^l}^H \end{bmatrix} \tag{10}$$

using symmetric-paired designs $\mathbf{G}$ and $\mathbf{L}$, arising from an equivalent QOD

$$\tilde{\mathbf{Q}}_{2^{l+1}\times 2^l}(z_1,\ldots,z_{2(l+1)}) = \begin{bmatrix} \mathbf{G}_{2^l} + \mathbf{L}_{2^l}j\,, \\ \mathbf{L}_{2^l}^H - \mathbf{G}_{2^l}^H j \end{bmatrix}.$$

Following remarks are crucial in the above realizations.

Remark 1: Note that the crucial difference that brings orthogonality for dual polarized antennas in QODs comes from the fact that the identity $\mathbf{L}_{2^l}^H\mathbf{G}_{2^l}j - j\mathbf{G}_{2^l}^H\mathbf{L}_{2^l} = \mathbf{O}$, only holds in the quaternion domain. On the other hand, complex STBC $\mathbf{C}_Q$ fails to attain full orthogonality due to absence of quaternionic coupling through j.

Remark 2: In [7], Mushtaq *et al.* analyzed the performance of those QODs in which COD $\mathbf{B}$ is a permuted version of $\mathbf{A}$ and obtained decoupled decoding only because the term γ was equal to a decoupled term λ_1. However, in the QOD construction (5), the term γ itself contains coupled terms of transmitted symbols.

We now present our system model that contains a new channel based on quaternionic structure. The novelty of this approach is that it immediately provides decoupled decoding for all above quasi-orthogonal STBCs. Before proceeding further, we note that the following result holds.

Corollary 2.3: For an arbitrary complex STBC $\mathbf{C}_q$, the product $\mathbf{P} = \mathbf{C}_q^H\mathbf{C}_q$, is necessarily Hermitian, i.e., $\mathbf{P}^H = \mathbf{P}$.

III. System Model and ML Decoding

Consider N_t dual-polarized transmit and two dual-polarized receive antennas in a MIMO system. The transmitting antennas transmit the data streams that are received in quaternionic form at the receiver. For a single dual-polarized receiving antenna, channel gain matrix in conventional system model [2] is demonstrated as

$$\mathbf{H}_c^{(m)} = \begin{bmatrix} h_{11}^{(m)} & h_{12}^{(m)} \\ h_{21}^{(m)} & h_{22}^{(m)} \end{bmatrix},$$

where $m = \{1,2,\ldots,N_t\}$. The diagonal terms $h_{11}^{(m)}$ and $h_{22}^{(m)}$ depict the channel gains in the received signals with the same transmitted polarization, whereas, the off-diagonal terms, $h_{12}^{(m)}$ and $h_{21}^{(m)}$, show the channel gains in the received signals with different polarization than the transmitted one. This change in polarization, also known as *cross polar scatter*, can be the result of polarization twist, scattering, and reflections between Tx and Rx antennas. Under the consideration of zero polarization with two receiving antennas, the channel gain matrix reduces to

$$\mathbf{H}^{(m)} = \begin{bmatrix} h_{11}^{(m,1)} & h_{11}^{(m,2)} \\ h_{22}^{(m,1)}j & h_{22}^{(m,2)}j \end{bmatrix},$$

where the superscript (m,r) indicates transmit $m = \{1,2,\ldots,N_t\}$ antenna along with receive $r = \{1,2\}$ antenna numbers, respectively.

This modified channel matrix considers zero cross polar scattering and is based on the exploitation of polarization diversity to induce quaternionic structure in the received signal at the receiver. The channel is assumed to be Rayleigh fading, which implies that each element of channel gain matrix is a complex Gaussian random variable (RV) with zero mean and unit variance. For this $N_t \times 2$ antenna arrangement, the received signal $\mathbf{R}$ can be written as

$$\mathbf{R} = \mathbf{C}_q \begin{bmatrix} \mathbf{H}^{(1)} \\ \vdots \\ \mathbf{H}^{(N_t)} \end{bmatrix} + \begin{bmatrix} n_{11} & n_{12} \\ \vdots & \vdots \\ n_{N_t1} & n_{N_t2} \end{bmatrix} = \mathbf{C}_q\mathbf{H} + \mathbf{N} \tag{11}$$

where $n_{k_1k_2} \forall k_1 = \{1,2,\ldots,N_t\}, k_2 = \{1,2\}$ represent quaternions $n_{k_1k_2} = n_{k_1k_2}^V + n_{k_1k_2}^H j$, constructed by combining the noise added in vertical, V, and horizontal, H, plane of receiving antennas, respectively. Individual components, i.e., $n_{k_1k_2}^V$ and $n_{k_1k_2}^H$ of noise $n_{k_1k_2}$ represent the entries of white noise as two dimensional independent and identically distributed (i.i.d.) complex Gaussian RVs with zero mean and identical variance per dimension. The complex code matrix $\mathbf{C}_q$ contains the transmitted symbols.

