Performance Analysis of LDPC-based Rate Adaptive Relays over Nakagami Channels

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***Abstract*—In this paper, link adaptation with decoupled code rates has been discussed in context of two-hop decode and forward (DAF) relay network. The source and the relay use variable rate low density parity check (LDPC) codes to encode the data, whereas the relay (in receiving mode) and the destination use the log domain iterative decoding algorithm to decode the data. Nakagami channels of different fading depths have been used. An algorithm has been designed to achieve the optimal performance by ensuring a target bit error rate (BER). Extensive Monte Carlo simulation results show that if decoupled code rates are used on both hops then a trade-off has to be maintained between system complexity, transmission delay and BER. More specifically, the system performs well in terms of lower BER, however, the complexity of the system also increases.**

***Index Terms*—Regenerative relaying, decode and forward relays, iterative decoding, Nakagami-m fading, code rates**

I. Introduction

Cooperative transmission (CT), where relay forwards the data of the source, has been proven to bring beneficial impacts on the performance of wireless systems [1]. A major benefit is the provision of diversity gain, which is required for performance enhancement in fading environments. Two hop networks with single relay between source and destination and multi-hop networks with multiple relays between source and destination have been studied extensively in literature [1]–[3], [9]–[14]. Multi-hop wireless systems have the ability to combat detrimental effects of wireless channel by dividing the transmission path between the source and the destination into multiple shorter paths that increase the average signal-to-noise ratio (SNR) on each hop and hence elevate the network coverage.

Despite all the advantages of multihop networks, the end-to-end success probability is the product of all the success probabilities of the intermediate hops which lowers the performance. Therefore in addition to performance improvement using the SNR advantage of CT, multiple hops employing the forward error correction (FEC) codes help to achieve a BER performance by exploiting the inherent properties of FEC. This can be done, e.g., by changing the rate of data transmission on different hops according to the quickly changing conditions of the wireless channel, which otherwise would be hard to maintain while having fixed data rates [5]. Low density parity check (LDPC) codes are considered best amongst all the available FEC codes because of their capacity approaching performance [8], [9] and iterative decoding mechanism. In the past decade, there has been lots of research carried out on the design of LDPC codes for the relay-based communications e.g., [2], [9], [18].

In many previous works, researchers have been using link adaptation techniques, or more specifically, adaptive coding and modulation (ACM) in cooperative networks. Andreas Müller et. al in [3] and [10], discuss the decode and forward (DAF) multi-hop systems without considering any particular encoding scheme on source and/or relay. The authors perform the hard decision decoding on both relay and destination nodes and adapt to different modulation schemes according to the changing channel conditions. The authors in [11] have adapted to different kinds of data rates and different modulation schemes in a non-FEC manner. Nevertheless, some people have carried out research by using different types of codes, rather than changing the code rates of the same type of codes. For instance, in [13], researchers use two different codes, namely, LDPC and Reed Solomon (RS), with same code rate. In [15], LDPC codes have been proposed with different puncturing and extension techniques, but these codes were used for single hop systems. Moreover these schemes come with a price of calculation of bits positions for punctured bits and sending this information to the receivers for successful decoding.

This paper focuses on link adaptation strategy of compatible code rates, where a single relay helps the source to deliver its information to the destination. Specifically, if the channel conditions on a link are better, higher code rates can be used, with poor channel conditions lower code rates shall be used.

The results are collected by using extensive Monte Carlo simulations and the results verify that an adaptive code rate system is more efficient than a fixed-rate one. Some of the results are the manifestation of the fact that performance of the relaying system gets better with better channel conditions i.e., an increased value of m. But this effect becomes invisible as the m gets higher and higher and after a certain value, the performance increase is almost negligible.

The remainder of the paper is organized as follows. In

978-1-5386-6358-5/18/$31.00 ©2018 IEEE

section II we discuss the system model with some details on LDPC encoding and decoding process. Perofmance analysis based on designed algorithm and simulation results have been presented in section III. We conclude the paper in section IV.

II. System Model

Consider a two hop wireless system, which includes three nodes: a source node S, a relay node R and a destination node D. There is no direct link between S and D and the source node communicates with the destination only via a relay, where the relay is assumed to operate in a regenerative decode-and-forward mode. We suppose that the relay not only decodes the source information but also regenerates the encoded sequence to send it further, hence, the name *"regenerative relaying"*.