A. ML Decoding Rule

For the proposed system, the ML decoding rule is equivalent to find the minimum of either the norm $\|\mathbf{R} - \mathbf{C}_q\mathbf{H}\|$ or its square. The squared norm criterion can be given as

$\min_{z_u}\left(\|\mathbf{R}-\mathbf{C}_q\mathbf{H}\|^2\right)=\Gamma^{(1)}+\Gamma^{(2)}+\Gamma^{(3)}$, where

$$\Gamma^{(1)}=\mathrm{tr}\big(\mathbf{R}^Q\mathbf{R}\big),\Gamma^{(2)}=-2\Re\Big(tr\big(\mathbf{R}^Q\mathbf{C}_q\mathbf{H}\big)\Big)$$
$$\Gamma^{(3)}=\mathrm{tr}\Big(\big(\mathbf{C}_q\mathbf{H}\big)^Q\mathbf{C}_q\mathbf{H}\Big) \tag{12}$$

where tr(.) denotes the trace operator. It can be seen that $\Gamma^{(2)}$ involves simple matrix multiplication and is a linear combination of transmitted symbols. However, it is the term $\Gamma^{(3)}$ that has been a matter of concern in the previous works of STBCs as it causes coupled decoding for nonorthogonal STBCs. In the following theorem, we show how the term $\Gamma^{(3)}$ can be simplified for the given system model using the proposed QOD construction giving $\mathbf{C}_Q$ or quasi-orthogonal STBC $\mathbf{C}_q$ to yield optimal decoupled decoding. It is natural to see that generally the matrix $\mathbf{P}$ contains both coupled and decoupled terms. We now some properties of the hollow matrices to prove the following theorem.

Theorem 3.1: Let p_{ab} denotes the entries of $\mathbf{P}=\mathbf{C}_q^H\mathbf{C}_q$, then for the channel matrix $\mathbf{H}$ in (11), decoding rule for any STBC $\mathbf{C}_q$ yields $\Gamma^{(3)}=\lambda\mathrm{tr}\big(\mathbf{H}^Q\mathbf{H}\big)$ if and only if $p_{a,b}$ contains coupled terms only when $a+b=2n+1\ \forall\, n\in\mathbb{N}$.

Proof: Since $\mathbf{P}$ is Hermitian, therefore, it can be decomposed into sum of a diagonal matrix and a combination of hollow Hermitian matrices with "k" distinct off-diagonal coupled entries as $\mathbf{P}=\mathbf{C}_q^H\mathbf{C}_q=\lambda\mathbf{I}+\sum_{e=1}^{k}\mathbf{F}^{(e)}$. It is straightforward to check that $\Gamma^{(3)}=\mathrm{tr}\big(\mathbf{H}^Q\mathbf{C}_q^H\mathbf{C}_q\mathbf{H}\big)$. Using the above relation, $\Gamma^{(3)}$ reduces to

$$\Gamma^{(3)}=\lambda\mathrm{tr}\big(\mathbf{H}^Q\mathbf{H}\big)+\sum_{e=0}^{k}\Big(\mathrm{tr}\big(\mathbf{H}^Q\mathbf{F}^{(e)}\mathbf{H}\big)\Big) \tag{13}$$

in which the second term vanishes by virtue of the following result. ■

Corollary 3.1: For the channel matrix $\mathbf{H}$ in (11) and a hollow Hermitian matrix $\mathbf{F}^{(e)}$, the product $\mathbf{H}^Q\mathbf{F}^{(e)}\mathbf{H}$, for each e, is hollow and consequently traceless, provided $f_{a,b}^{(e)}$ comprises of coupled terms only when $a+b=2n+1$.

Proof: Suppose $m=1$, therefore the channel matrix $\mathbf{H}^{(1)}$ is of order 2. The complex code matrix results into a hollow matrix $\mathbf{F}^{(1)}$ with $f_{12}^{(1)}=\zeta$, where the term ζ being coupled can be decomposed as $\mathbf{F}^{(1)}=\left[\begin{smallmatrix}0&\zeta\\ \zeta^*&0\end{smallmatrix}\right]=\Re(\zeta)\left[\begin{smallmatrix}0&1\\ 1&0\end{smallmatrix}\right]+\Im(\zeta)\left[\begin{smallmatrix}0&i\\ -i&0\end{smallmatrix}\right]=\Re(\zeta)\boldsymbol{\Upsilon}_1+\Im(\zeta)\boldsymbol{\Upsilon}_2$. It is easy to verify that both $\mathbf{H}^{(1)Q}\boldsymbol{\Upsilon}_1\mathbf{H}^{(1)}$ and $\mathbf{H}^{(1)Q}\boldsymbol{\Upsilon}_2\mathbf{H}^{(1)}$ are hollow matrices, hence, tracefree. It is pointed out that $\mathbf{H}^{(1)Q}\boldsymbol{\Upsilon}_2\mathbf{H}^{(1)}$, fails to be hollow for complex channel matrices which is why in this construction, the decoupled decoding becomes a characteristic due to quaternionic channel (11). We now proceed further. For higher number of antennas, the number of rows of the channel matrix $\mathbf{H}$ increases with the pattern that second row of each channel component $\mathbf{H}^{(m)}$ contains quaternion coupling. On the other hand, the matrix $\boldsymbol{\Upsilon}_1$ now becomes a hollow square symmetric matrix of higher orders whose main role in the multiplication $\boldsymbol{\Upsilon}_1\mathbf{H}$ is to swap two rows of $\mathbf{H}$, corresponding to two non-zero entries of $\boldsymbol{\Upsilon}_1$, while rest of the rows of $\boldsymbol{\Upsilon}_1\mathbf{H}$ contain only zeros. It is now necessary to swap complex rows with quaternion rows which is not possible if ath row is replaced with $a+2$, $a+4$, etc., rows. Consequently, $\mathbf{H}^Q\boldsymbol{\Upsilon}_1\mathbf{H}$ is a multiplication of those two rows with their conjugate elements in $\mathbf{H}^Q$, resulting in a 2×2 hollow matrix provided $a+b=2n+1$. Similarly, $\mathbf{H}^Q\boldsymbol{\Upsilon}_2\mathbf{H}$ is a hollow Hermitian 2×2 matrix. This complete proof of the Corollary 3.1. ■

Proof: Since, the above result holds true for each e, therefore, second term vanishes in (13), provided $a+b=2n+1$. ■

The above theorem can be applied immediately to those quasi STBCs $\mathbf{C}_Q$ which result from QODs to unveil decoupled decoding in such codes.

Lemma 3.1: For the channel matrix $\mathbf{H}$ in (11) and $\mathbf{C}_Q$ of QOD construction $\mathbf{Q}$ in (8), $\Gamma^{(3)}$ term of ML decoding rule simplifies to $\Gamma^{(3)}=\lambda tr\big(\mathbf{H}^Q\mathbf{H}\big)$.

Proof: The term $\Gamma^{(3)}$, is given by

$$\Gamma^{(3)}=\mathrm{tr}\left(\mathbf{H}^Q(\lambda_1\mathbf{I}+\gamma\mathbf{E})\mathbf{H}\right)=\mathrm{tr}\left(\lambda_1\mathbf{H}^Q\mathbf{H}+\gamma\mathbf{H}^Q\mathbf{E}\mathbf{H}\right)$$
$$\overset{(a)}{=}\lambda_1\mathrm{tr}\big(\mathbf{H}^Q\mathbf{H}\big)+\gamma\mathrm{tr}\left(\mathbf{H}^Q\mathbf{E}\mathbf{H}\right) \tag{14}$$

where (a) follows from the fact that $\gamma=\gamma^*$. In $\mathbf{C}_Q^H\mathbf{C}_Q$, coupled terms will exist adjacent to diagonal terms only and diagonal being even sum of a and b would contain odd indices adjacent to it. Therefore, (14) will reduce to $\Gamma^{(3)}=\lambda\mathrm{tr}\big(\mathbf{H}^Q\mathbf{H}\big)$. ■

We use quadrature phase shift keying (QPSK) modulation for data transmission and due to its equal symbol energy characteristics, decoding rule (12) further simplifies to

$$\min_{z_u}\left(\|\mathbf{R}-\mathcal{C}_q\mathbf{H}\|^2\right)=\min\left(\Gamma^{(2)}\right). \tag{15}$$

We will be using this expression in the following corollary to decode the transmitted data symbols of the rate one QOD (7).