In this paper, time-division based half-duplex relaying has been applied over Nakagami-m channels, where m determines the fading severity and may or may not be identical on both hops. . Each transmission is assigned a time slot, (TS). Ideally if S and R transmit with the same code rate, then in the first time slot, i.e., TS-1, the source sends the data to the relay. In TS-2, S remains silent and R transmits the decoded and re-encoded sequence to the destination D. We assume that the source and the relay transmit with the same transmit power, P, such that the received signals at the relay and the destination are given as

$$y_r = \sqrt{P} x_s h_{sr} + n_r, \tag{1}$$

$$y_d = \sqrt{P} x_r h_{rd} + n_d, \tag{2}$$

In the above equations, x_s and x_r are transmitted symbols from source and relay, whereas the channel coefficients between source to relay ($S-R$) and relay to destination ($R-D$) links are given as h_{sr} and h_{rd}, respectively. Nakagami-m fading is considered on both hops, with the fading parameter m either same or different on both links. In (1) and (2), n_r and n_d are the additive white Gaussian noise (AWGN) symbols, added on $S-R$ and $R-D$ links, respectively. Moreover $n_i \sim CN(0, {\sigma_i}^2)$, and $i \in \{r, d\}$.

Further, it has been assumed that all the nodes are equipped with single antennas and the wireless channel on both links undergoes a flat-fading. The channel coefficients h_{sr} and h_{rd} are independent and identically distributed (i.i.d) and follow the Nakagami fading, each with a probability density function (PDF) described as

$$f(x) = \frac{2}{\Gamma(m)} \left(\frac{m}{\Omega}\right)^m x^{2m-1} exp\left(-\frac{m}{\Omega}x^2\right), m \geq 0.5 \tag{3}$$

In (3), Ω is the instantaneous power of the random variable, i.e., channel here. Moreover, we assume that all the nodes have perfect channel state information (CSI) of links on which they receive data, thus facilitating the perfect coherent detection. LDPC codes are used at the source to encode the information sequence and the log-domain sum product algorithm (SPA) is used to decode the LDPC coded sequence over the intermediate relay node and finally on the destination. Log domain calculations change the multiplications into additions, hence require less storage space and easier implementation.

A. Low Density Parity Check Codes

LDPC codes are a class of linear block codes. Since their inception, lots of research has been done on LDPC codes to find capacity limits, encoder/decoder designs and their implementation for different channel models. They work in close proximity of 0.04 dB of Shanon limit at BER of 10^{-6} for a code length of 10^7 [8].

Encoding LDPC Codes

We generate an (n, k) irregular LDPC code by keeping the code rate (R_c) variable, where code rate can be defined as k/n, i.e, for every k bits of useful information, LDPC codes generate a total of n bits, of which $(n-k)$ are redundant. The codes used in this paper are given in TableI. These redundant bits are also called *check bits* and are used to detect and correct the errors on the receiver node.

At the source with $(n-k)$ rows and k columns, we generate a sparse parity check matrix, $\mathbf{H}$ and a corresponding generator matrix $\mathbf{G}$ is generated to encode the given sequence as $\mathbf{x_s} = \mathbf{u} \times \mathbf{G_1}$, where $\mathbf{u}$ is the input sequence, $\mathbf{x_s}$ is modulated sequence using M-ary quadrature amplitude modulation i.e., M-QAM with $M = \{4, 16, 64, ...\}$ and is transmitted by the source in TS-1. At the relay station, this received sequence is decoded using log domain SPA. The decoded sequence $\mathbf{u'}$ at the relay is encoded again as $\mathbf{x_r} = \mathbf{u'} \times \mathbf{G_2}$, and after M-QAM modulation, it is transmitted by the relay in TS-2. The destination, then decodes the received information using the log-domain SPA again. Please note that $\mathbf{G_1}$ and $\mathbf{G_2}$ may be same or different.

In this paper, variable rate LDPC codes have been considered, i.e., if S sends information at a certain code rate R_c, then R receives the sequence and decodes it using the same parity check matrix, $\mathbf{H}$. Before forwarding the sequence to the destination, we ought to make some assumptions here, which are as follows

i - Ideally CSI is already known to the receiver nodes, i.e., relay and destination. Based on this knowledge of CSI, a *code rate index* is fed back to the transmitting node on the control channels. Eventually, just one bit of information is sent on the feed back channels.

ii- We divide the transmission of data into three different scenarios, as under:

Case1: When channel conditions are same on both hops, same data rate is used. No data will be stored temporally on the intermediate node.

Case2: If the channel conditions are worse on $R-D$ link than $S-R$ link, more redundant bits will be needed to maintain the target BER, δ_0, at the destination, hence, the R_c will be lowered. In this case remaining bits will be stored in the relay buffer and will be sent in the next time slots.