Corollary 3.3: The decoupled decoding rule for the QOD presented in (7) is given by

$$\min_{z_1}-2\sum_{k=1}^{2}\Re\left\{z_1\left(r_{1k}^Qh_{11}^{(1,k)}+r_{3k}^Qh_{22}^{(1,k)}j\right)-z_1^*\left(r_{2k}^Qh_{11}^{(2,k)}+r_{4k}^Qh_{22}^{(2,k)}j\right)\right\}$$
$$\min_{z_2}-2\sum_{k=1}^{2}\Re\left\{z_2\left(r_{1k}^Qh_{11}^{(2,k)}+r_{3k}^Qh_{22}^{(2,k)}j\right)+z_2^*\left(r_{2k}^Qh_{11}^{(1,k)}+r_{4k}^Qh_{22}^{(1,k)}j\right)\right\}$$
$$\min_{z_3}-2\sum_{k=1}^{2}\Re\left\{z_3\left(r_{1k}^Qh_{22}^{(1,k)}j+r_{3k}^Qh_{11}^{(1,k)}\right)-z_3^*\left(r_{2k}^Qh_{22}^{(2,k)}j+r_{4k}^Qh_{11}^{(2,k)}\right)\right\}$$
$$\min_{z_4}-2\sum_{k=1}^{2}\Re\left\{z_4\left(r_{1k}^Qh_{22}^{(2,k)}j+r_{3k}^Qh_{11}^{(2,k)}\right)+z_4^*\left(r_{2k}^Qh_{22}^{(1,k)}j+r_{4k}^Qh_{11}^{(1,k)}\right)\right\}.$$

The following QOD of rate 5/4 has been presented in [6] to highlight the issue of decoupled decoding in QODs, i.e.,

$$\mathbf{Q}_2 = \begin{bmatrix} x_3 + x_1 i & x_0 + x_2 i & x_3 + x_4 i & x_1 + x_2 i \\ x_3 + x_4 i & -x_1 + x_2 i & -x_3 - x_4 i & x_0 - x_2 i \end{bmatrix}. \quad (16)$$

To simplify $\Gamma^{(3)}$ for (16), we observe that

$$\mathbf{C}_{q_2}^H \mathbf{C}_{q_2} = \begin{bmatrix} \lambda & \gamma_1 & 0 & \gamma_2 \\ \gamma_1^* & \lambda & \gamma_2^* & 0 \\ 0 & \gamma_2 & \lambda & \gamma_3 \\ \gamma_2^* & 0 & \gamma_3^* & \lambda \end{bmatrix} \quad (17)$$

where $\gamma_1 = x_3x_0 + x_3x_2 i - x_4x_0 i + x_2x_4 - x_1x_3 + x_1x_4 i + x_2x_3 i + x_2x_4$, $\gamma_2 = x_1x_3 - x_1x_4 i + x_3x_0 - x_0x_4 i$, $\gamma_3 = x_1x_3 + x_2x_3 i - x_1x_4 i - x_3x_0 + x_2x_3 i + x_0x_4 i$. It is important to note that all five real symbols x_0, x_1, x_2, x_3, and x_4 are coupled and therefore, would involve 2^5 iterations to decode each block at the receiver end. However, it is straightforward to see that (17) satisfies the constraints given in the Theorem 3.1, therefore, we obtain the following linear decoding rule for this high rate QOD or quasi-orthogonal STBC.

Corollary 3.4: The decoupled decoding rule for the QOD presented in (16) is given by

$$\min_{x_0} -2\sum_{k=1}^{2} \Re\left\{x_0 \left(r_{1k}^Q h_{22}^{(1,k)} j + r_{2k}^Q h_{22}^{(2,k)} j\right)\right\}$$

$$\min_{x_1} -2\sum_{k=1}^{2} \Re\left\{x_1 \left(r_{1k}^Q h_{22}^{(2,k)} j - r_{2k}^Q h_{22}^{(1,k)} j\right)\right\}$$

$$\min_{x_2} -2\sum_{k=1}^{2} \Re\left\{x_2 i \left(\left(r_{1k}^Q \left(h_{22}^{(1,k)} j + h_{22}^{(2,k)} j\right) + r_{2k}^Q \left(h_{22}^{(1,k)} j - h_{22}^{(2,k)}\right)\right)\right\}$$