Case3: If the channel conditions are better on $R-D$ link than $S-R$ link, R_c can be increased, i.e., more information can be sent with lesser redundant bits. In this case, bits from previous transmissions, stored in the buffer can be grouped together with bits coming from the S, encoded with a modified parity check matrix. The modified parity check matrix will calculate the information and check bits according to the new code rate. They are sent to the destination. Destination decodes the longer sequence with the modified parity check matrix.

iii - The buffer at the relay is of infinite length, therefore, no overflow occurs ever.

iv - Our system can support 5 data rates initially and adapts to any data rate (Table I), depending on channel conditions.

TABLE I
DIFFERENT DATA RATES R_c FOR LDPC, CODE LENGTH N = 312

Code Rate R_c	Info. Bits k	Check Bits $(n-k)$	CodeRate Index r_{index}
1/4	78	234	0
1/2	156	156	1
3/4	234	78	2
5/6	260	52	3
7/8	273	39	4

Decoding on LDPC Codes

The received vectors $\mathbf{y_r}$ and $\mathbf{y_d}$ in (1) and (2) are valid codewords only if they satisfy the following condition

$$\mathbf{H}\mathbf{y_i}^T = \mathbf{0}, i \in \{r, d\} \tag{4}$$

i.e., $\mathbf{H}\mathbf{y_i}^T$ is a zero vector of dimensions $(N \times 1)$. For the received bits, the input bit probabilities are called the *a priori probabilities*, since they are known before running the LDPC decoder [17]. The decoder is also provided with the $\mathbf{H}$ matrix and for the estimate of codeword bit c_i, the content of every bit (variable nodes (column bits) and check nodes (row bits)) in $\mathbf{H}$ has to be updated in each iteration. Decoder computes apostriori probability (APP), since iterative decoders require APP as metrics rather than direct observations of the channels [16], of each c_i, i.e.

$$P_i = P_r(c_i = 1|y_i), \tag{5}$$

APP is the probability that each codeword bit value is '1', conditional that all the y_i's have been received and that it meets all parity check constraints. In case of sum-product decoding these probabilities can be expressed as log-likelihood ratios [17], given as follows

$$L(c_i) \triangleq \log_e\left(\frac{P_r(c_i = 0|y)}{P_r(c_i = 1|y)}\right) \tag{6}$$

By applying the Baye's theorem on (6) and considering the conditional Gaussian distribution for the received symbols $\mathbf{y_i}$ in AWGN [17]

$$L(c_i) = \underbrace{\frac{2}{\sigma^2} y_i}_{intrinsic} + \underbrace{\log_e\left(\frac{p(c_i = 0)}{p(c_i = 1)}\right)}_{extrinsic} \tag{7}$$

In (7), the first part on the right hand side gives the *apriori* or *intrinsic probability*, which is the original probability of the input bits, independent of the code constraints whereas second part represents the *likelihood* or *extrinsic information*, which is the observation of all the parity checks. The estimated APP L_i of each bit is the sum of intrinsic and extrinsic LLR's, expressed as

$$L_i = \underbrace{R_i}_{intrinsic} + \underbrace{\sum_{j \in A_i} E_{i,j}}_{extrinsic}, \tag{8}$$

Given the knowledge of the channel properties, R_i is the initial message, $L_{i,j}$ sent from variable node i to the check node j, hence R_i is actually the LLR of the received signal y_i for the first iteration only, whereas for the rest of the iterations $E_{i,j}$ are calculated that is the estimated extrinsic LLRs. The extrinsic message from check node j to bit node i is the LLR of the probability that parity-check j is satised if bit i is assumed to be a 1. For each bit, hard decision is made i.e. y_i is 1 if $L_i \leq 0$ and 0 if $L_i > 0$.

Decoding terminates if $\mathbf{y_i}$ is a valid codeword i.e. $\mathbf{H}\mathbf{y_i}^T = \mathbf{0}$, or if the maximum number of iterations has reached.

III. PERFORMANCE ANALYSIS

When same code rate is used on both hops, then theres no need to buffer the extra bits on relays. However the major drawback is that the performance of the system is dominated by the poor hop. If the channel conditions on that certain hop are poor then over all performance will be degraded and spectral efficiency will be lowered, no matter how good are the conditions on the other hop. Such a hop becomes the bottle neck of the system and both the nodes have to transmit with the same rate, i.e, most robust data rate [3].

In our proposed scheme, by keeping the modulation order fixed on both hops, we use decoupled rate adaptation on these hops, which ensures the performance criteria in such a way that the system can maintain an end-to-end target BER, δ_0, along with a desired spectral efficiency. On the other hand, the temporal storage of data in buffer and using more than one time slots for the transmission of data from relay to destination might add further delay in overall time, required for the transmission of data from source to the destination [3].