$$\min_{x_3} -2\sum_{k=1}^{2} \Re\left\{x_3 \left(\left(r_{1k}^Q \left(h_{11}^{(1,k)} + h_{11}^{(2,k)}\right) + r_{2k}^Q \left(h_{11}^{(1,k)} - h_{11}^{(2,k)}\right)\right)\right\}$$

$$\min_{x_4} -2\sum_{k=1}^{2} \Re\left\{x_4 i \left(r_{1k}^Q (h_{11}^{(1,k)} + h_{11}^{(2,k)}) + r_{2k}^Q \left(h_{11}^{(1,k)} - h_{11}^{(2,k)}\right)\right)\right\}. \quad (18)$$

To further elaborate the significance of Theorem 3.1, we present the following QOD of rate 2 as a counter example which fails to generate quasi-code that satisfies the theorem. It indicates how Theorem 3.1, can be applied successfully by permuting few columns. The QOD of $\mathbf{C}_Q$, given in [12], which is

$$\mathbf{Q}_3 = \begin{bmatrix} z_1 + z_2 j & z_4 + z_3 j \\ z_2^* - z_1^* j & -z_3^* + z_4^* \end{bmatrix} \quad (19)$$

fails to provide decoupled decoding using decoding rule (12). That is because its resultant $\mathbf{C}_{Q_3}^H \mathbf{C}_{Q_3}$, contains nonzero entries $\gamma_1 = z_1^* z_4 - z_2 z_3^*$, $\gamma_4 = z_2^* z_3 - z_1 z_4^*$ at (1, 3) and (2, 4) positions, respectively. Therefore, it does not satisfy Theorem 3.1 even when $\mathbf{Q}_3$ is a valid QOD. Remarkably, we can attain decoupled decoding by swapping some column to get

$$\mathbf{C}_{q_3} = \begin{bmatrix} z_1 & z_3 & z_2 & z_4 \\ z_2^* & -z_4^* & -z_1^* & z_3^* \end{bmatrix}. \quad (20)$$

Note that STBC $\mathbf{C}_{q_3}$ fails to generate a viable QOD. Surprisingly, it obeys Theorem 3.1 and, therefore, obtains decoupled decoding.

Corollary 3.5: The decoupled decoding rule for the QOD presented in (20) is given by

$$\min_{z_1} -2\sum_{k=1}^{2} \Re\left\{z_1 \left(r_{1k}^Q h_{11}^{(1,k)}\right) - z_1^* \left(r_{2k}^Q h_{11}^{(2,k)}\right)\right\}$$

$$\min_{z_2} -2\sum_{k=1}^{2} \Re\left\{z_2 \left(r_{1k}^Q h_{11}^{(2,k)}\right) + z_2^* \left(r_{2k}^Q h_{11}^{(1,k)}\right)\right\}$$

$$\min_{z_3} -2\sum_{k=1}^{2} \Re\left\{z_3 \left(r_{1k}^Q h_{22}^{(1,k)} j\right) + z_3^* \left(r_{2k}^Q h_{22}^{(2,k)} j\right)\right\}$$

$$\min_{z_4} -2\sum_{k=1}^{2} \Re\left\{z_4 \left(r_{1k}^Q h_{22}^{(2,k)} j\right) - z_4^* \left(r_{2k}^Q h_{22}^{(1,k)} j\right)\right\}.$$

An important application of the proposed method is possible in the next generation communication networks, LTE-A. As mentioned in [13] and [14], the codes which are used in LTE-A communication network requires odd time slots for signal transmission due to OFDM frame. Keeping this in view, they have proposed three timeslots based-codes for which linear decoder has been designed. To extend the applications of our proposed system, we applied our decoder on the codes given in [13]. For successful linear decoding of a code with our proposed system model, it must satisfy Theorem 3.1. It is found that this is indeed the case as

$$\mathbf{P} = \begin{bmatrix} x_1x_1^* + x_2x_2^* + x_3x_3^* & x_3x_3^* \\ x_3x_3^* & x_1x_1^* + x_2x_2^* + x_3x_3^* \end{bmatrix} \quad (21)$$

for the hybrid code given for two single-polarized transmit antennas given in Fig. 3 of [13]. As there are no coupled terms in $\mathbf{P}$, therefore, it would result in complete decoupled decoding. Furthermore, if we check the proposed code of [13] for Theorem 3.1, we observe that $\mathbf{P}$ would have coupled terms only at the index of $(1, 2)$ and $(2, 1)$, where $1 + 2 = 2 + 1 = 2n + 1$ for $n = 1$, hence, it satisfies constraint of Theorem 3.1 and results in decoupled decoding. For a more brief exposition of our argument, we consider hybrid code construction and generate a hybrid QOD code using equation (20) of [13] as follows:

$$\mathbf{C}_{q_4} = \begin{bmatrix} z_1 & z_3 & z_2 & z_4 \\ z_2^* & -z_4^* & -z_1^* & z_3^* \\ z_5 & 0 & z_5 & 0 \end{bmatrix}. \quad (22)$$

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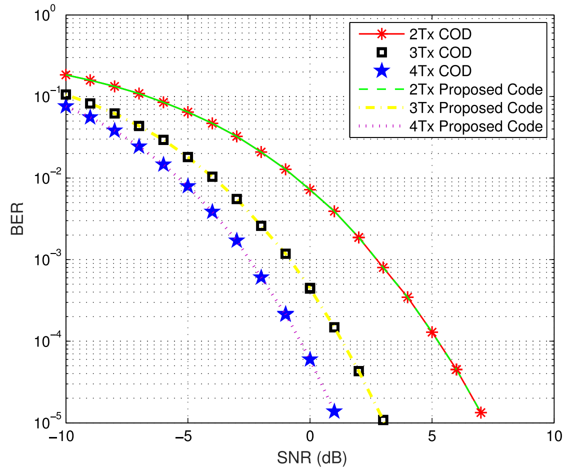

Fig. 1. BER versus SNR comparison of the ML coupled and decoupled decoding for 2, 3, and 4 dual-polarized transmit antennas.

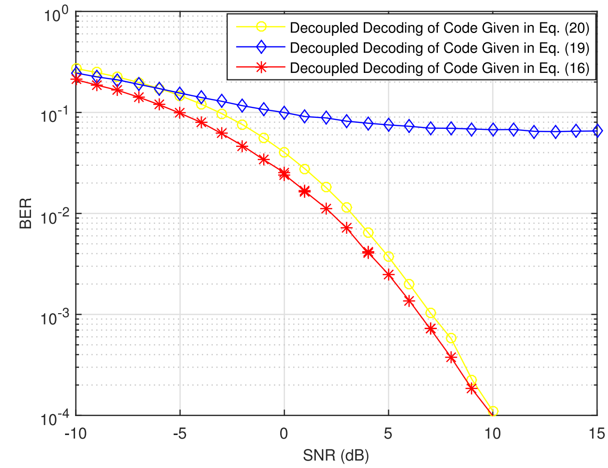

Fig. 2. BER versus SNR comparison of codes given in (20), (19), and (16) with the decoding rule [6].

The proposed codes in [13] have code rate 1 and are designed in the configuration of two transmit antennas. However, the above code based on quaternions has code rate $5/3$, with four transmit antennas and provides decoupled decoding as given in the following corollary.

Corollary 3.6: The decoupled decoding rule for the QOD presented in (22) is given by

$$\min_{z_1} -2\sum_{k=1}^{2}\Re\left\{z_1\left(r_{1k}^{Q}h_{11}^{(1,k)}\right)-z_1^*\left(r_{2k}^{Q}h_{11}^{(2,k)}\right)\right\}$$

$$\min_{z_2} -2\sum_{k=1}^{2}\Re\left\{z_2\left(r_{1k}^{Q}h_{11}^{(2,k)}\right)+z_2^*\left(r_{2k}^{Q}h_{11}^{(1,k)}\right)\right\}$$

$$\min_{z_3} -2\sum_{k=1}^{2}\Re\left\{z_3\left(r_{1k}^{Q}h_{22}^{(1,k)}j\right)+z_3^*\left(r_{2k}^{Q}h_{22}^{(2,k)}j\right)\right\}$$

$$\min_{z_4} -2\sum_{k=1}^{2}\Re\left\{z_4\left(r_{1k}^{Q}h_{22}^{(2,k)}j\right)-z_4^*\left(r_{2k}^{Q}h_{22}^{(1,k)}j\right)\right\}$$

$$\min_{z_5} -2\Re\left\{z_5\left(r_{31}^{Q}h_{11}^{(1,1)}j\right)+\left(r_{31}^{Q}h_{11}^{(2,1)}\right)\right\}.$$