In Algorithm 1, during the very first transmission, source transmits with the most robust data rate r_0. Here *code rate index* is r_{index}, where $index \in \{0, 1, ..., 5\}$ and r_0 is the lowest index corresponding to $R_c = 1/4$ and r_5 represents the $R_c = 7/8$. Relay receives the data, decodes and forwards again with r_0. On feed back channels, rather than sending the CSI, just one bit, a code rate index, based on knowledge of CSI on receiving nodes is fedback to the relay and source respectively. This transmission takes 2 time slots.

In further transmissions, if instantaneous BER δ_{inst} is greater than δ_0, then *code rate index* is lowered. After receiving the r_{index} from relay node, source node encodes with the corresponding R_c, highest rate to maintain the target BER δ_0. Exactly in the similar manner, when relay receives the *code rate index*, r_{index}, it looks for the corresponding code

Algorithm 1 Code Rate Selection

buf = [] ▷ Initialization
$\delta_0 = 10^{-3}$ ▷ Target BER
if $t = 1$ **then** ▷ First Transmission
  $r_{index} = r_0$
else ▷ 2nd/further Transmissions
  if $\delta_{inst} > \delta_0$ **then**
    if $r_{index} = r_0$ **then**
    $r_{index} = r_{index}$
    else
    $r_{index} = r_{index} - 1$
    end if
    $u'' = []$
    buf= y_i ▷ y_i is received data, stored in buffer
    Split the data into n shorter data streams
    for i = 1:n **do**
      function CODEGENERATOR(M, N, y_i)
        return (u, H)
      end function
      function DECODER(L_i, H) ▷ L_i are log liklihood ratios
        return u'
      end function
    $u'' = [u'', u']$ ▷ Appending the data
    end for
  else
    $r_{index} = r_{index} + 1$
    $buf = buf - bi$ ▷ bi, stored bits in buffer
    function CODEGENERATOR(M, N, y_i)
      return (u, H)
    end function
    $u' = [u, b_i]$ ▷ Appending the data
    function DECODER(L_i, H)
      return u''
    end function
  end if
end if

rate and encodes the message with the new R_c. At the end of the transmission, destination and relay again send the feed backs to their respective source nodes. This sequence keeps repeating until source/relay still have data to transfer. When all the data has been transmitted from source to destination, BER is calculated from end-to-end and checked against overall average BER δ_0.

A. Simulation Results

For Monte Carlo simulations, we have used different code lengths of LDPC codes, but for every length n, 10,000 frames have been delivered to the destination with 20 decoding iterations. Five different code rates have been used with M-QAM, $M \in \{4, 16\}$ and a fading factor $m \in \{1, 3\}$ has been used to collect the results. Figure 1 shows the results with 4-QAM when there is no buffering of data on the relays and same code rates were used on both links. For two different values of m, the BER curves show that the performance degrades as the code rate increases. On the otherhand, it becomes inevitable to have a lower BER for $m = 1$, i.e., Rayleigh fading, when data rates are higher. For code rates of 3/4, 5/6 and 7/8, there is negligible difference in the BER performance for $m = 1$. Better performance is achieved as the m changes from 1 to 3. In Figure 2, the same trends can be observed with 16-QAM. However the BER perfomance deteriorates as a concequence of using higher order modulation.

Figure 3 shows the variation of the effective bits received at the destination in one frame, when a fixed data rate of 3/4 is maintained on the first hop and variable rate is chosen on the second. For instance, for the lowest rate of 1/4 on second hop, we need 4TS, and 3 times more bits have to be transfered on second hop. Remaining bits are stored on relay that not only adds to the latency in transmission but also the complexity of processing. We can see that as the code rate increases, we have to send less redundant bits, resulting in reduced latency, with no buffering of data on the intermediate hop. Therefore, it always will be a trade-off between a highly reliable system with very low BER at a low SNR, where latency is no issue *or* a system with an acceptable BER performance where late response is intolerable.

Contour of the destination BER against SNRs on both hops is shown in Figure 4. It can be observed that if a higher SNR can be achieved on the first hop, then to maintain a certain target BER of δ_0, a lower average SNR on second hop should still work to maintain a consistent performance. This conforms with the idea that by breaking down the transmission path into shorter paths, if we have good SNR on one link, we can still maintain a certain end-to-end BER with worsened conditions on the other hop.