IV. Simulation Results

For simulations, we develop QODs for 2, 3, and 4 dual-polarized transmit antennas with code rates 1, 3/4, and 3/4 while their complex counterparts have rates $3/4$, $1/2$, and $1/2$, respectively. The first rate 1 QOD for 2 dual-polarized transmit antennas may be compared with the state-of-art low-dispersion STBC of rate 1 given in (66) and (67) in [14] for four single-polarized transmit antennas. Comparison of the bit error rate (BER) curves of their codes given in Figs. 7 and 8 in [14] with BER curve of our QOD in Fig. 1 shows that our proposed QOD performs better. It is pertinent to mention that their BER curves for both codes also rely on maintaining a distance between transmit antennas $d_t = \lambda_c$ and $d_t = \lambda_c/4$, while our QOD is designed for two colocated antennas. We now use the decoupled decoding rule (15) to decode the transmitted data and the channel coefficients are already known at the receiver, whereas white noise is added in each polarization independently.

Fig. 1 demonstrates the optimal performance of the proposed low-complexity ML decoder. All the BER curves are derived where it can be seen that performance of the proposed decoder is optimal for smaller as well as higher order QODs. In addition to this, the BER curves also exhibit the phenomenon of diminishing returns with the increase in the number of transmit antennas. Both the codes, i.e., the proposed design and the CODs, for 2Tx/2Rx, 3Tx/2Rx, and 4Tx/2Rx antenna arrangements yield the value of 7, 3, and 1 dB signal-to-noise ratio (SNR) at a target BER of 10^{-5}, respectively. Hence, for zero-cross polarization environment, the proposed construction outperforms CODs in terms of code rates. Fig. 2 represents the BER curves derived for the QOD of rate 5/4 and 2 proposed in [6] and [12], respectively, where the aim is to highlight the importance of the proposed Theorem 3.1. In the zero-cross polar scattering environment, not every code provides decoupled decoding as it can be observed from the code $\mathbf{Q}_3$, for which the proposed decoder fails to yield decoupled decoding. However, for other codes, i.e., [6] and [12], which satisfy Theorem 3.1, we achieve optimal performance with least decoder complexity. Moreover, we have achieved optimal low-complexity solution for the code (16) which is given in [6] as a problem statement.

The decoupled decoding makes decoder independent of the number of unique transmitted symbols, denoted as ζ, in one QOD. On the other hand, complexity of coupled decoding increases exponentially with the increase in the number of unique symbols, as it has to perform computations for all combinations of complex symbols present in the constellation of modulation, for example, for QPSK modulation, it would be 4^ζ. Therefore, the complexity of the proposed decoder, calculated in terms of floating-point operations (FLOPs), comes out to be O(4(2N)(t)(2)), whereas, coupled ML decoder complexity remains O(4^ζ(2N)(t)(2)), where N represents the single-polarized transmitting antennas, t signifies the timeslots required to transmit a code block and $2N(t)(2)$ denotes the number of computations required to calculate the term $\mathcal{C}(Q)H$ of the proposed complexity ML decoder. For example, for the QOD presented in (7) of the manuscript, $\|\mathbf{R} - \mathbf{C}_q\mathbf{H}\|^2$ would require 16 384 number of operations, whereas linear equation based solution provided in Corollary 3.3. would decode the transmitted code in 256 number of FLOPs, which reduces systems complexity by 98.4%. Therefore, the benchmark result of our work has been the provision of decoupled decoding for the proposed codes for which the last decoder would had failed.

V. Concluding Remarks

In this paper, we have proposed a new construction of QODs that can generate codes for any number of transmit antennas using quaternion algebra, which provides decoupled decoding for quasi-orthogonal codes under the assumption of zero-cross polarization. Simulation results verified the optimal performance of the ML decoder for the proposed codes and also indicated that the proposed quaternion-based codes and CODs provide same diversity for the given system model; however, the proposed codes provided better code rates. Brief comparative analysis of code rates, decoding mechanisms, and relative BERs for a wide variety of quasi-orthogonal codes deems necessary and is left as a future work. Moreover, we intend to explore the behavior of QODs in the presence of channel correlation in future besides asynchronous transmissions under frequency selective fading channels as proposed in [15] and [16]. A new model based on channel correlations in terms of quaternions is recently proposed in [17], which we intend to use in future.

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