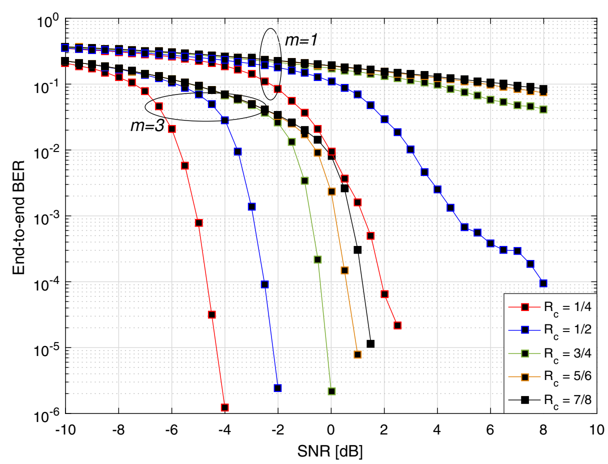

Fig. 1. BER performance at the destination node for LDPC code length N = 312, 4-QAM, m=1, m=3

IV. Conclusion

We have discussed variable rate DAF relays with LDPC codes in this paper. An algorithm which changes the code rate on both hops according to the channel conditions has

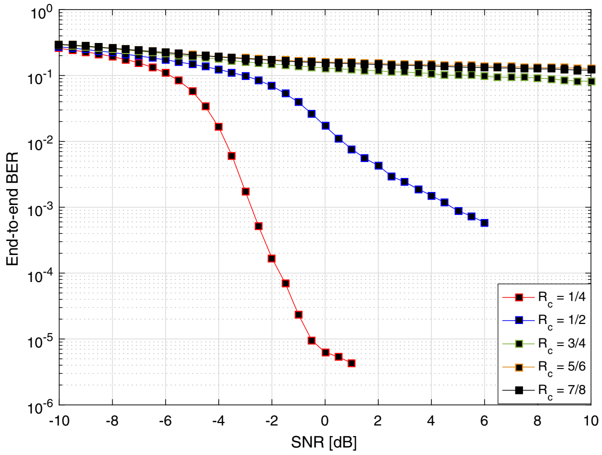

Fig. 2. BER performance at the destination node for LDPC code length N = 312, 16-QAM, m=3

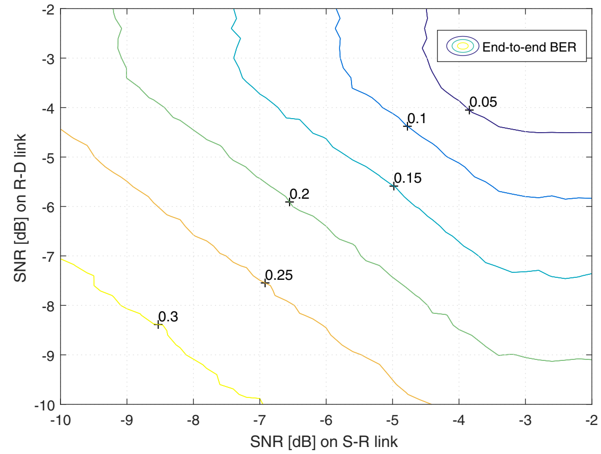

Fig. 4. A contour of SNR on two-hop relaying system

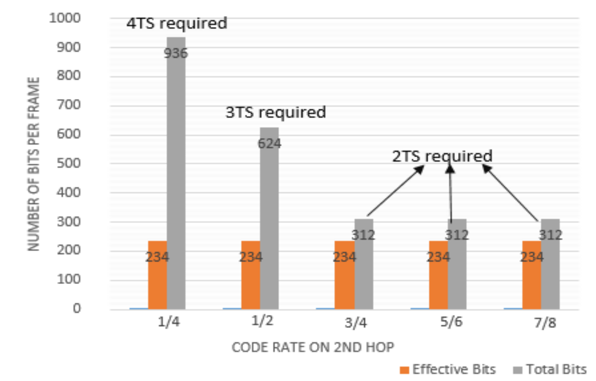

Fig. 3. Two-hop system with an LDPC code, fixed code rate on first hop = 3/4 and a variable rate on second hop

been designed. Initially our system supports five different code rates and 4-QAM and 16-QAM modulations. Based on simulated results we can see that the fading effects can be mitigated not only by the use of regenerative relays but also by link adaptive techniques of variable rate on two hop wireless networks. We conclude that by applying link adaptation we can attain better performance, with an increased complexity and transmission delay for lower rate, whereas for higher rates complexity of processing at relay and overall delay reduces, hence this scenario needs a trade-off between complexity and BER performance.

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